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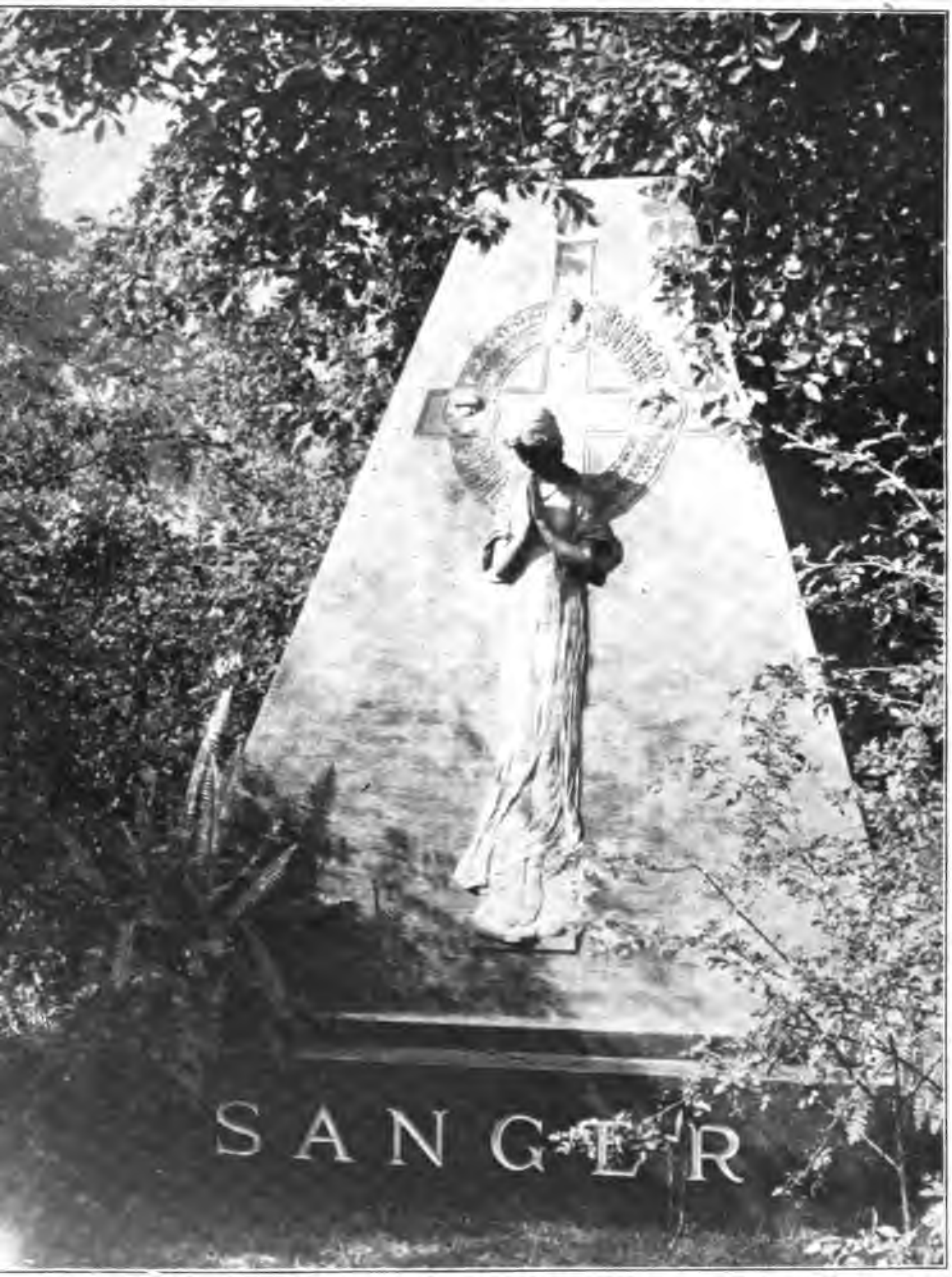

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THE editors have freely consulted the standard technical literature of America and Europe in the preparation of these volumes. They desire to express their indebtedness particularly to the following eminent authorities whose well-known works should be in the library of every one connected with building.

Grateful acknowledgment is here made also for the invaluable co-operation of the foremost architects, engineers, and builders in making these volumes thoroughly representative of the very best and latest practice in the design and construction of buildings; also for the valuable drawings and data, suggestions, criticisms, and other courtesies.

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Foreword

THE rapid evolution of constructive methods in recent years, as illustrated in the use of steel and concrete, and the increased size and complexity of buildings, has created the necessity for an authority which shall embody accumulated experience and approved practice along a variety of correlated lines. The Cyclopedia of Architecture, Carpentry, and Building is designed to fill this acknowledged need.

¶ There is no industry that compares with Building in the close interdependence of its subsidiary trades. The Architect, for example, who knows nothing of Steel or Concrete construction is to-day as much out of place on important work as the Contractor who cannot make intelligent estimates, or who understands nothing of his legal rights and responsibilities. A carpenter must now know something of Masonry, Electric Wiring, and, in fact, all other trades employed in the erection of a building; and the same is true of all the craftsmen whose handiwork will enter into the completed structure.

¶ Neither pains nor expense have been spared to make the present work the most comprehensive and authoritative on the subject of Building and its allied industries. The aim has been, not merely to create a work which will appeal to the trained expert, but one that will commend itself also to the beginner and the self-taught, practical man by giving him a working knowledge of the principles and methods, not only of his own particular trade, but of all other branches of the Building Industry as well. The various sections have been prepared especially for home study, each written by an acknowledged authority on the subject. The arrangement of matter is such as to carry the student forward by easy stages. Series of review questions are inserted in each volume, enabling the reader to test his knowledge and make it a permanent possession. The illustrations have been selected with unusual care to elucidate the text.

¶ The work will be found to cover many important topics on which little information has heretofore been available. This is especially apparent in such sections as those on Steel, Concrete, and Reinforced Concrete Construction; Building Superintendence; Estimating; Contracts and Specifications, including the principles and methods of awarding and executing Government contracts; and Building Law.

¶ The method adopted in the preparation of the work is that which the American School of Correspondence has developed and employed so successfully for many years. It is not an experiment, but has stood the severest of all tests—that of practical use—which has demonstrated it to be the best method yet devised for the education of the busy working man.

¶ In conclusion, grateful acknowledgment is due the staff of authors and collaborators, without whose hearty co-operation this work would have been impossible.

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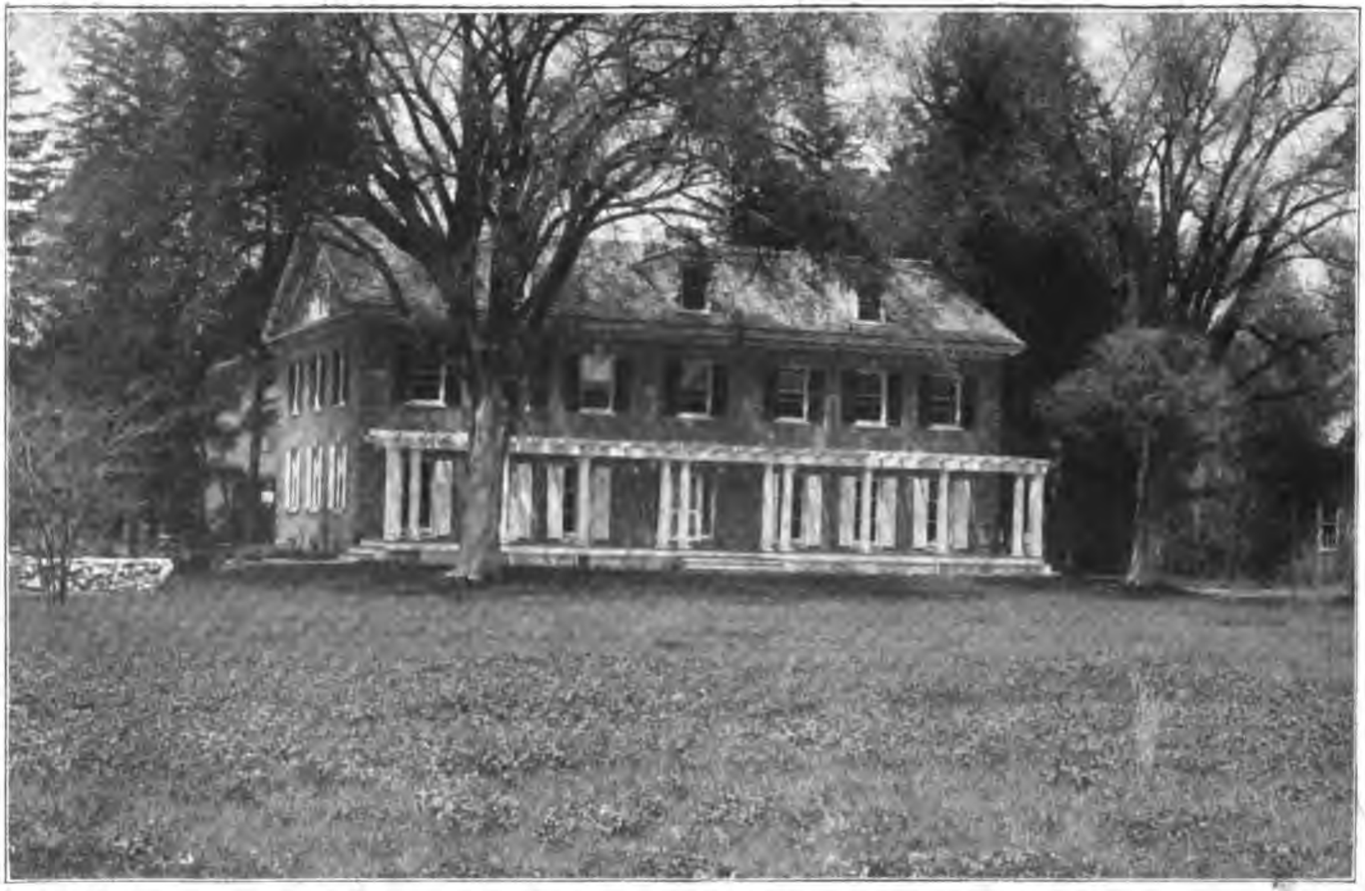

GARDEN FRONT OF RESIDENCE AT DEDHAM, MASS.
Frank Chouteau Brown, Architect, Boston, Mass.
Street Front and Plans are also shown in this Volume.

STRENGTH OF MATERIALS.

PART I.

SIMPLE STRESS.

1. Stress. When forces are applied to a body they tend in a greater or less degree to break it. Preventing or tending to prevent the rupture, there arise, generally, forces between every two adjacent parts of the body. Thus, when a load is suspended by means of an iron rod, the rod is subjected to a downward pull at its lower end and to an upward pull at its upper end, and these two forces tend to pull it apart. At any cross-section of the rod the iron on either side "holds fast" to that on the other, and these forces which the parts of the rod exert upon each other prevent the tearing of the rod. For example, in Fig. 1, let *a* represent the rod and its suspended load, 1,000 pounds; then the pull on the lower end equals 1,000 pounds. If we neglect the weight of the rod, the pull on the upper end is also 1,000 pounds, as shown in Fig. 1 (*b*); and the upper part A exerts on the lower part B an upward pull Q equal to 1,000 pounds, while the lower part exerts on the upper a force P also equal to 1,000 pounds. These two forces, P and Q, prevent rupture of the rod at the "section" C; at any other section there are two forces like P and Q preventing rupture at that section.

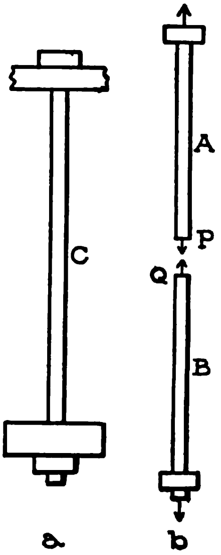

Fig. 1.

By *stress at a section* of a body is meant the force which the part of the body on either side of the section exerts on the other. Thus, the stress at the section C (Fig. 1) is P (or Q), and it equals 1,000 pounds.

2. Stresses are usually expressed (in America) in pounds, sometimes in tons. Thus the stress P in the preceding article is

1,000 pounds, or ½ ton. Notice that this value has nothing to do with the size of the cross-section on which the stress acts.

3. Kinds of Stress. (a) When the forces acting on a body (as a rope or rod) are such that they tend to tear it, the stress at any cross-section is called a *tension* or a *tensile stress*. The stresses P and Q, of Fig. 1, are tensile stresses. Stretched ropes, loaded "tie rods" of roofs and bridges, etc., are under tensile stress.

(b.) When the forces acting on a body (as a short post, brick, etc.) are such that they tend to crush it, the stress at any section at right angles to the direction of the crushing forces is called a *pressure* or a *compressive stress*. Fig. 2 (*a*) represents a loaded post, and Fig. 2 (*b*) the upper and lower parts. The upper part presses down on B, and the lower part presses up on A, as shown. P or Q is the compressive stress in the post at section C. Loaded posts, or struts, piers, etc., are under compressive stress.

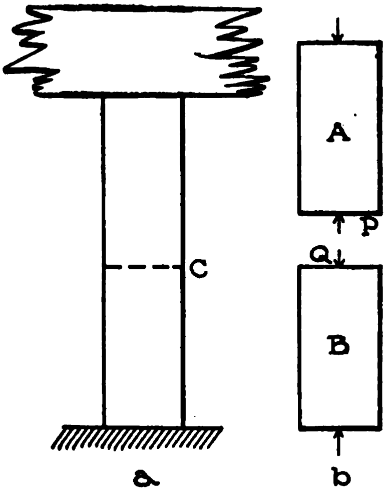

Fig. 2.

(c.) When the forces acting on a body (as a rivet in a bridge joint) are such that they tend to cut or " shear " it across, the stress at a section along which there is a tendency to cut is called a *shear* or a *shearing stress*. This kind of stress takes its name from the act of cutting with a pair of shears. In a material which is being cut in this way, the stresses that are being "overcome" are shearing stresses. Fig. 3 (*a*) represents a riveted joint, and Fig. 3 (*b*) two parts of the rivet. The forces applied to the joint are such that A tends to slide to the left, and B to the right; then B exerts on A a force P toward the right, and A on B a force Q toward the left as shown. P or Q is the shearing stress in the rivet.

Tensions, Compressions and Shears are called *simple stresses*. Forces may act upon a body so as to produce a combination of simple stresses on some section; such a combination is called a *complex* *stress.* The stresses in beams are usually complex. There are other terms used to describe stress; they will be defined farther on.

4. **Unit-Stress.** It is often necessary to specify not merely the amount of the entire stress which acts on an area, but also the amount which acts on each unit of area (square inch for example). By unit-stress is meant stress per unit area.

To find the value of a unit-stress: *Divide the whole stress by the whole area of the section on which it acts, or over which it is distributed.* Thus, let

P denote the value of the whole stress,
A the area on which it acts, and
S the value of the unit-stress; then

$$S = \frac{P}{A}, \text{ also } P = AS. \qquad (1)$$

Strictly these formulas apply only when the stress P is uniform, that is, when it is uniformly distributed over the area, each square inch for example sustaining the same amount of stress. When the stress is not uniform, that is, when the stresses on different square inches are not equal, then P÷A equals the *average value* of the unit-stress.

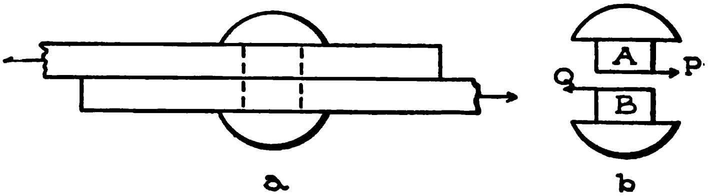

Fig. 3.

5. Unit-stresses are usually expressed (in America) in pounds per square inch, sometimes in tons per square inch. If P and A in equation 1 are expressed in pounds and square inches respectively, then S will be in pounds per square inch; and if P and A are expressed in tons and square inches, S will be in tons per square inch.

Examples. 1. Suppose that the rod sustaining the load in Fig. 1 is 2 square inches in cross-section, and that the load weighs 1,000 pounds. What is the value of the unit-stress?

Here $P = 1,000$ pounds, $A = 2$ square inches; hence.

$$S = \frac{1,000}{2} = 500 \text{ pounds per square inch.}$$

2. Suppose that the rod is one-half square inch in cross-section. What is the value of the unit-stress ?

$A = \frac{1}{2}$ square inch, and, as before, $P = 1,000$ pounds; hence

$$S = 1,000 \div \frac{1}{2} = 2,000 \text{ pounds per square inch.}$$

Notice that one must always divide the whole stress by the area to get the unit-stress, whether the area is greater or less than one.

6. Deformation. Whenever forces are applied to a body it changes in size, and usually in shape also. This change of size and shape is called deformation. Deformations are usually measured in inches; thus, if a rod is stretched 2 inches, the "elongation" = 2 inches.

7. Unit-Deformation. It is sometimes necessary to specify not merely the value of a total deformation but its amount per unit length of the deformed body. Deformation per unit length of the deformed body is called unit-deformation.

To find the value of a unit-deformation: *Divide the whole deformation by the length over which it is distributed.* Thus, if

D denotes the value of a deformation,
l the length,
s the unit-deformation, then

$$s = \frac{D}{l}, \text{ also } D = ls. \qquad (2)$$

Both D and l should always be expressed in the same unit.

Example. Suppose that a 4-foot rod is elongated $\frac{1}{2}$ inch. What is the value of the unit-deformation?

Here $D = \frac{1}{2}$ inch, and $l = 4$ feet $= 48$ inches;
hence $s = \frac{1}{2} \div 48 = \frac{1}{96}$ inch per inch.

That is, each inch is elongated $\frac{1}{96}$ inch.

Unit-elongations are sometimes expressed in per cent. To express an elongation in per cent: *Divide the elongation in inches by the original length in inches, and multiply by 100.*

8. Elasticity. Most solid bodies when deformed will regain more or less completely their natural size and shape when the de-forming forces cease to act. This property of regaining size and shape is called elasticity.

We may classify bodies into kinds depending on the degree of elasticity which they have, thus:

1. *Perfectly elastic* bodies; these will regain their original form and size no matter how large the applied forces are if less than breaking values. Strictly there are no such materials, but rubber, practically, is perfectly elastic.
2. *Imperfectly elastic* bodies; these will fully regain their original form and size if the applied forces are not too large, and practically even if the loads are large but less than the breaking value. Most of the constructive materials belong to this class.
3. *Inelastic* or *plastic* bodies; these will not regain in the least their original form when the applied forces cease to act. Clay and putty are good examples of this class.

9. Hooke's Law, and Elastic Limit. If a gradually increasing force is applied to a perfectly elastic material, the deformation increases proportionally to the force; that is, if P and P′ denote two values of the force (or stress), and D and D′ the values of the deformation produced by the force,

$$\text{then } P:P'::D:D'.$$

This relation is also true for imperfectly elastic materials, provided that the loads P and P′ do not exceed a certain limit depending on the material. Beyond this limit, the deformation increases much faster than the load; that is, if within the limit an addition of 1,000 pounds to the load produces a stretch of 0.01 inch, beyond the limit an equal addition produces a stretch larger and usually much larger than 0.01 inch.

Beyond this limit of proportionality a part of the deformation is permanent; that is, if the load is removed the body only partially recovers its form and size. The permanent part of a deformation is called *set.*

The fact that for most materials the deformation is proportional to the load within certain limits, is known as Hooke's Law. The unit-stress within which Hooke's law holds, or above which the deformation is not proportional to the load or stress, is called *elastic limit.*

10. Ultimate Strength. By ultimate tensile, compressive, or shearing strength of a material is meant the greatest tensile, compressive, or shearing unit-stress which it can withstand.

As before mentioned, when a material is subjected to an increasing load the deformation increases faster than the load beyond the elastic limit, and much faster near the stage of rupture. Not only do tension bars and compression blocks elongate and shorten respectively, but their cross-sectional areas change also; tension bars thin down and compression blocks "swell out" more or less. The value of the ultimate strength for any material is ascertained by subjecting a specimen to a gradually increasing tensile, compressive, or shearing stress, as the case may be, until rupture occurs, and measuring the greatest load. *The breaking load divided by the area of the original cross-section sustaining the stress, is the value of the ultimate strength.*

Example. Suppose that in a tension test of a wrought-iron rod $\frac{1}{2}$ inch in diameter the greatest load was 12,540 pounds. What is the value of the ultimate strength of that grade of wrought iron?

The original area of the cross-section of the rod was

$$0.7854\ (\text{diameter})^2 = 0.7854 \times \tfrac{1}{4} = 0.1964 \text{ square inches; hence}$$

the ultimate strength equals

$$12{,}540 \div 0.1964 = 63{,}850 \text{ pounds per square inch.}$$

11. Stress-Deformation Diagram. A "test" to determine the elastic limit, ultimate strength, and other information in regard to a material is conducted by applying a gradually increasing load until the specimen is broken, and noting the deformation corresponding to many values of the load. The first and second columns of the following table are a record of a tension test on a steel rod one inch in diameter. The numbers in the first column are the values of the pull, or the loads, at which the elongation of the specimen was measured. The elongations are given in the second column. The numbers in the third and fourth columns are the values of the unit-stress and unit-elongation corresponding to the values of the load opposite to them. The numbers in the third column were obtained from those in the first by dividing the latter by the area of the cross-section of the rod, 0.7854 square inches. Thus,

$$3{,}930 \div 0.7854 = 5{,}000$$
$$7{,}850 \div 0.7854 = 10{,}000, \text{ etc.}$$

Total Pull in pounds, P	Deformation in inches, D	Unit-Stress in pounds per square inch, S	Unit-Deformation, s
3930	0.00136	5000	0.00017
7850	.00280	10000	.00035
11780	.00404	15000	.00050
15710	.00538	20000	.00067
19635	.00672	25000	.00084
23560	.00805	30000	.00101
27490	.00942	35000	.00118
31415	.01080	40000	.00135
35345	.01221	45000	.00153
39270	.0144	50000	.00180
43200	.0800	55000	.0100
47125	.1622	60000	.0202
51050	.201	65000	.0251
54980	.281	70000	.0351
58910	.384	75000	.048
62832	.560	80000	.070
65200	1.600	83000	.200

The numbers in the fourth column were obtained by dividing those in the second by the length of the specimen (or rather the length of that part whose elongation was measured), 8 inches. Thus,

$$0.00136 \div 8 = 0.00017,$$
$$.00280 \div 8 = .00035, \text{ etc.}$$

Looking at the first two columns it will be seen that the elongations are practically proportional to the loads up to the ninth load, the increase of stretch for each increase in load being about 0.00135 inch; but beyond the ninth load the increases of stretch are much greater. Hence the elastic limit was reached at about the ninth load, and its value is about 45,000 pounds per square inch. The greatest load was 65,200 pounds, and the corresponding unit-stress, 83,000 pounds per square inch, is the ultimate strength.

Nearly all the information revealed by such a test can be well represented in a diagram called a *stress-deformation diagram*. It is made as follows: Lay off the values of the unit-deformation (fourth column) along a horizontal line, according to some convenient scale, from some fixed point in the line. At the points on the horizontal line representing the various unit-elongations, lay off perpendicular distances equal to the corresponding unit-stresses. Then connect by a smooth curve the upper ends of all those distances, last distances laid off. Thus, for instance, the highest unit-elongation (0.20) laid off from *o* (Fig. 4) fixes the point *a*, and a perpendicular distance to represent the highest unit-stress (83,000) fixes the point *b*. All the points so laid off give the curve *ocb*. The part *oc*, within the elastic limit, is straight and nearly vertical while the remainder is curved and more or less horizontal, especially toward the point of rupture *b*. Fig. 5 is a typical stress-deformation diagram for timber, cast iron, wrought iron, soft and hard steel, in tension and compression.

12. Working Stress and Strength, and Factor of Safety. The greatest unit-stress in any part of a structure when it is sustaining its loads is called the *working stress* of that part. If it is under tension, compression and shearing stresses, then the corresponding highest unit-stresses in it are called its working stress in tension, in compression, and in shear respectively; that is, we speak of as many working stresses as it has kinds of stress.

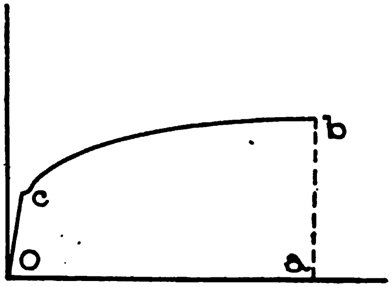

Fig. 4.

By *working strength* of a material to be used for a certain purpose is meant the highest unit-stress to which the material ought to be subjected when so used. Each material has a working strength for tension, for compression, and for shear, and they are in general different.

By *factor of safety* is meant the ratio of the ultimate strength of a material to its working stress or strength. Thus, if

S_u denotes ultimate strength,
S_w denotes working stress or strength, and
f denotes factor of safety, then

$$f = \frac{S_u}{S_w}; \text{ also } S_w = \frac{S_u}{f}. \qquad (3)$$

When a structure which has to stand certain loads is about to be designed, it is necessary to select working strengths or factors of safety for the materials to be used. Often the selection is a matter of great importance, and can be wisely performed only by an experienced engineer, for this is a matter where hard-and-fast rules should not govern but rather the judgment of the expert. But there are certain principles to be used as guides in making a selection, chief among which are:

1. The working strength should be considerably below the elastic limit. (Then the deformations will be small and not permanent.)

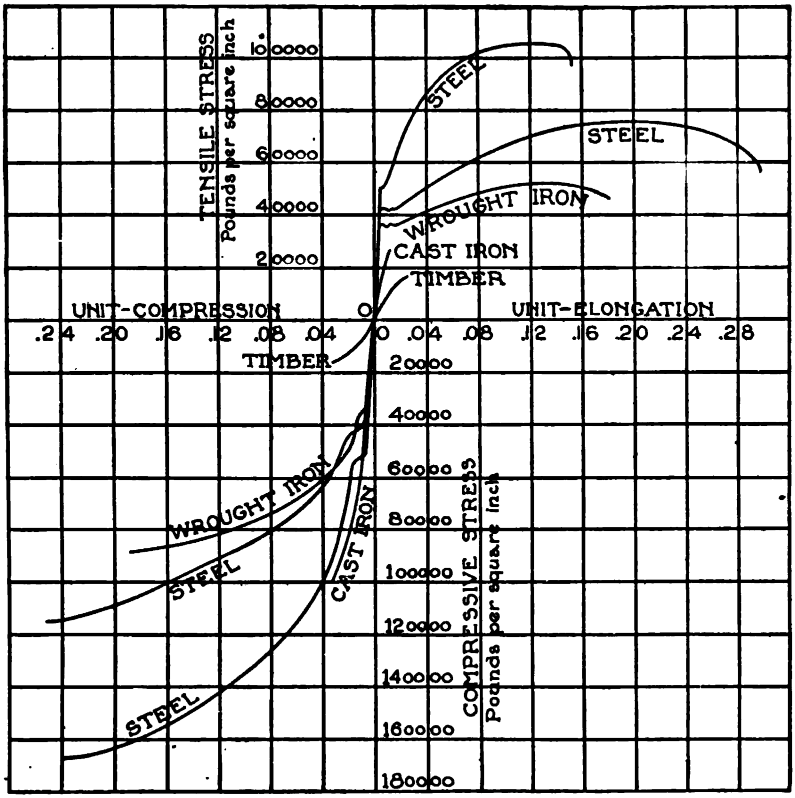

Fig. 5. (After Johnson.)

2. The working strength should be smaller for parts of a structure sustaining varying loads than for those whose loads are steady. (Actual experiments have disclosed the fact that the strength of a specimen depends on the kind of load put upon it, and that in a general way it is less the less steady the load is.)

3. The working strength must be taken low for non-uniform material, where poor workmanship may be expected, when the

loads are uncertain, etc. Principles 1 and 2 have been reduced to figures or formulas for many particular cases, but the third must remain a subject for display of judgment, and even good guessing in many cases.

The following is a table of factors of safety* which will be used in the problems:

Factors of Safety.

Materials.	For steady stress. (Buildings.)	For varying stress. (Bridges.)	For shocks. (Machines.)
Timber	8	10	15
Brick and stone	15	25	30
Cast iron	6	15	20
Wrought iron	4	6	10
Steel	5	7	15

They must be regarded as average values and are not to be adopted in every case in practice.

Examples. 1. A wrought-iron rod 1 inch in diameter sustains a load of 30,000 pounds. What is its working stress? If its ultimate strength is 50,000 pounds per square inch, what is its factor of safety?

The area of the cross-section of the rod equals 0.7854× (diameter)^2=0.7854×1^2=0.7854 square inches. Since the whole stress on the cross-section is 30,000 pounds, equation 1 gives for the unit working stress

$$S=\frac{30{,}000}{0.7854}=38{,}197 \text{ pounds per square inch.}$$

Equation 3 gives for factor of safety

$$f=\frac{50{,}000}{38{,}197}=1.3$$

2. How large a steel bar or rod is needed to sustain a steady pull of 100,000 pounds if the ultimate strength of the material is 65,000 pounds?

The load being steady, we use a factor of safety of 5 (see table above); hence the working strength to be used (see equation 3) is

$$S=\frac{65{,}000}{5}=13{,}000 \text{ pounds per square inch.}$$

The proper area of the cross-section of the rod can now be computed from equation 1 thus:

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*Taken from Merriman's "Mechanics of Materials."

$$A = \frac{P}{S} = \frac{100,000}{13,000} = 7.692 \text{ square inches.}$$

A bar 2×4 inches in cross-section would be a little stronger than necessary. To find the diameter (d) of a round rod of sufficient strength, we write $0.7854\, d^2 = 7.692$, and solve the equation for d; thus:

$$d^2 = \frac{7.692}{0.7854} = 9.794, \text{ or } d = 3.129 \text{ inches.}$$

3. How large a steady load can a short timber post safely sustain if it is 10×10 inches in cross-section and its ultimate compressive strength is 10,000 pounds per square inch ?

According to the table (page 12) the proper factor of safety is 8, and hence the working strength according to equation 3 is

$$S = \frac{10,000}{8} = 1,250 \text{ pounds per square inch.}$$

The area of the cross-section is 100 square inches; hence the safe load (see equation 1) is

$$P = 100 \times 1,250 = 125,000 \text{ pounds.}$$

4. When a hole is punched through a plate the shearing strength of the material has to be overcome. If the ultimate shearing strength is 50,000 pounds per square inch, the thickness of the plate $\frac{1}{2}$ inch, and the diameter of the hole $\frac{3}{4}$ inch, what is the value of the force to be overcome ?

The area shorn is that of the cylindrical surface of the hole or the metal punched out; that is

$$3.1416 \times \text{diameter} \times \text{thickness} = 3.1416 \times \tfrac{3}{4} \times \tfrac{1}{2} = 1.178 \text{ sq. in.}$$

Hence, by equation 1, the total shearing strength or resistance to punching is

$$P = 1.178 \times 50,000 = 58,900 \text{ pounds.}$$

STRENGTH OF MATERIALS UNDER SIMPLE STRESS.

13. Materials in Tension. Practically the only materials used extensively under tension are timber, wrought iron and steel, and to some extent cast iron.

14. Timber. A successful tension test of wood is difficult, as the specimen usually crushes at the ends when held in the testing machine, splits, or fails otherwise than as desired. Hence the tensile strengths of woods are not well known, but the following may be taken as approximate average values of the ultimate strengths of the woods named, when "dry out of doors."

Hemlock,	7,000	pounds per square inch.	
White pine,	8,000	"	"
Yellow pine, long leaf,	12,000	"	"
" " , short leaf,	10,000	"	"
Douglas spruce,	10,000	"	"
White oak,	12,000	"	"
Red oak,	9,000	"	"

15. Wrought Iron. The process of the manufacture of wrought iron gives it a "grain," and its tensile strengths along and across the grain are unequal, the latter being about three-fourths of the former. The ultimate tensile strength of wrought iron along the grain varies from 45,000 to 55,000 pounds per square inch. Strength along the grain is meant when not otherwise stated.

The strength depends on the size of the piece, it being greater for small than for large rods or bars, and also for thin than for thick plates. The elastic limit varies from 25,000 to 40,000 pounds per square inch, depending on the size of the bar or plate even more than the ultimate strength. Wrought iron is very ductile, a specimen tested in tension to destruction elongating from 5 to 25 per cent of its length.

16. Steel. Steel has more or less of a grain but is practically of the same strength in all directions. To suit different purposes, steel is made of various grades, chief among which may be mentioned rivet steel, sheet steel (for boilers), medium steel (for bridges and buildings), rail steel, tool and spring steel. In general, these grades of steel are hard and strong in the order named, the ultimate tensile strength ranging from about 50,000 to 160,000 pounds per square inch.

There are several grades of structural steel, which may be described as follows:*

1. Rivet steel:

Ultimate tensile strength, 48,000 to 58,000 pounds per square inch.
Elastic limit, not less than one-half the ultimate strength.
Elongation, 26 per cent.
Bends 180 degrees flat on itself without fracture.

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*Taken from "Manufacturer's Standard Specifications."

2. Soft steel:

Ultimate tensile strength, 52,000 to 62,000 pounds per square inch.
Elastic limit, not less than one-half the ultimate strength.
Elongation, 25 per cent.
Bends 180 degrees flat on itself.

3. Medium steel:

Ultimate tensile strength, 60,000 to 70,000 pounds per square inch.
Elastic limit, not less than one-half the ultimate strength.
Elongation, 22 per cent.
Bends 180 degrees to a diameter equal to the thickness of the specimen without fracture.

17. **Cast Iron.** As in the case of steel, there are many grades of cast iron. The grades are not the same for all localities or districts, but they are based on the appearance of the fractures, which vary from coarse dark grey to fine silvery white.

The ultimate tensile strength does not vary uniformly with the grades but depends for the most part on the percentage of "combined carbon" present in the iron. This strength varies from 15,000 to 35,000 pounds per square inch, 20,000 being a fair average.

Cast iron has no well-defined elastic limit (see curve for cast iron, Fig. 5). Its ultimate elongation is about one per cent.

EXAMPLES FOR PRACTICE.

1. A steel wire is one-eighth inch in diameter, and the ultimate tensile strength of the material is 150,000 pounds per square inch. How large is its breaking load? Ans. 1,840 pounds.

2. A wrought-iron rod (ultimate tensile strength 50,000 pounds per square inch) is 2 inches in diameter. How large a steady pull can it safely bear? Ans. 39,270 pounds.

18. **Materials in Compression.** Unlike the tensile, the compressive strength of a specimen or structural part depends on its dimension in the direction in which the load is applied, for, in compression, a long bar or rod is weaker than a short one. At present we refer only to the strength of short pieces such as do not bend under the load, the longer ones (columns) being discussed farther on.

Different materials break or fail under compression, in two very different ways:

1. Ductile materials (structural steel, wrought iron, etc.),

and wood compressed across the grain, do not fail by breaking into two distinct parts as in tension, but the former bulge out and flatten under great loads, while wood splits and mashes down. There is no particular point or instant of failure under increasing loads, and such materials have no definite ultimate strength in compression.

2. Brittle materials (brick, stone, hard steel, cast iron, etc.), and wood compressed along the grain, do not mash gradually, but fail suddenly and have a definite ultimate strength in compression. Although the surfaces of fracture are always much inclined to the direction in which the load is applied (about 45 degrees), the ultimate strength is computed by dividing the total breaking load by the cross-sectional area of the specimen.

The principal materials used under compression in structural work are timber, wrought iron, steel, cast iron, brick and stone.

19. Timber. As before noted, timber has no definite ultimate compressive strength across the grain. The U. S. Forestry Division has adopted certain amounts of compressive *deformation* as marking stages of failure. Three per cent compression is regarded as "a working limit allowable," and fifteen per cent as "an extreme limit, or as failure." The following (except the first) are values for compressive strength from the Forestry Division Reports, all in pounds per square inch:

	Ultimate strength along the grain.	3% Compression across the grain
Hemlock	6,000	
White pine	5,400	700
Long-leaf yellow pine	8,000	1,260
Short-leaf yellow pine	6,500	1,050
Douglas spruce	5,700	800
White oak	8,500	2,200
Red oak	7,200	2,300

20. Wrought Iron. The elastic limit of wrought iron, as before noted, depends very much upon the size of the bars or plate, it being greater for small bars and thin plates. Its value for compression is practically the same as for tension, 25,000 to 40,000 pounds per square inch.

21. Steel. The hard steels have the highest compressive strength; there is a recorded value of nearly 400,000 pounds per square inch, but 150,000 is probably a fair average.

The elastic limit in compression is practically the same as in tension, which is about 60 per cent of the ultimate tensile strength, or, for structural steel, about 25,000 to 42,000 pounds per square inch.

22. Cast Iron. This is a very strong material in compression, in which way, principally, it is used structurally. Its ultimate strength depends much on the proportion of "combined carbon" and silicon present, and varies from 50,000 to 200,000 pounds per square inch, 90,000 being a fair average. As in tension, there is no well-defined elastic limit in compression (see curve for cast iron, Fig. 5).

23. Brick. The ultimate strengths are as various as the kinds and makes of brick. For soft brick, the ultimate strength is as low as 500 pounds per square inch, and for pressed brick it varies from 4,000 to 20,000 pounds per square inch, 8,000 to 10,000 being a fair average. The ultimate strength of good paving brick is still higher, its average value being from 12,000 to 15,000 pounds per square inch.

24. Stone. Sandstone, limestone and granite are the principal building stones. Their ultimate strengths in pounds per square inch are about as follows:

Sandstone,*	5,000	to 16,000,	average	8,000.
Limestone,*	8,000	" 16,000,	"	10,000.
Granite,	14,000	" 24,000,	"	16,000.

*Compression at right angles to the "bed" of the stone.

EXAMPLES FOR PRACTICE.

1. A limestone 12×12 inches on its bed is used as a pier cap, and bears a load of 120,000 pounds. What is its factor of safety? Ans. 12.

2. How large a post (short) is needed to sustain a steady load of 100,000 pounds if the ultimate compressive strength of the wood is 10,000 pounds per square inch? Ans. 10×10 inches.

25. Materials in Shear. The principal materials used under shearing stress are timber, wrought iron, steel and cast iron. Partly on account of the difficulty of determining shearing strengths, these are not well known.

26. Timber. The ultimate shearing strengths of the more important woods *along the grain* are about as follows:

Hemlock,	300	pounds per square inch.
White pine,	400	" "
Long-leaf yellow pine,	850	" "
Short-leaf " "	775	" "
Douglas spruce,	500	" "
White oak,	1,000	" "
Red oak,	1,100	" "

Wood rarely fails by shearing across the grain. Its ultimate shearing strength in that direction is probably four or five times the values above given.

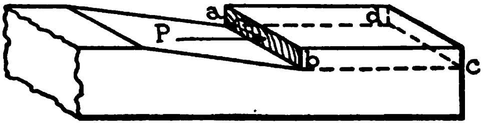


Fig. 6 *a*.

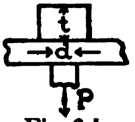


Fig. 6 *b*.

27. Metals. The ultimate shearing strength of wrought iron, steel, and cast iron is about 80 per cent of their respective ultimate tensile strengths.

EXAMPLES FOR PRACTICE.

1. How large a pressure P (Fig. 6 *a*) exerted on the shaded area can the timber stand before it will shear off on the surface *abcd*, if $ab = 6$ inches and $bc = 10$ inches, and the ultimate shearing strength of the timber is 400 pounds per square inch?

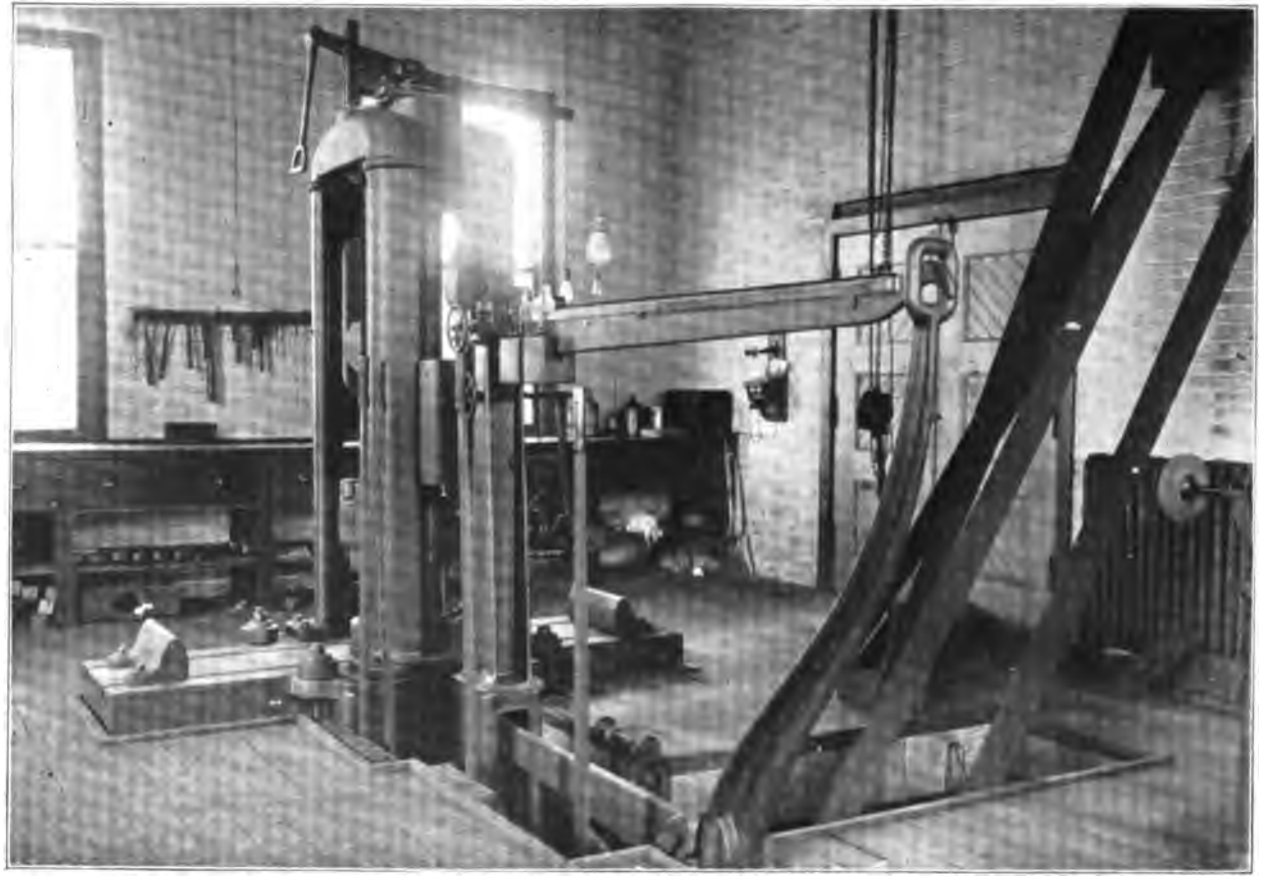
Ans. 24,000 pounds.

2. When a bolt is under tension, there is a tendency to tear the bolt and to "strip" or shear off the head. The shorn area would be the surface of the cylindrical hole left in the head. Compute the tensile and shearing unit-stresses when P (Fig. 6 *b*) equals 30,000 pounds, $d = 2$ inches, and $t = 3$ inches.

Ans. { Tensile unit-stress, 9,550 pounds per square inch.
Shearing unit-stress, 1,595 pounds per square inch.

REACTIONS OF SUPPORTS.

28. Moment of a Force. By moment of a force with respect to a point is meant its tendency to produce rotation about that point. Evidently the tendency depends on the magnitude of the force and on the perpendicular distance of the line of action of the force from the point: the greater the force and the perpendicular distance, the greater the tendency; hence *the moment*



A 300,000-lb. RIEHLE TESTING MACHINE
The Chicago Physical Testing Laboratory of Robert W. Hunt & Co., Engineers, Chicago, New York, Pittsburg, and London.

of a force with respect to a point equals the product of the force and the perpendicular distance from the force to the point.

The point with respect to which the moment of one or more forces is taken is called an *origin* or *center of moments*, and the perpendicular distance from an origin of moments to the line of action of a force is called the *arm* of the force with respect to that origin. Thus, if F_1 and F_2 (Fig. 7) are forces, their arms with respect to O′ are a_1' and a_2' respectively, and their moments are $F_1a'_1$ and $F_2a'_2$. With respect to O″ their arms are a_1'' and a_2'' respectively, and their moments are F_1a_1'' and F_2a_2''.

If the force is expressed in pounds and its arm in feet, the moment is in foot-pounds; if the force is in pounds and the arm in inches, the moment is in inch-pounds.

29. A *sign* is given to the moment of a force for convenience; the rule used herein is as follows: *The moment of a force about a point is positive or negative according as it tends to turn the body about that point in the clockwise or counter-clockwise direction*.*

Thus the moment (Fig. 7)

of F_1 about O′ is negative, about O″ positive;
" F_2 " O′ " " , about O″ negative.

30. Principle of Moments. In general, a single force of proper magnitude and line of action can balance any number of forces. That single force is called the *equilibrant* of the forces, and the single force that would balance the equilibrant is called the *resultant* of the forces. Or, otherwise stated, the resultant of any number of forces is a force which produces the same effect. It can be proved that—*The algebraic sum of the moments of any number of forces with respect to a point, equals the moment of their resultant about that point.*

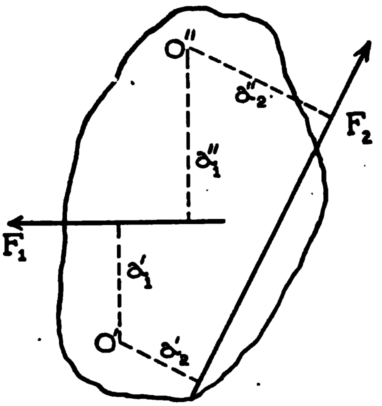


Fig. 7.

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*By clockwise direction is meant that in which the hands of a clock rotate; and by counter-clockwise, the opposite direction.

This is a useful principle and is called "principle of moments."

31. All the forces acting upon a body which is at rest are said to be *balanced* or *in equilibrium.* No force is required to balance such forces and hence their equilibrant and resultant are zero.

Since their resultant is zero, *the algebraic sum of the moments of any number of forces which are balanced or in equilibrium equals zero.*

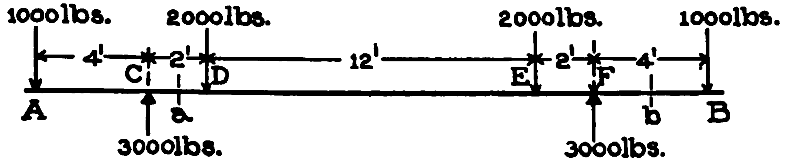


Fig. 8.

This is known as the principle of moments for forces in equilibrium; for brevity we shall call it also "the principle of moments."

The principle is easily verified in a simple case. Thus, let AB (Fig. 8) be a beam resting on supports at C and F. It is evident from the symmetry of the loading that each reaction equals one-half of the whole load, that is, $\frac{1}{2}$ of 6,000=3,000 pounds. (We neglect the weight of the beam for simplicity.)

With respect to C, for example, the moments of the forces are, taking them in order from the left:

−1,000 × 4 = − 4,000 foot-pounds
3,000 × 0 = 0 "
2,000 × 2 = 4,000 "
2,000 × 14 = 28,000 "
−3,000 × 16 = −48,000 "
1,000 × 20 = 20,000 "

The algebraic sum of these moments is seen to equal zero.

Again, with respect to B the moments are:

− 1,000 × 24 = − 24,000 foot-pounds
3,000 × 20 = 60,000 "
−2,000 × 18 = −36,000 "
−2,000 × 6 = − 12,000 "
3,000 × 4 = 12,000 "
1,000 × 0 = 0 "

The sum of these moments also equals zero. In fact, no matter where the center of moments is taken, it will be found in this and any other balanced system of forces that the algebraic sum of their moments equals zero. The chief use that we shall make of this principle is in finding the supporting forces of loaded beams.

32. Kinds of Beams. A *cantilever beam* is one resting on one support or fixed at one end, as in a wall, the other end being free.

A *simple beam* is one resting on two supports.

A *restrained beam* is one fixed at both ends; a beam fixed at one end and resting on a support at the other is said to be restrained at the fixed end and simply supported at the other.

A *continuous beam* is one resting on more than two supports.

33. Determination of Reactions on Beams. The forces which the supports exert on a beam, that is, the "supporting forces," are called *reactions*. We shall deal chiefly with simple beams. The reaction on a cantilever beam supported at one point evidently equals the total load on the beam.

When the loads on a horizontal beam are all vertical (and this is the usual case), the supporting forces are also vertical and *the sum of the reactions equals the sum of the loads.* This principle is sometimes useful in determining reactions, but in the case of simple beams the principle of moments is sufficient. The general method of determining reactions is as follows:

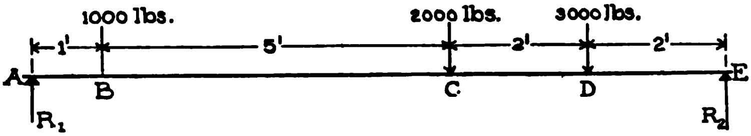


Fig. 9.

1. Write out two equations of moments for all the forces (loads and reactions) acting on the beam with origins of moments at the supports.
2. Solve the equations for the reactions.
3. As a check, try if the sum of the reactions equals the sum of the loads.

Examples. 1. Fig. 9 represents a beam supported at its ends and sustaining three loads. We wish to find the reactions due to these loads.

Let the reactions be denoted by R_1 and R_2 as shown; then the moment equations are:

For origin at A,

$$1{,}000\times1+2{,}000\times6+3{,}000\times8-R_2\times10=0.$$

For origin at E,

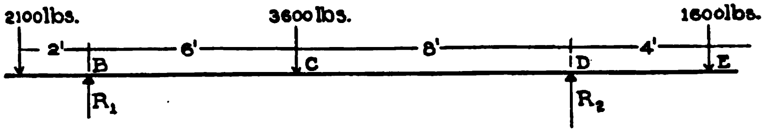


Fig. 10.

$$R_1\times10-1{,}000\times9-2{,}000\times4-3{,}000\times2=0.$$

The first equation reduces to

$$10\,R_2=1{,}000+12{,}000+24{,}000=37{,}000;\text{ or}$$

$R_2=3{,}700$ pounds.

The second equation reduces to

$$10\,R_1=9{,}000+8{,}000+6{,}000=23{,}000;\text{ or}$$

$R_1=2{,}300$ pounds.

The sum of the loads is 6,000 pounds and the sum of the reactions is the same; hence the computation is correct.

2. Fig. 10 represents a beam supported at B and D (that is, it has overhanging ends) and sustaining three loads as shown. We wish to determine the reactions due to the loads.

Let R_1 and R_2 denote the reactions as shown; then the moment equations are:

For origin at B,

$$-2{,}100\times2+0+3{,}600\times6-R_2\times14+1{,}600\times18=0.$$

For origin at D,

$$-2{,}100\times16+R_1\times14-3{,}600\times8+0+1{,}600\times4=0.$$

The first equation reduces to

$$14\,R_2=-4{,}200+21{,}600+28{,}800=46{,}200;\text{ or}$$

$R_2=3{,}300$ pounds.

The second equation reduces to

$$14\,R_1=33{,}600+28{,}800-6{,}400=56{,}000;\text{ or}$$

$R_1=4{,}000$ pounds.

The sum of the loads equals 7,300 pounds and the sum of the reactions is the same; hence the computation checks.

3. What are the total reactions in example 1 if the beam weighs 400 pounds?

(1.) Since we already know the reactions due to the loads (2,300 and 3,700 pounds at the left and right ends respectively (see illustration 1 above), we need only to compute the reactions due to the weight of the beam and add. Evidently the reactions due to the weight equal 200 pounds each; hence the

left reaction $= 2{,}300+200=2{,}500$ pounds, and the
right " $= 3{,}700+200=3{,}900$ " .

(2.) Or, we might compute the reactions due to the loads and weight of the beam together and directly. In figuring the moment due to the weight of the beam, we imagine the weight as concentrated at the middle of the beam; then its moments with respect to the left and right supports are (400×5) and $-(400\times5)$ respectively. The moment equations for origins at A and E are like those of illustration 1 except that they contain one more term, the moment due to the weight; thus they are respectively:

$$1{,}000\times1+2{,}000\times6+3{,}000\times8-R_2\times10+400\times5=0,$$
$$R_1\times10-1{,}000\times9-2{,}000\times4-3{,}000\times2-400\times5=0.$$

The first one reduces to

$$10\ R_2=39{,}000,\text{ or } R_2=3{,}900 \text{ pounds;}$$

and the second to

$$10\ R_1=25{,}000,\text{ or } R_1=2{,}500 \text{ pounds.}$$

4. What are the total reactions in example 2 if the beam weighs 42 pounds per foot?

As in example 3, we might compute the reactions due to the weight and then add them to the corresponding reactions due to the loads (already found in example 2), but we shall determine the total reactions due to load and weight directly.

The beam being 20 feet long, its weight is 42×20, or 840 pounds. Since the middle of the beam is 8 feet from the left and 6 feet from the right support, the moments of the weight with respect to the left and right supports are respectively:

$$840\times8=6{,}720,\text{ and} -840\times6=-5{,}040 \text{ foot-pounds.}$$

The moment equations for all the forces applied to the beam for origins at B and D are like those in example 2, with an additional term, the moment of the weight; they are respectively:

$$-2{,}100\times2+0+3{,}600\times6-R_2\times14+1{,}600\times18+6{,}720=0,$$
$$-2{,}100\times16+R_1\times14-3{,}600\times8+0+1{,}600\times4-5{,}040=0.$$

The first equation reduces to

$$14\ R_2 = 52{,}920, \text{ or } R_2 = 3{,}780 \text{ pounds,}$$

and the second to

$$14\ R_1 = 61{,}040, \text{ or } R_1 = 4{,}360 \text{ pounds.}$$

The sum of the loads and weight of beam is 8,140 pounds; and since the sum of the reactions is the same, the computation checks.

EXAMPLES FOR PRACTICE.

1. AB (Fig. 11) represents a simple beam supported at its ends. Compute the reactions, neglecting the weight of the beam.

Ans. { Right reaction = 1,443.75 pounds.
Left reaction = 1,556.25 pounds.

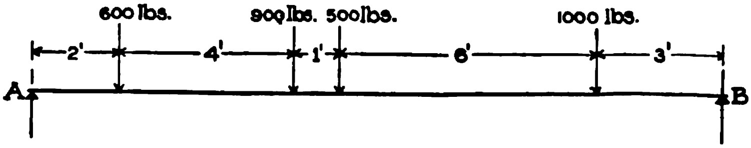


Fig. 11.

2. Solve example 1 taking into account the weight of the beam, which suppose to be 400 pounds.

Ans. { Right reaction = 1,643.75 pounds.
Left reaction = 1,756.25 pounds.

3. Fig. 12 represents a simple beam weighing 800 pounds supported at A and B, and sustaining three loads as shown. What are the reactions ?

Ans. { Right reaction = 2,014.28 pounds.
Left reaction = 4,785.72 pounds.

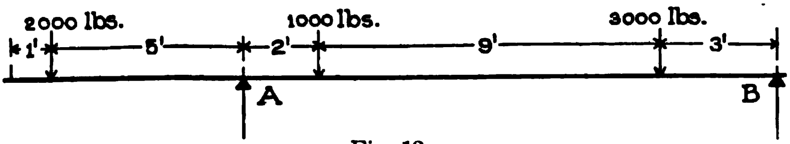


Fig. 12.

4. Suppose that in example 3 the beam also sustains a uniformly distributed load (as a floor) over its entire length, of 500 pounds per foot. Compute the reactions due to all the loads and the weight of the beam.

Ans. { Right reaction = 4,871.43 pounds. ?
Left reaction = 11,928.57 pounds.

EXTERNAL SHEAR AND BENDING MOMENT.

On almost every cross-section of a loaded beam there are three kinds of stress, namely tension, compression and shear. The first two are often called *fibre stresses* because they act along the real fibres of a wooden beam or the imaginary ones of which we may suppose iron and steel beams composed. Before taking up the subject of these stresses in beams it is desirable to study certain quantities relating to the loads, and on which the stresses in a beam depend. These quantities are called *external shear* and *bending moment*, and will now be discussed.

34. External Shear. By external shear at (or for) any section of a loaded beam is meant the algebraic sum of all the loads (including weight of beam) and reactions on *either side* of the section. This sum is called external shear because, as is shown later, it equals the shearing stress (internal) at the section. For brevity, we shall often say simply "shear" when external shear is meant.

35. Rule of Signs. In computing external shears, it is customary to give the plus sign to the reactions and the minus sign to the loads. But in order to get the same sign for the external shear whether computed from the right or left, we *change the sign* of the sum when computed from the loads and reactions *to the right.* Thus for section *a* of the beam in Fig. 8 the algebraic sum is, when computed from the left,

$$-1{,}000+3{,}000 = +2{,}000 \text{ pounds;}$$

and when computed from the right,

$$-1{,}000+3{,}000-2{,}000-2{,}000 = -2{,}000 \text{ pounds.}$$

The external shear at section *a* is +2,000 pounds.

Again, for section *b* the algebraic sum is, when computed from the left,

$$-1{,}000+3{,}000-2{,}000-2{,}000+3{,}000 = +1{,}000 \text{ pounds;}$$

and when computed from the right, $-1{,}000$ pounds.

The external shear at the section is +1,000 pounds.

It is usually convenient to compute the shear at a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left sides of the section.

36. Units for Shears. It is customary to express external shears in pounds, but any other unit for expressing force and weight (as the ton) may be used.

37. Notation. We shall use V to stand for external shear at any section, and the shear at a particular section will be denoted by that letter subscripted; thus V_1, V_2, etc., stand for the shears at sections one, two, etc., feet from the left end of a beam.

The shear has different values just to the left and right of a support or concentrated load. We shall denote such values by V' and V''; thus V_5' and V_5'' denote the values of the shear at sections a little less and a little more than 5 feet from the left end respectively.

Examples. 1. Compute the shears for sections one foot apart in the beam represented in Fig. 9, neglecting the weight of the beam. (The right and left reactions are 3,700 and 2,300 pounds respectively; see example 1, Art. 33.)

All the following values of the shear are computed from the left. The shear just to the right of the left support is denoted by V_0'', and $V_0'' = 2{,}300$ pounds. The shear just to the left of B is denoted by V_1', and since the only force to the left of the section is the left reaction, $V_1' = 2{,}300$ pounds. The shear just to the right of B is denoted by V_1'', and since the only forces to the left of this section are the left reaction and the 1,000-pound load, $V_1'' = 2{,}300 - 1{,}000 = 1{,}300$ pounds. To the left of all sections between B and C, there are but two forces, the left reaction and the 1,000-pound load; hence the shear at any of those sections equals $2{,}300 - 1{,}000 = 1{,}300$ pounds, or

$$V_2 = V_3 = V_4 = V_5 = V_6' = 1{,}300 \text{ pounds.}$$

The shear just to the right of C is denoted by V_6''; and since the forces to the left of that section are the left reaction and the 1,000- and 2,000-pound loads,

$$V_6'' = 2{,}300 - 1{,}000 - 2{,}000 = -700 \text{ pounds.}$$

Without further explanation, the student should understand that

$$\begin{aligned} V_7 &= +2{,}300 - 1{,}000 - 2{,}000 = -700 \text{ pounds,} \\ V_8' &= -700, \\ V_8'' &= +2{,}300 - 1{,}000 - 2{,}000 - 3{,}000 = -3{,}700, \\ V_9 &= V_{10}' = -3{,}700, \\ V_{10}'' &= +2{,}300 - 1{,}000 - 2{,}000 - 3{,}000 + 3{,}700 = 0 \end{aligned}$$

2. A simple beam 10 feet long, and supported at each end, weighs 400 pounds, and bears a uniformly distributed load of 1,600 pounds. Compute the shears for sections two feet apart.

Evidently each reaction equals one-half the sum of the load and weight of the beam, that is, $\frac{1}{2}$ (1,600+400) = 1,000 pounds. To the left of a section 2 feet from the left end, the forces acting on the beam consist of the left reaction, the load on that part of the beam, and the weight of that part; then since the load and weight of the beam *per foot* equal 200 pounds,

$$V_2 = 1{,}000 - 200 \times 2 = 600 \text{ pounds.}$$

To the left of a section four feet from the left end, the forces are the left reaction, the load on that part of the beam, and the weight; hence

$$V_4 = 1{,}000 - 200 \times 4 = 200 \text{ pounds.}$$

Without further explanation, the student should see that

$$\begin{aligned} V_6 &= 1{,}000 - 200 \times 6 = -200 \text{ pounds,} \\ V_8 &= 1{,}000 - 200 \times 8 = -600 \text{ pounds,} \\ V_{10}' &= 1{,}000 - 200 \times 10 = -1{,}000 \text{ pounds,} \\ V_{10}'' &= 1{,}000 - 200 \times 10 + 1{,}000 = 0. \end{aligned}$$

3. Compute the values of the shear in example 1, taking into account the weight of the beam (400 pounds). (The right and left reactions are then 3,900 and 2,500 pounds respectively; see example 3, Art. 33.)

We proceed just as in example 1, except that in each computation we include the weight of the beam to the left of the section (or to the right when computing from forces to the right). The weight of the beam being 40 pounds per foot, then (computing from the left)

$$\begin{aligned} V_0'' &= +2{,}500 \text{ pounds,} \\ V_1' &= +2{,}500 - 40 = +2{,}460, \\ V_1'' &= +2{,}500 - 40 - 1{,}000 = +1{,}460, \\ V_2 &= +2{,}500 - 1{,}000 - 40 \times 2 = +1{,}420, \\ V_3 &= +2{,}500 - 1{,}000 - 40 \times 3 = +1{,}380, \\ V_4 &= +2{,}500 - 1{,}000 - 40 \times 4 = +1{,}340, \\ V_5 &= +2{,}500 - 1{,}000 - 40 \times 5 = +1{,}300, \\ V_6' &= +2{,}500 - 1{,}000 - 40 \times 6 = +1{,}260, \\ V_6'' &= +2{,}500 - 1{,}000 - 40 \times 6 - 2{,}000 = -740, \\ V_7 &= +2{,}500 - 1{,}000 - 2{,}000 - 40 \times 7 = -780, \end{aligned}$$

$$
\begin{aligned}
V_8' &= +2{,}500-1{,}000-2{,}000-40\times 8 = -820,\\
V_8'' &= +2{,}500-1{,}000-2{,}000-40\times 8-3{,}000 = -3{,}820,\\
V_9 &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 9 = -3{,}860,\\
V'_{10} &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 10 = -3{,}900,\\
V''_{10} &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 10 + 3{,}900 = 0.
\end{aligned}
$$

Computing from the right, we find, as before, that

$$
\begin{aligned}
V_7 &= -(3{,}900-3{,}000-40\times 3) = -780 \text{ pounds},\\
V_8' &= -(3{,}900-3{,}000-40\times 2) = -820,\\
V_8'' &= -(3{,}900-40\times 2) = -3{,}820,
\end{aligned}
$$

etc., etc.

EXAMPLES FOR PRACTICE.

1. Compute the values of the shear for sections of the beam represented in Fig. 10, neglecting the weight of the beam. (The right and left reactions are 3,300 and 4,000 pounds respectively; see example 2, Art. 33.)

$$
\text{Ans.}\left\{
\begin{aligned}
&V_1 = V_2' = -2{,}100 \text{ pounds},\\
&V_2'' = V_3 = V_4 = V_5 = V_6 = V_7 = V'_8 = +1{,}900,\\
&V_8'' = V_9 = V_{10} = V_{11} = V_{12} = V_{13} = V_{14} = V_{15} = V'_{16} = -1{,}700,\\
&V''_{16} = V_{17} = V_{18} = V_{19} = V'_{20} = +1{,}600.
\end{aligned}\right.
$$

2. Solve the preceding example, taking into account the weight of the beam, 42 pounds per foot. (The right and left reactions are 3,780 and 4,360 pounds respectively; see example 4, Art. 33.)

$$
\text{Ans.}\left\{
\begin{array}{lll}
V_0'' = -2{,}100 \text{ lbs.} & V_7 = +1{,}966 \text{ lbs.} & V_{14} = -1{,}928 \text{ lbs.}\\
V_1 = -2{,}142 & V_8' = +1{,}924 & V_{15} = -1{,}970\\
V_2' = -2{,}184 & V_8'' = -1{,}676 & V_{16}' = -2{,}012\\
V_2'' = +2{,}176 & V_9 = -1{,}718 & V_{16}'' = +1{,}768\\
V_3 = +2{,}134 & V_{10} = -1{,}760 & V_{17} = +1{,}726\\
V_4 = +2{,}092 & V_{11} = -1{,}802 & V_{18} = +1{,}684\\
V_5 = +2{,}050 & V_{12} = -1{,}844 & V_{19} = +1{,}642\\
V_6 = +2{,}008 & V_{13} = -1{,}886 & V_{20}' = +1{,}600
\end{array}\right.
$$

3. Compute the values of the shear at sections one foot apart in the beam of Fig. 11, neglecting the weight. (The right and left reactions are 1,444 and 1,556 pounds respectively; see example 1, Art. 33.)

$$\text{Ans.}\left\{\begin{array}{l} V_0''=V_1=V_2'=+1{,}556 \text{ pounds}, \\ V_2''=V_3=V_4=V_5=V_6'=+956, \\ V_6''=V_7'=+56, \\ V_7''=V_8=V_9=V_{10}=V_{11}=V_{12}=V_{13}'=-444, \\ V_{13}''=V_{14}=V_{15}=V_{16}'=-1{,}444. \end{array}\right.$$

4. Compute the vertical shear at sections one foot apart in the beam of Fig. 12, taking into account the weight of the beam, 800 pounds, and a distributed load of 500 pounds per foot. (The right and left reactions are 4,870 and 11,930 pounds respectively; see examples 3 and 4, Art. 33.)

$$\text{Ans.}\left\{\begin{array}{lll} V_0 = 0 & V_7 = +6{,}150 \text{ lbs.} & V_{15} = +830 \text{ lbs.} \\ V_1' = -540 \text{ lbs.} & V_8' = +5{,}610 & V_{16} = +290 \\ V_1'' = -2{,}540 & V_8'' = +4{,}610 & V_{17}' = -250 \\ V_2 = -3{,}080 & V_9 = +4{,}070 & V_{17}'' = -3{,}250 \\ V_3 = -3{,}620 & V_{10} = +3{,}530 & V_{18} = -3{,}790 \\ V_4 = -4{,}160 & V_{11} = +2{,}990 & V_{19} = -4{,}330 \\ V_5 = -4{,}700 & V_{12} = +2{,}450 & V_{20}' = -4{,}870 \\ V_6' = -5{,}240 & V_{13} = +1{,}910 & V_{20}'' = 0 \\ V_6'' = +6{,}690 & V_{14} = +1{,}370 & \end{array}\right.$$

38. Shear Diagrams. The way in which the external shear varies from section to section in a beam can be well represented by means of a diagram called a *shear diagram.* To construct such a diagram for any loaded beam,

1. Lay off a line equal (by some scale) to the length of the beam, and mark the positions of the supports and the loads. (This is called a "base-line.")

2. Draw a line such that the distance of any point of it from the base equals (by some scale) the shear at the corresponding section of the beam, and so that the line is above the base where the shear is positive, and below it where negative. (This is called a *shear line*, and the distance from a point of it to the base is called the "ordinate" from the base to the shear line at that point.)

We shall explain these diagrams further by means of illustrative examples.

Examples. 1. It is required to construct the shear diagram for the beam represented in Fig. 13, *a* (a copy of Fig. 9).

Lay off A'E' (Fig. 13, *b*) to represent the beam, and mark the positions of the loads B', C' and D'. In example 1, Art. 37, we computed the values of the shear at sections one foot apart; hence we lay off ordinates at points on A'E' one foot apart, to represent those shears.

Use a scale of 4,000 pounds to one inch. Since the shear for any section in AB is 2,300 pounds, we draw a line *ab* parallel to the base 0.575 inch (2,300÷4,000) therefrom; this is the shear line for the portion AB. Since the shear for any section in BC equals 1,300 pounds, we draw a line *b'c* parallel to the base and 0.325 inch (1,300÷4,000) therefrom; this is the shear line for the portion BC. Since the shear for any section in CD is –700 pounds, we draw a line *c'd* below the base and 0.175 inch (700÷4,000) therefrom; this is the shear line for the portion CD. Since the shear for any section in DE equals –3,700 lbs., we draw a line *d'e* below the base and 0.925 inch (3,700÷4,000) therefrom; this is the shear line for the portion DE. Fig. 13, *b*, is the required shear diagram.

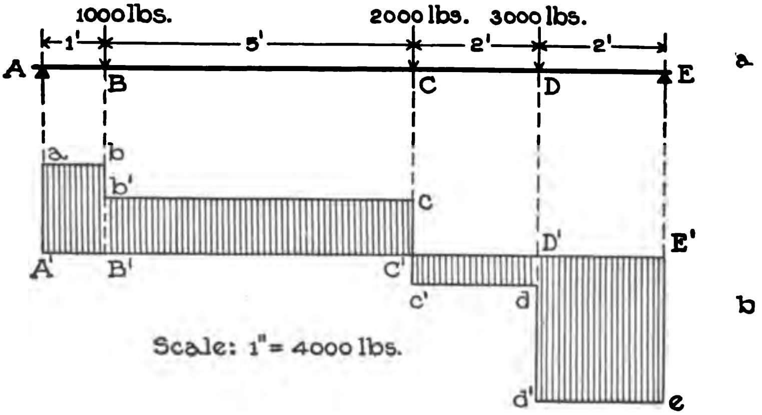


Fig. 13.

2. It is required to construct the shear diagram for the beam of Fig. 14, *a* (a copy of Fig. 9), taking into account the weight of the beam, 400 pounds.

The values of the shear for sections one foot apart were computed in example 3, Art. 37, so we have only to erect ordinates at the various points on a base line A'E' (Fig. 14, *b*), equal to those values. We shall use the same scale as in the preceding illustration, 4,000 pounds to an inch. Then the lengths of the ordinates corresponding to the values of the shear (see example 3, Art. 37) are respectively:

$$2{,}500 \div 4{,}000 = 0.625 \text{ inch}$$
$$2{,}460 \div 4{,}000 = 0.615 \text{ ``}$$
$$1{,}460 \div 4{,}000 = 0.365 \text{ ``}$$
$$\text{etc.} \qquad \text{etc.}$$

Laying these ordinates off from the base (upwards or downwards according as they correspond to positive or negative shears), we get *ab*, *b'c*, *c'd*, and *d'e* as the shear lines.

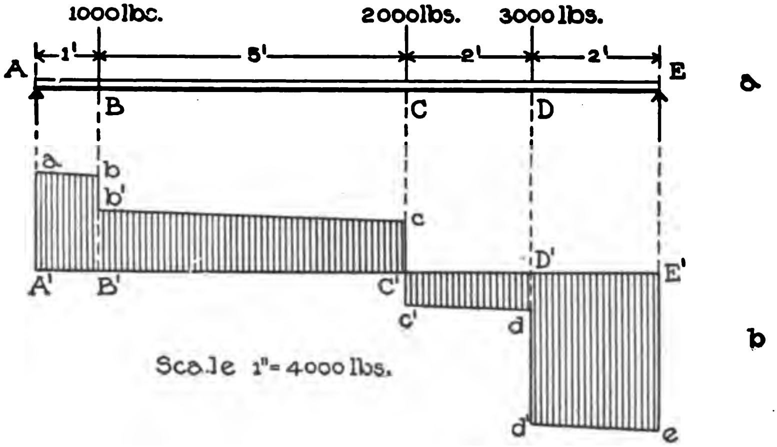


Fig. 14.

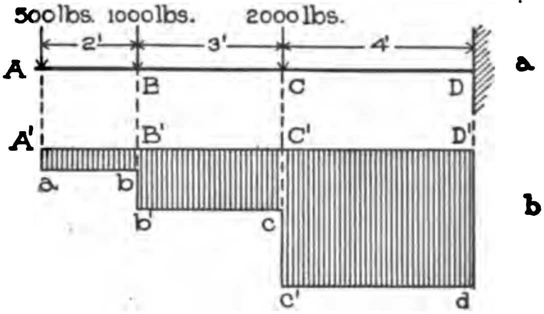
3. It is required to construct the shear diagram for the cantilever beam represented in Fig. 15, *a*, neglecting the weight of the beam.

The value of the shear for any section in AB is -500 pounds; for any section in BC, $-1{,}500$ pounds; and for any section in CD, $-3{,}500$ pounds. Hence the shear lines are *ab*, *b'c*, *c'd*. The scale being 5,000 pounds to an inch,

$$A'a = 500 \div 5{,}000 = 0.1 \text{ inch,}$$
$$B'b' = 1{,}500 \div 5{,}000 = 0.3 \text{ ``}$$
$$C'c' = 3{,}500 \div 5{,}000 = 0.7 \text{ ``}$$

The shear lines are all below the base because all the values of the shear are negative.

4. Suppose that the cantilever of the preceding illustration sustains also a uniform load of 200 pounds per foot (see Fig. 16, *a*). Construct a shear diagram.



Scale 1" = 5000 lbs.

Fig. 15.

First, we compute the values of the shear at several sections. Thus

$$V_0'' = -500 \text{ pounds},$$
$$V_1 = -500 - 200 = -700,$$
$$V_2' = -500 - 200 \times 2 = -900,$$
$$V_2'' = -500 - 200 \times 2 - 1{,}000 = -1{,}900,$$
$$V_3 = -500 - 1{,}000 - 200 \times 3 = -2{,}100,$$
$$V_4 = -500 - 1{,}000 - 200 \times 4 = -2{,}300,$$
$$V_5' = -500 - 1{,}000 - 200 \times 5 = -2{,}500,$$
$$V_5'' = -500 - 1{,}000 - 200 \times 5 - 2{,}000 = -4{,}500,$$
$$V_6 = -500 - 1{,}000 - 2{,}000 - 200 \times 6 = -4{,}700,$$
$$V_7 = -500 - 1{,}000 - 2{,}000 - 200 \times 7 = -4{,}900,$$
$$V_8 = -500 - 1{,}000 - 2{,}000 - 200 \times 8 = -5{,}100,$$
$$V_9 = -500 - 1{,}000 - 2{,}000 - 200 \times 9 = -5{,}300.$$

The values, being negative, should be plotted downward. To a scale of 5,000 pounds to the inch they give the shear lines *ab*, *b'c*, *c'd* (Fig. 16, *b*).

EXAMPLES FOR PRACTICE.

1. Construct a shear diagram for the beam represented in Fig. 10, neglecting the weight of the beam (see example 1, Art. 37).

2. Construct the shear diagram for the beam represented in Fig. 11, neglecting the weight of the beam (see example 3, Art. 37).

3. Construct the shear diagram for the beam of Fig. 12 when it sustains, in addition to the loads represented, its own weight, 800 pounds, and a uniform load of 500 pounds per foot (see example 4, Art. 37).

4. Figs. *a*, cases 1 and 2, Table B (page 55), represent two cantilever beams, the first bearing a concentrated load P at the free end, and the second a uniform load W. Figs. *b* are the corresponding shear diagrams. Take P and W equal to 1,000 pounds, and satisfy yourself that the diagrams are correct.

5. Figs. *a*, cases 3 and 4, same table, represent simple beams supported at their ends, the first bearing a concentrated load P at the middle, and the second a uniform load W. Figs. *b* are the corresponding shear diagrams. Take P and W equal to 1,000 pounds, and satisfy yourself that they are correct.

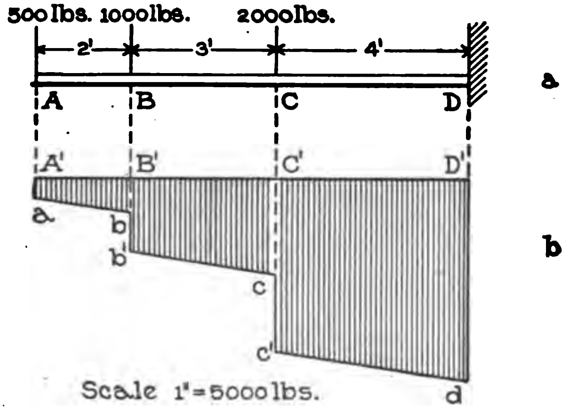


Fig. 16.

39. Maximum Shear. It is sometimes desirable to know the greatest or maximum value of the shear in a given case. This value can always be found with certainty by constructing the shear diagram, from which the maximum value of the shear is evident at a glance. In any case it can most readily be computed if one knows the section for which the shear is a maximum. The student should examine all the shear diagrams in the preceding articles and those that he has drawn, and see that

1. *In cantilevers fixed in a wall, the maximum shear occurs at the wall.*

2. *In simple beams, the maximum shear occurs at a section next to one of the supports.*

By the use of these propositions one can determine the value of the maximum shear without constructing the whole shear diagram. Thus, it is easily seen (referring to the diagrams, page 55) that for a

Cantilever, end load P, maximum shear = P
" , uniform load W, " " = W
Simple beam, middle load P, " " = ½P
" " , uniform " W, " " = ½W

40. Bending Moment. By bending moment at (or for) a section of a loaded beam, is meant the algebraic sum of the moments of all the loads (including weight of beam) and reactions to the left or right of the section with respect to any point in the section.

41. Rule of Signs. We follow the rule of signs previously stated (Art. 29) that the moment of a force which tends to produce clockwise rotation is plus, and that of a force which tends to produce counter-clockwise rotation is minus; but in order to get the same sign for the bending moment whether computed from the right or left, we *change the sign* of the sum of the moments when computed from the loads and reactions *on the right.* Thus for section *a*, Fig. 8, the algebraic sums of the moments of the forces are:

when computed from tne left,

$$-1{,}000\times5+3{,}000\times1=-2{,}000 \text{ foot-pounds};$$

and when computed from the right,

$$1{,}000\times19-3{,}000\times15+2{,}000\times13+2{,}000\times1=+2{,}000 \text{ foot-pounds.}$$

The bending moment at section *a* is –2,000 foot-pounds.

Again, for section *b*, the algebraic sums of the moments of the forces are:

when computed from the left,

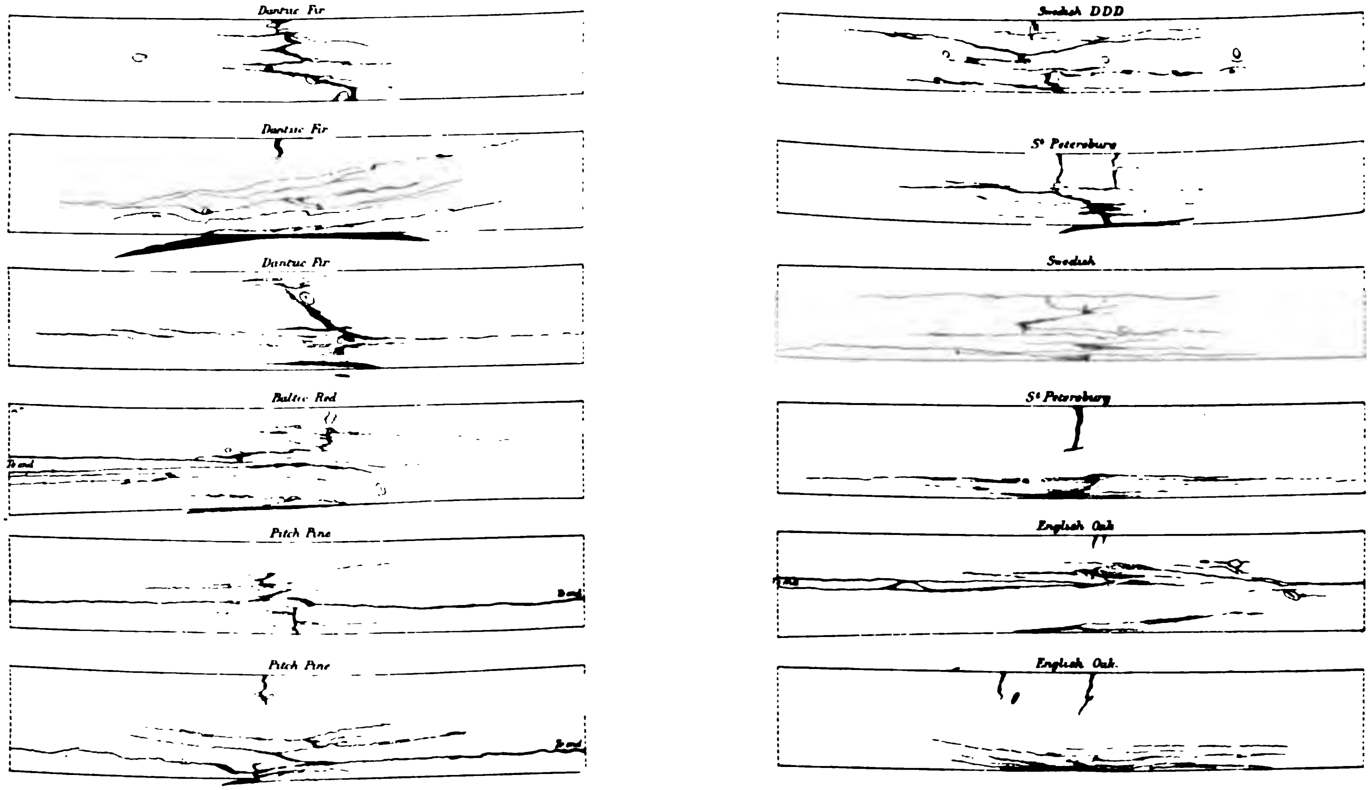
$$-1{,}000\times22+3{,}000\times18-2{,}000\times16-2{,}000\times4+3{,}000\times2=-2{,}000 \text{ foot-pounds};$$

and when computed from the right,

$$1{,}000\times2=+2{,}000 \text{ foot-pounds.}$$

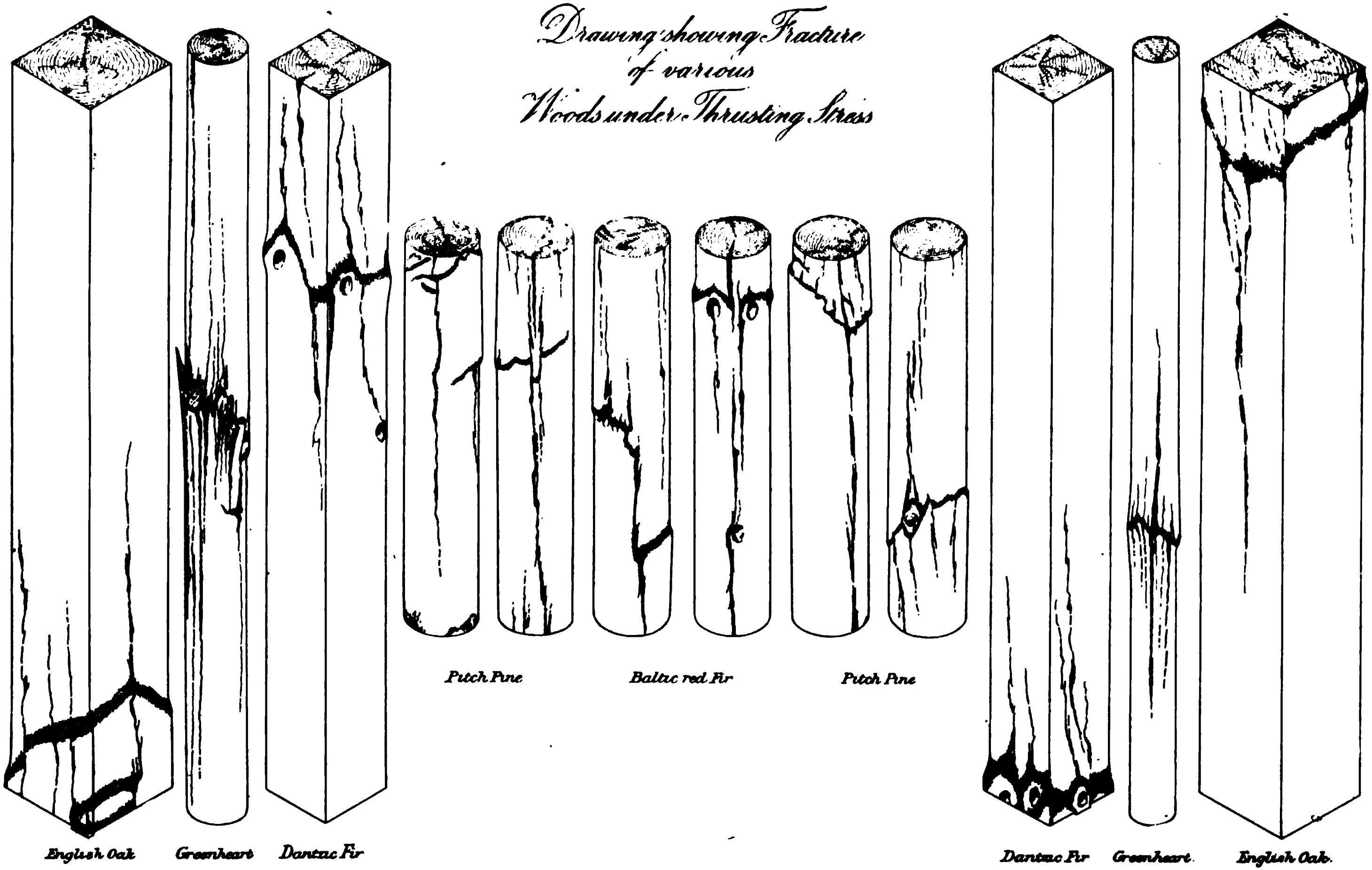
The bending moment at the section is –2,000 foot-pounds.

Drawing showing Fracture of various Woods under Bending Stress



DRAWING SHOWING FRACTURE OF VARIOUS WOODS UNDER BENDING STRESS

Reproduced from "Strength and Properties of Materials," by W. G. Kirkaldy.



DRAWING SHOWING FRACTURE OF VARIOUS WOODS UNDER THRUSTING STRESS

Reproduced from "Strength and Properties of Materials," by W. G. Kirkaldy.

It is usually convenient to compute the bending moment for a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left side of the section.

42. Units. It is customary to express bending moments in inch-pounds, but often the foot-pound unit is more convenient. *To reduce foot-pounds to inch-pounds, multiply by twelve.*

43. Notation. We shall use M to denote bending moment at any section, and the bending moment at a particular section will be denoted by that letter subscripted; thus M_1, M_2, etc., denote values of the bending moment for sections one, two, etc., feet from the left end of the beam.

Examples. 1. Compute the bending moments for sections one foot apart in the beam represented in Fig. 9, neglecting the weight of the beam. (The right and left reactions are 3,700 and 2,300 pounds respectively. See example 1, Art. 33.)

Since there are no forces acting on the beam to the left of the right support, $M_0=0$. To the left of the section one foot from the left end there is but one force, the left reaction, and its arm is one foot; hence $M_1=+2{,}300\times 1=2{,}300$ foot-pounds. To the left of a section two feet from the left end there are two forces, 2,300 and 1,000 pounds, and their arms are 2 feet and 1 foot respectively; hence $M_2=+2{,}300\times 2-1{,}000\times 1=3{,}600$ foot-pounds. At the left of all sections between C and D there are only two forces, 2,300 and 1,000 pounds; hence

$$
\begin{aligned}
M_3&=+2{,}300\times 3-1{,}000\times 2=+4{,}900 \text{ foot-pounds,}\\
M_4&=+2{,}300\times 4-1{,}000\times 3=+6{,}200 \quad \text{"}\\
M_5&=+2{,}300\times 5-1{,}000\times 4=+7{,}500 \quad \text{"}\\
M_6&=+2{,}300\times 6-1{,}000\times 5=+8{,}800 \quad \text{"}
\end{aligned}
$$

To the right of a section seven feet from the left end there are two forces, the 3,000-pound load and the right reaction (3,700 pounds), and their arms with respect to an origin in that section are respectively one foot and three feet; hence

$$M_7=-(-3{,}700\times 3+3{,}000\times 1)=+8{,}100 \text{ foot-pounds.}$$

To the right of any section between E and D there is only one force, the right reaction; hence

$$M_8=-(-3{,}700\times 2)=7{,}400 \text{ foot-pounds,}$$
$$M_9=-(-3{,}700\times 1)=3{,}700 \quad \text{“}$$

Clearly $M_{10}=0$.

2. A simple beam 10 feet long and supported at its ends weighs 400 pounds, and bears a uniformly distributed load of 1,600 pounds. Compute the bending moments for sections two feet apart.

Each reaction equals one-half the whole load, that is, $\frac{1}{2}$ of $(1{,}600+400)=1{,}000$ pounds, and the load per foot including weight of the beam is 200 pounds. The forces acting on the beam to the left of the first section, two feet from the left end, are the left reaction (1,000 pounds) and the load (including weight) on the part of the beam to the left of the section (400 pounds). The arm of the reaction is 2 feet and that of the 400-pound force is 1 foot (the distance from the middle of the 400-pound load to the section). Hence

$$M_2=+1{,}000\times 2-400\times 1=+1{,}600 \text{ foot-pounds.}$$

The forces to the left of the next section, 4 feet from the left end, are the left reaction and all the load (including weight of beam) to the left (800 pounds). The arm of the reaction is 4 feet, and that of the 800-pound force is 2 feet; hence

$$M_4=+1{,}000\times 4-800\times 2=+2{,}400 \text{ foot-pounds.}$$

Without further explanation the student should see that

$$M_6=+1{,}000\times 6-1{,}200\times 3=+2{,}400 \text{ foot-pounds,}$$
$$M_8=+1{,}000\times 8-1{,}600\times 4=+1{,}600 \quad \text{“}$$

Evidently $M_0=M_{10}=0$.

3. Compute the values of the bending moment in example 1, taking into account the weight of the beam, 400 pounds. (The right and left reactions are respectively 3,900 and 2,500 pounds; see example 3, Art. 33.)

We proceed as in example 1, except that the moment of the weight of the beam to the left of each section (or to the right when computing from forces to the right) must be included in the respective moment equations. Thus, computing from the left,

$$
\begin{aligned}
M_0 &= 0\\
M_1 &= +2{,}500\times 1 - 40\times \tfrac{1}{2} = +2{,}480 \text{ foot-pounds},\\
M_2 &= +2{,}500\times 2 - 1{,}000\times 1 - 80\times 1 = +3{,}920,\\
M_3 &= +2{,}500\times 3 - 1{,}000\times 2 - 120\times 1\tfrac{1}{2} = +5{,}320,\\
M_4 &= +2{,}500\times 4 - 1{,}000\times 3 - 160\times 2 = +6{,}680,\\
M_5 &= +2{,}500\times 5 - 1{,}000\times 4 - 200\times 2\tfrac{1}{2} = +8{,}000,\\
M_6 &= +2{,}500\times 6 - 1{,}000\times 5 - 240\times 3 = +9{,}280.
\end{aligned}
$$

Computing from the right,

$$
\begin{aligned}
M_7 &= -(-3{,}900\times 3 + 3{,}000\times 1 + 120\times 1\tfrac{1}{2}) = +8{,}520,\\
M_8 &= -(-3{,}900\times 2 + 80\times 1) = +7{,}720,\\
M_9 &= -(-3{,}900\times 1 + 40\times \tfrac{1}{2}) = +3{,}880,\\
M_{10} &= 0.
\end{aligned}
$$

EXAMPLES FOR PRACTICE.

1. Compute the values of the bending moment for sections one foot apart, beginning one foot from the left end of the beam represented in Fig. 10, neglecting the weight of the beam. (The right and left reactions are 3,300 and 4,000 pounds respectively; see example 2, Art. 33.)

$$
\text{Ans. (in foot-pounds)}\left\{
\begin{array}{llll}
M_1 = -2{,}100 & M_6 = +3{,}400 & M_{11} = +2{,}100 & M_{16} = -6{,}400\\
M_2 = -4{,}200 & M_7 = +5{,}300 & M_{12} = +400 & M_{17} = -4{,}800\\
M_3 = -2{,}300 & M_8 = +7{,}200 & M_{13} = -1{,}300 & M_{18} = -3{,}200\\
M_4 = -400 & M_9 = +5{,}500 & M_{14} = -3{,}000 & M_{19} = -1{,}600\\
M_5 = +1{,}500 & M_{10} = +3{,}800 & M_{15} = -4{,}700 & M_{20} = 0
\end{array}
\right.
$$

2. Solve the preceding example, taking into account the weight of the beam, 42 pounds per foot. (The right and left reactions are 3,780 and 4,360 pounds respectively; see example 4, Art. 33.)

$$
\text{Ans. (in foot-pounds)}\left\{
\begin{array}{llll}
M_1 = -2{,}121 & M_6 = +4{,}084 & M_{11} = +2{,}799 & M_{16} = -6{,}736\\
M_2 = -4{,}284 & M_7 = +6{,}071 & M_{12} = +976 & M_{17} = -4{,}989\\
M_3 = -2{,}129 & M_8 = +8{,}016 & M_{13} = -889 & M_{18} = -3{,}284\\
M_4 = -16 & M_9 = +6{,}319 & M_{14} = -2{,}796 & M_{19} = -1{,}621\\
M_5 = +2{,}055 & M_{10} = +4{,}580 & M_{15} = -4{,}745 & M_{20} = 0
\end{array}
\right.
$$

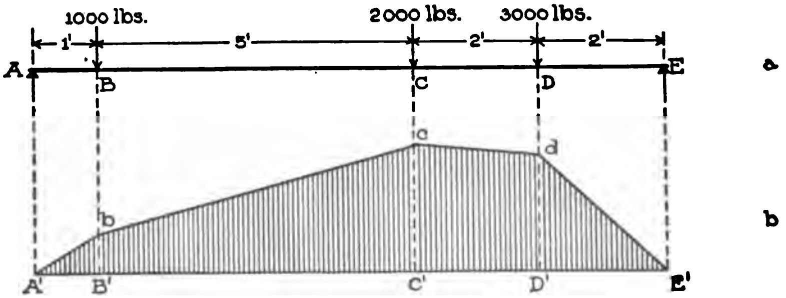
3. Compute the bending moments for sections one foot apart, of the beam represented in Fig. 11, neglecting the weight. (The right and left reactions are 1,444 and 1,556 pounds respectively; see example 1, Art. 33.)

Ans. (in foot-pounds)
$M_1 = +1,556$ $M_5 = +5,980$ $M_9 = +6,104$ $M_{13} = +4,328$
$M_2 = +3,112$ $M_6 = +6,936$ $M_{10} = +5,660$ $M_{14} = +2,884$
$M_3 = +4,068$ $M_7 = +6,992$ $M_{11} = +5,216$ $M_{15} = +1,440$
$M_4 = +5,024$ $M_8 = +6,548$ $M_{12} = +4,772$ $M_{16} = 0$

4 Compute the bending moments at sections one foot apart in the beam of Fig. 12, taking into account the weight of the beam, 800 pounds, and a uniform load of 500 pounds per foot. (The right and left reactions are 4,870 and 11,930 pounds respectively; see Exs. 3 and 4, Art. 33.)

Ans. (in foot-pounds)
$M_1 = -270$ $M_6 = -19,720$ $M_{11} = +3,980$ $M_{16} = 12,180$
$M_2 = -3,080$ $M_7 = -13,300$ $M_{12} = +6,700$ $M_{17} = 12,200$
$M_3 = -6,430$ $M_8 = -7,420$ $M_{13} = +8,880$ $M_{18} = 8,680$
$M_4 = -10,320$ $M_9 = -3,080$ $M_{14} = +10,520$ $M_{19} = 4,620$
$M_5 = -14,750$ $M_{10} = +720$ $M_{15} = +11,620$ $M_{20} = 0$

44. Moment Diagrams. The way in which the bending moment varies from section to section in a loaded beam can be well represented by means of a diagram called a *moment diagram*. To construct such a diagram for any loaded beam,



Scale: 1 = 10000 ft.-lbs

Fig. 17.

1. Lay off a base-line just as for a shear diagram (see Art. 38).

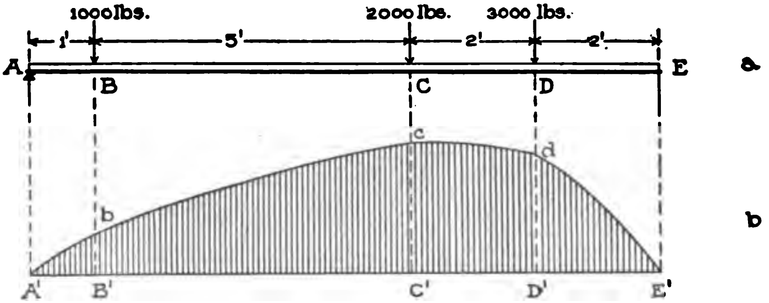
2. Draw a line such that the distance from any point of it to the base-line equals (by some scale) the value of the bending moment at the corresponding section of the beam, and so that the line is above the base where the bending moment is positive and below it where it is negative. (This line is called a "moment line.")

Examples. 1. It is required to construct a moment diagram for the beam of Fig. 17, *a* (a copy of Fig. 9), loaded as there shown.

Lay off A′E′ (Fig. 17, *b*) as a base. In example 1, Art. 43, we computed the values of the bending moment for sections one foot apart, so we erect ordinates at points of A′E′ one foot apart, to represent the bending moments.

We shall use a scale of 10,000 foot-pounds to the inch; then the ordinates (see example 1, Art. 43, for values of M) will be:

One foot from left end, 2,300÷10,000 = 0.23 inch,
Two feet " " " 3,600÷10,000 = 0.36 "
Three " " " " 4,900÷10,000 = 0.49 "
Four " " " " 6,200÷10,000 = 0.62 "
etc., etc.



Scale: 1" = 10000 ft-lbs.

Fig. 18.

Laying these ordinates off, and joining their ends in succession, we get the line A′*bcd*E′, which is the bending moment line. Fig. 17, *b*, is the moment diagram.

2. It is required to construct the moment diagram for the beam, Fig. 18, *a* (a copy of Fig. 9), taking into account the weight of the beam, 400 pounds.

The values of the bending moment for sections one foot apart were computed in example 3, Art. 43. So we have only to lay off ordinates equal to those values, one foot apart, on the base A′E′ (Fig. 18, *b*).

To a scale of 10,000 foot-pounds to the inch the ordinates (see example 3, Art. 43, for values of M) are:

At left end, 0
One foot from left end, 2,480÷10,000=0.248 inch
Two feet " " " 3,920÷10,000=0.392 "
Three " " " " 5,320÷10,000=0.532 "
Four " " " " 6,680÷10,000=0.668 "

Laying these ordinates off at the proper points, we get A'*bcd*E as the moment line.

3. It is required to construct the moment diagram for the cantilever beam represented in Fig. 19, *a*, neglecting the weight of the beam. The bending moment at B equals

$$-500 \times 2 = -1,000 \text{ foot-pounds;}$$

at C,

$$-500 \times 5 - 1,000 \times 3 = -5,500;$$

and at D,

$$-500 \times 9 - 1,000 \times 7 - 2,000 \times 4 = -19,500.$$

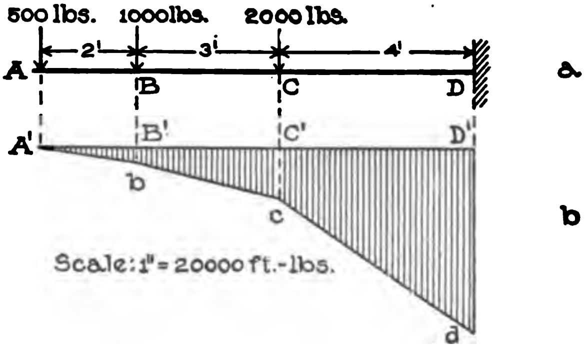


Fig. 19.

Using a scale of 20,000 foot-pounds to one inch, the ordinates in the bending moment diagram are:

At B, 1,000÷20,000=0.05 inch,
" C, 5,500÷20,000=0.275 "
" D, 19,500÷20,000=0.975 "

Hence we lay these ordinates off, and downward because the bending moments are negative, thus fixing the points *b*, *c* and *d*. The bending moment at A is zero; hence the moment line connects A *b*, *c* and *d*. Further, the portions A*b*, *bc* and *cd* are straight, as can be shown by computing values of the bending moment for sections in AB, BC and CD, and laying off the corresponding ordinates in the moment diagram.

4. Suppose that the cantilever of the preceding illustration sustains also a uniform load of **100** pounds per foot (see Fig. 20, *a*). Construct a moment diagram.

First, we compute the values of the bending moment at several sections; thus,

$M_1=-500\times 1-100\times \frac{1}{2}=-550$ foot-pounds,
$M_2=-500\times 2-200\times 1=-1{,}200$,
$M_3=-500\times 3-1{,}000\times 1-300\times 1\frac{1}{2}=-2{,}950$,
$M_4=-500\times 4-1{,}000\times 2-400\times 2=-4{,}800$,
$M_5=-500\times 5-1{,}000\times 3-500\times 2\frac{1}{2}=-6{,}750$,
$M_6=-500\times 6-1{,}000\times 4-2{,}000\times 1-600\times 3=-10{,}800$,
$M_7=-500\times 7-1{,}000\times 5-2{,}000\times 2-700\times 3\frac{1}{2}=-14{,}950$,
$M_8=-500\times 8-1{,}000\times 6-2{,}000\times 3-800\times 4=-19{,}200$,
$M_9=-500\times 9-1{,}000\times 7-2{,}000\times 4-900\times 4\frac{1}{2}=-23{,}550$.

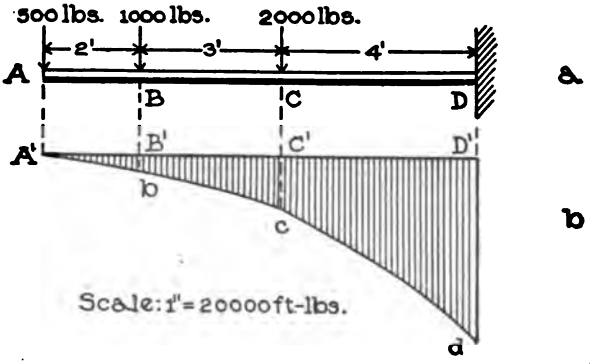


Fig. 20.

These values all being negative, the ordinates are all laid off downwards. To a scale of 20,000 foot-pounds to one inch, they fix the moment line *A'bcd*.

EXAMPLES FOR PRACTICE.

1. Construct a moment diagram for the beam represented in Fig. 10, neglecting the weight of the beam. (See example 1, Art. 43).

2. Construct a moment diagram for the beam represented in Fig. 11, neglecting the weight of the beam. (See example 3, Art. 43).

3. Construct the moment diagram for the beam of Fig. 12

when it sustains, in addition to the loads represented and its own weight (800 pounds), a uniform load of 500 pounds per foot. (See example 4, Art. 43.)

4. Figs. *a*, cases 1 and 2, page 55, represent two cantilever beams, the first bearing a load P at the free end, and the second a uniform load W. Figs. *b* are the corresponding moment diagrams. Take P and W equal to 1,000 pounds, and l equal to 10 feet, and satisfy yourself that the diagrams are correct.

5. Figs. *a*, cases 3 and 4, page 55, represent simple beams on end supports, the first bearing a middle load P, and the other a uniform load W. Figs. *b* are the corresponding moment diagrams. Take P and W equal to 1,000 pounds, and l equal to 10 feet, and satisfy yourself that the diagrams are correct.

45. Maximum Bending Moment. It is sometimes desirable to know the greatest or maximum value of the bending moment in a given case. This value can always be found with certainty by constructing the moment diagram, from which the maximum value of the bending moment is evident at a glance. But in any case, it can be most readily computed if one knows the section for which the bending moment is greatest. If the student will compare the corresponding shear and moment diagrams which have been constructed in foregoing articles (Figs. 13 and 17, 14 and 18, 15 and 19, 16 and 20), and those which he has drawn, he will see that—*The maximum bending moment in a beam occurs where the shear changes sign.*

By the help of the foregoing principle we can readily compute the maximum moment in a given case. We have only to construct the shear line, and observe from it where the shear changes sign; then compute the bending moment for that section. If a simple beam has one or more overhanging ends, then the shear changes sign more than once—twice if there is one overhanging end, and three times if two. In such cases we compute the bending moment for each section where the shear changes sign; the largest of the values of these bending moments is the maximum for the beam.

The section of maximum bending moment in a cantilever fixed at one end (as when built into a wall) is always at the wall.

Thus, without reference to the moment diagrams, it is readily seen that,

for a cantilever whose length is l,
with an end load P, the maximum moment is Pl,
“ a uniform “ W, “ “ “ “ $\frac{1}{2}Wl$.

Also by the principle, it is seen that,

for a beam whose length is l, on end supports,
with a middle load P, the maximum moment is $\frac{1}{4}Pl$,
“ uniform “ W, “ “ “ “ $\frac{1}{8}Wl$.

46. Table of Maximum Shears, Moments, etc. Table B on page 55 shows the shear and moment diagrams for eight simple cases of beams. The first two cases are built-in cantilevers; the next four, simple beams on end supports; and the last two, restrained beams built in walls at each end. In each case l denotes the length.

CENTER OF GRAVITY AND MOMENT OF INERTIA.

It will be shown later that the strength of a beam depends partly on the form of its cross-section. The following discussion relates principally to cross-sections of beams, and the results reached (like shear and bending moment) will be made use of later in the subject of strength of beams.

47. Center of Gravity of an Area. The student probably knows what is meant by, and how to find, the center of gravity of any flat disk, as a piece of tin. Probably his way is to balance the piece of tin on a pencil point, the point of the tin at which it so balances being the center of gravity. (Really it is midway between the surfaces of the tin and over the balancing point.) The center of gravity of the piece of tin, is also that point of it through which the resultant force of gravity on the tin (that is, the weight of the piece) acts.

By “center of gravity” of a plane area of any shape we mean that point of it which corresponds to the center of gravity of a piece of tin when the latter is cut out in the shape of the area. The center of gravity of a quite irregular area can be found most readily by balancing a piece of tin or stiff paper cut in the shape of the area. But when an area is simple in shape, or consists of parts which are simple, the center of gravity of the whole can be found readily by computation, and such a method will now be described.

48. Principle of Moments Applied to Areas. Let Fig. 21 represent a piece of tin which has been divided off into any number of parts in any way, the weight of the whole being W, and that of the parts W_1, W_2, W_3, etc. Let C_1, C_2, C_3, etc., be the centers of gravity of the parts, C that of the whole, and c_1, c_2, c_3, etc., and c the distances from those centers of gravity respectively to some line (L L) in the plane of the sheet of tin. When the tin is lying in a horizontal position, the moment of the weight of the entire piece about L L is Wc, and the moments of the parts are W_1c_1, W_2c_2, etc. Since the weight of the whole is the resultant of the weights of the parts, the moment of the weight of the whole equals the sum of the moments of the weights of the parts; that is,

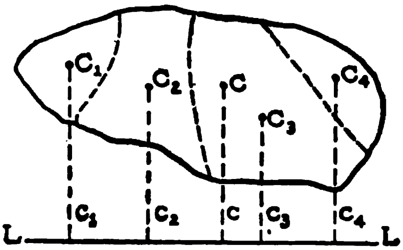


Fig. 21.

$$Wc = W_1c_1 + W_2c_2 + \text{etc.} \ldots$$

Now let A_1, A_2, etc. denote the areas of the parts of the pieces of tin, and A the area of the whole; then since the weights are proportional to the areas, we can replace the W's in the preceding equation by corresponding A's, thus:

$$Ac = A_1c_1 + A_2c_2 + \text{etc.} \ldots \qquad (4)$$

If we call the product of an area and the distance of its center of gravity from some line in its plane, the "moment" of the area with respect to that line, then the preceding equation may be stated in words thus:

The moment of an area with respect to any line equals the algebraic sum of the moments of the parts of the area.

If all the centers of gravity are on one side of the line with respect to which moments are taken, then all the moments should be given the plus sign; but if some centers of gravity are on one side and some on the other side of the line, then the moments of the areas whose centers of gravity are on one side should be given the same sign, and the moments of the others the opposite sign. The foregoing is the principle of moments for areas, and it is the basis of all rules for finding the center of gravity of an area.

To find the center of gravity of an area which can be divided up into simple parts, we write the principle in forms of equations for two different lines as "axes of moments," and then solve the equations for the unknown distances of the center of gravity of the whole from the two lines. We explain further by means of specific examples.

Examples. 1. It is required to find the center of gravity of Fig. 22, *a*, the width being uniformly one inch.

The area can be divided into two rectangles. Let C_1 and C_2 be the centers of gravity of two such parts, and C the center of gravity of the whole. Also let *a* and *b* denote the distances of C from the two lines OL′ and OL″ respectively.

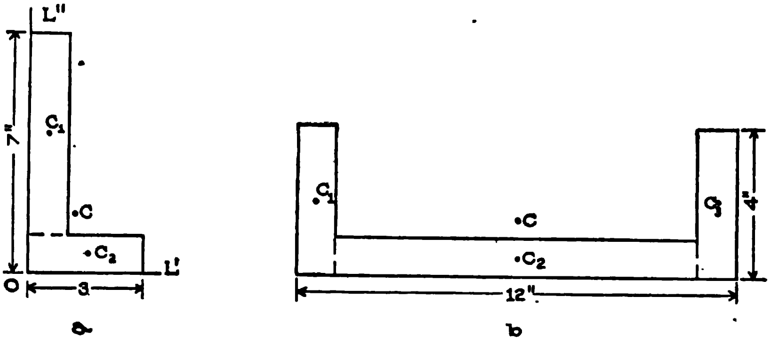


Fig. 22.

The areas of the parts are 6 and 3 square inches, and their arms with respect to OL′ are 4 inches and ½ inch respectively, and with respect to OL″ ½ inch and 1½ inches. Hence the equations of moments with respect to OL′ and OL″ (the whole area being 9 square inches) are:

$$9 \times a = 6 \times 4 + 3 \times \tfrac{1}{2} = 25.5,$$
$$9 \times b = 6 \times \tfrac{1}{2} + 3 \times 1\tfrac{1}{2} = 7.5.$$

Hence,
$$a = 25.5 \div 9 = 2.83 \text{ inches},$$
$$b = 7.5 \div 9 = 0.83 \text{ “}.$$

2. It is required to locate the center of gravity of Fig. 22, *b*, the width being uniformly one inch.

The figure can be divided up into three rectangles. Let C_1, C_2 and C_3 be the centers of gravity of such parts, C the center of gravity of the whole; and let a denote the (unknown) distance of C from the base. The areas of the parts are 4, 10 and 4 square inches, and their "arms" with respect to the base are 2, ½ and 2 inches respectively; hence the equation of moments with respect to the base (the entire area being 18 square inches) is:

$$18 \times a = 4 \times 2 + 10 \times \tfrac{1}{2} + 4 \times 2 = 21.$$

Hence, $a = 21 \div 18 = 1.17$ inches.

From the symmetry of the area it is plain that the center of gravity is midway between the sides.

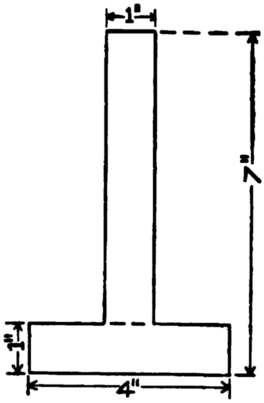


Fig. 23.

EXAMPLE FOR PRACTICE.

1. Locate the center of gravity of Fig. 23.

Ans. 2.6 inches above the base.

49. Center of Gravity of Built-up Sections. In Fig. 24 there are represented cross-sections of various kinds of rolled steel, called "shape steel," which is used extensively in steel construction. Manufacturers of this material publish "handbooks" giving full information in regard thereto, among other things, the position of the center of gravity of each cross section. With such a handbook available, it is therefore not necessary actually to compute the position of the center of gravity of any section, as we did in the preceding article; but sometimes several shapes are riveted together to

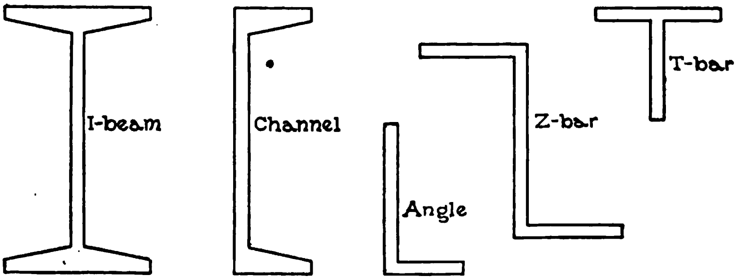


Fig. 24.

make a "built-up" section (see Fig. 25), and then it may be necessary to compute the position of the center of gravity of the section.

Example. It is desired to locate the center of gravity of the section of a built-up beam represented in Fig. 25. The beam consists of two channels and a plate, the area of the cross-section of a channel being 6.03 square inches.

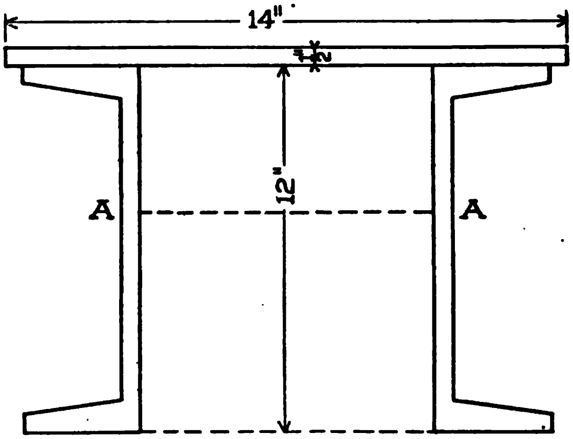


Fig. 25.

Evidently the center of gravity of each channel section is 6 inches, and that of the plate section is $12\frac{1}{4}$ inches, from the bottom. Let c denote the distance of the center of gravity of the whole section from the bottom; then since the area of the plate section is 7 square inches, and that of the whole section is 19.06,

$$19.06\times c = 6.03\times 6 + 6.03\times 6 + 7\times 12\tfrac{1}{4} = 158.11.$$

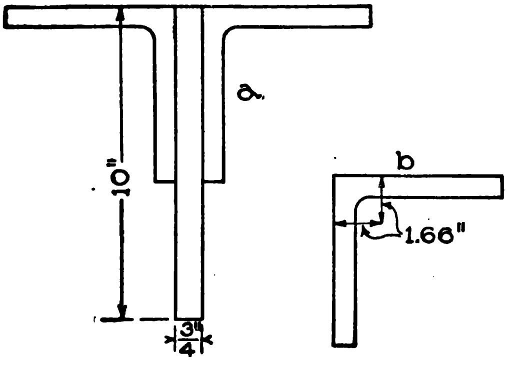


Fig. 26.

Hence, $c = 158.11 \div 19.06 = 8.30$ inches.

EXAMPLES FOR PRACTICE.

1. Locate the center of gravity of the built-up section of

Fig. 26, *a*, the area of each "angle" being 5.06 square inches, and the center of gravity of each being as shown in Fig. 26, *b*.

Ans. Distance from top, 3.08 inches.

2. Omit the left-hand angle in Fig. 26, *a*, and locate the center of gravity of the remainder.

Ans. { Distance from top, 3.65 inches,
" " left side, 1.19 inches.

50. Moment of Inertia. If a plane area be divided into an infinite number of infinitesimal parts, then the sum of the products obtained by multiplying the area of each part by the square of its distance from some line is called the *moment of inertia* of the area with respect to the line. The line to which the distances are measured is called the *inertia-axis;* it may be taken anywhere in the plane of the area. In the subject of beams (where we have sometimes to compute the moment of inertia of the cross-section of a beam), the inertia-axis is taken through the center of gravity of the section and horizontal.

An approximate value of the moment of inertia of an area can be obtained by dividing the area into small parts (not infinitesimal), and adding the products obtained by multiplying the area of each part by the square of the distance from its center to the inertia-axis.

Example. If the rectangle of Fig. 27, *a*, is divided into 8 parts as shown, the area of each is one square inch, and the distances from the axis to the centers of gravity of the parts are $\frac{1}{2}$ and $1\frac{1}{2}$ inches. For the four parts lying nearest the axis the product (area times distance squared) is:

$1\times(\frac{1}{2})^2=\frac{1}{4}$; and for the other parts it is
$1\times(1\frac{1}{2})^2=\frac{9}{4}$.

Hence the approximate value of the moment of inertia of the area with respect to the axis, is

$$4(\tfrac{1}{4})+4(\tfrac{9}{4})=10.$$

If the area is divided into 32 parts, as shown in Fig. 27, *b*, the area of each part is $\frac{1}{4}$ square inch. For the eight of the little squares farthest away from the axis, the distance from their centers of gravity to the axis is $1\frac{3}{4}$ inches; for the next eight it is $1\frac{1}{4}$; for the next eight $\frac{3}{4}$; and for the remainder $\frac{1}{4}$ inch. Hence an approximate value of the moment of inertia of the rectangle with respect to the axis is:

$$8\times\tfrac{1}{4}\times(1\tfrac{3}{4})^2+8\times\tfrac{1}{4}\times(1\tfrac{1}{4})^2+8\times\tfrac{1}{4}\times(\tfrac{3}{4})^2+8\times\tfrac{1}{4}\times(\tfrac{1}{4})^2=10\tfrac{1}{2}.$$

If we divide the rectangle into still smaller parts and form the products

$$(\text{small area})\times(\text{distance})^2,$$

and add the products just as we have done, we shall get a larger answer than $10\frac{1}{2}$. The smaller the parts into which the rectangle is divided, the larger will be the answer, but it will never be larger than $10\frac{2}{3}$. This $10\frac{2}{3}$ is the sum corresponding to a division of the rectangle into an infinitely large number of parts (infinitely small) and it is the exact value of the moment of inertia of the rectangle with respect to the axis selected.

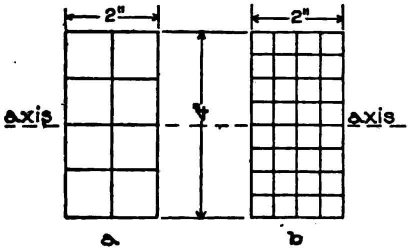


Fig. 27.

There are short methods of computing the exact values of the moments of inertia of simple figures (rectangles, circles, etc.,), but they cannot be given here since they involve the use of difficult mathematics. The foregoing method to obtain approximate values of moments of inertia is used especially when the area is quite irregular in shape, but it is given here to explain to the student the *meaning* of the moment of inertia of an area. He should understand now that the moment of inertia of an area is simply a name for such sums as we have just computed. The name is not a fitting one, since the sum has nothing whatever to do with inertia. It was first used in this connection because the sum is very similar to certain other sums which had previously been called moments of inertia.

51. Unit of Moment of Inertia. The product (area×distance^2) is really the product of four lengths, two in each factor; and since a moment of inertia is the sum of such products, a moment of inertia is also the product of four lengths. Now the product of two lengths is an area, the product of three is a volume, and the product of four is moment of inertia—unthinkable in the way in which we can think of an area or volume, and therefore the source of much difficulty to the student. The units of these quantities (area, volume, and moment of inertia) are respectively:

the square inch, square foot, etc.,
" cubic " , cubic " " ,
" biquadratic inch, biquadratic foot, etc.;

but the biquadratic inch is almost exclusively used in this connection; that is, the inch is used to compute values of moments of inertia, as in the preceding illustration. It is often written thus: Inches^4.

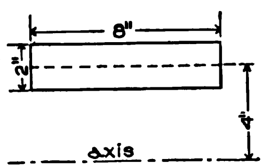


Fig. 23.

52. Moment of Inertia of a Rectangle. Let b denote the base of a rectangle, and a its altitude; then by higher mathematics it can be shown that the moment of inertia of the rectangle with respect to a line through its center of gravity and parallel to its base, is $\frac{1}{12} ba^3$.

Example. Compute the value of the moment of inertia of a rectangle 4×12 inches with respect to a line through its center of gravity and parallel to the long side.

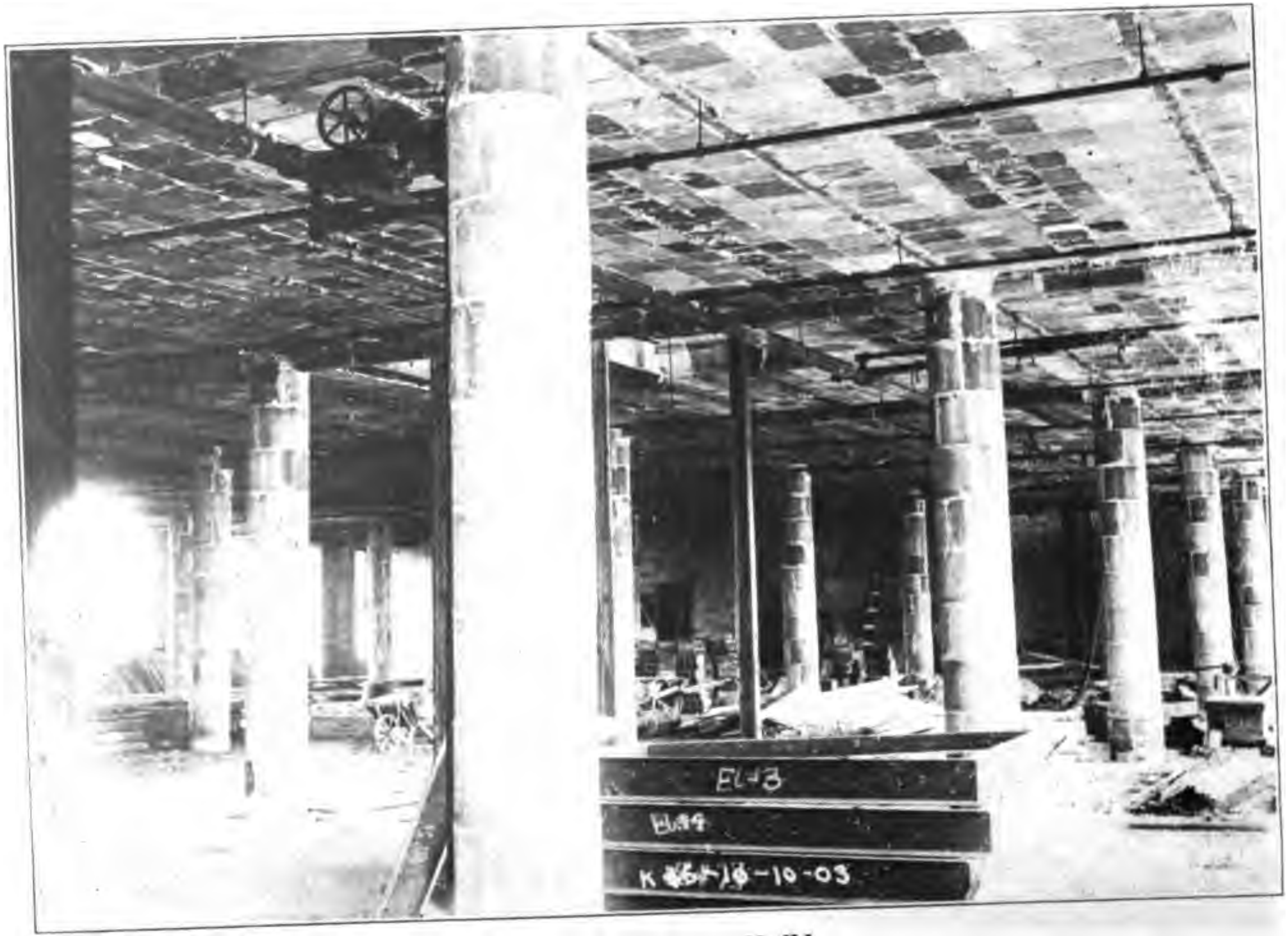
Here $b = 12$, and $a = 4$ inches ; hence the moment of inertia desired equals

$$\tfrac{1}{12}(12 \times 4^3) = 64 \text{ inches}^4.$$

EXAMPLE FOR PRACTICE.

1. Compute the moment of inertia of a rectangle 4×12 inches with respect to a line through its center of gravity and parallel to the short side. Ans. 576 inches^4.

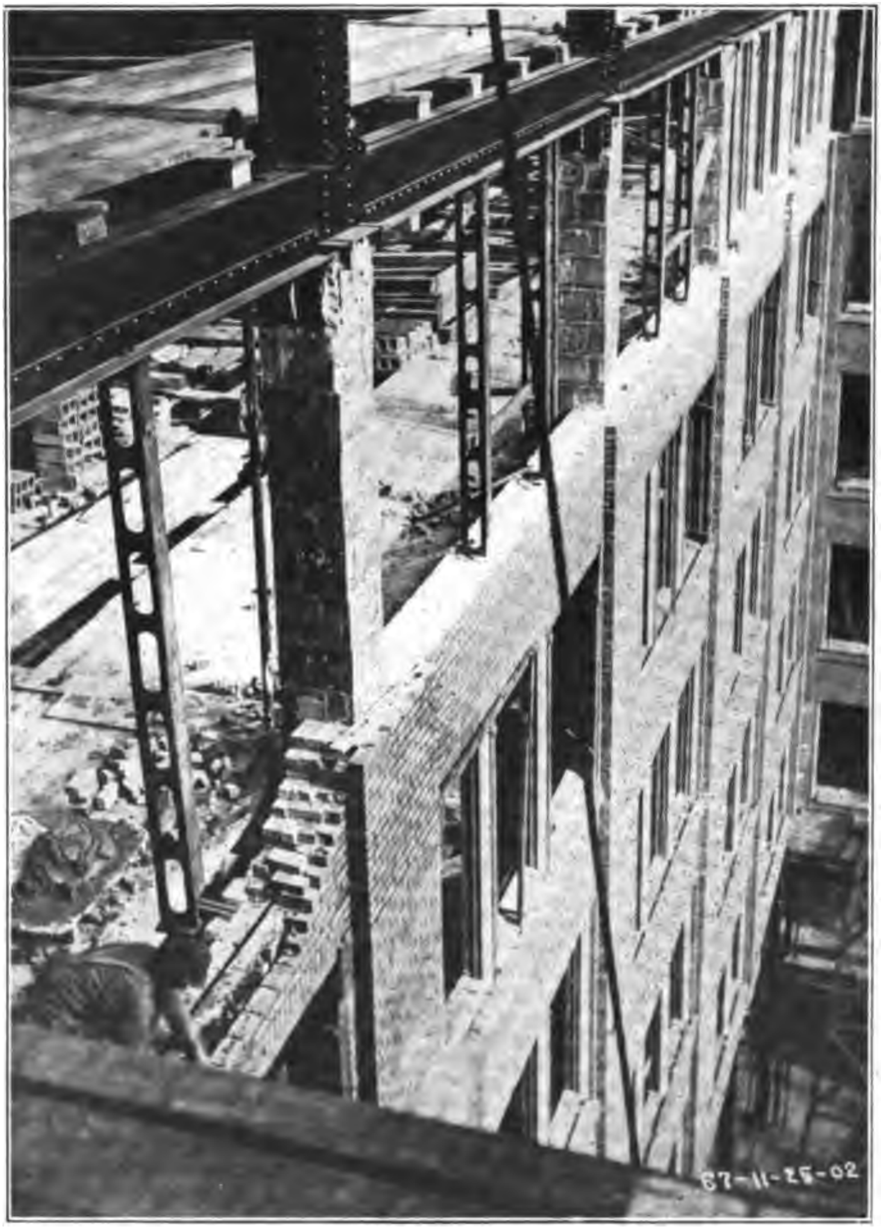
53. Reduction Formula. In the previously mentioned "handbooks" there can be found tables of moments of inertia of all the cross-sections of the kinds and sizes of rolled shapes made. The inertia-axes in those tables are always taken through the center of gravity of the section, and usually parallel to some edge of the section. Sometimes it is necessary to compute the moment of inertia of a "rolled section" with respect to some other axis, and if the two axes (that is, the one given in the tables, and the other) are parallel, then the desired moment of inertia can be easily computed from the one given in the tables by the following rule:



KENT BUILDING, CHICAGO, ILL.

Pond & Pond, Architects; E. C. & R. M. Shankland, Engineers.

This Picture Shows a Method of Good Fireproof Construction, of Burnt Tile. View Taken Just before Plastering was Done. Pipes Along Ceiling are Part of Sprinkler System; this is Used in Warehouses to Put Out Incipient Fires.



LA SALLE STATION, L. S. & M. S. AND C. R. I. & P. RAILROADS, CHICAGO, ILL.

Frost & Granger, Architects; E. C. & R. M. Shankland, Engineers.

Steel Frame Showing Method of Attaching Angle-Iron Shelf to Spandrel Beam. This Occurs at Each Floor Line, and Supports the Brick Walls. Note the Cast-Iron Supports in Window Mullions to Stiffen the Window Opening. Sometimes these are Made of Steel. They are Fastened to the Floor Beams at Top and Bottom.

The moment of inertia of an area with respect to any axis equals the moment of inertia with respect to a parallel axis through the center of gravity, plus the product of the area and the square of the distance between the axes.

Or, if I denotes the moment of inertia with respect to any axis; I_0 the moment of inertia with respect to a parallel axis through the center of gravity; A the area; and d the distance between the axes, then

$$I = I_0 + Ad^2 \ldots \qquad (5)$$

Example. It is required to compute the moment of inertia of a rectangle 2×8 inches with respect to a line parallel to the long side and 4 inches from the center of gravity.

Let I denote the moment of inertia sought, and I_0 the moment of inertia of the rectangle with respect to a line parallel to the long side and through the center of gravity (see Fig. 28). Then

$I_0 = \frac{1}{12} ba^3$ (see Art. 52); and,

since $b = 8$ inches and $a = 2$ inches,

$I_0 = \frac{1}{12}(8\times 2^3) = 5\frac{1}{3}$ biquadratic inches.

The distance between the two inertia-axes is 4 inches, and the area of the rectangle is 16 square inches, hence equation 5 becomes

$$I = 5\tfrac{1}{3} + 16\times 4^2 = 261\tfrac{1}{3} \text{ biquadratic inches.}$$

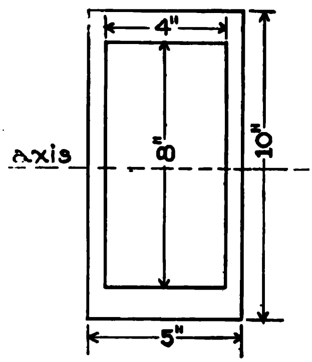


Fig. 29.

EXAMPLE FOR PRACTICE.

1. The moment of inertia of an "angle" $2\frac{1}{2}\times 2\times \frac{1}{2}$ inches (lengths of sides and width respectively) with respect to a line through the center of gravity and parallel to the long side, is 0.64 inches^4. The area of the section is 2 square inches, and the distance from the center of gravity to the long side is 0.63 inches. (These values are taken from a "handbook".) It is required to compute the moment of inertia of the section with respect to a line parallel to the long side and 4 inches from the center of gravity. Ans. 32.64 inches^4.

54. Moment of Inertia of Built-up Sections. As before stated, beams are sometimes "built up" of rolled shapes (angles, channels, etc.). The moment of inertia of such a section with respect to a definite axis is computed by adding the moments of inertia of the parts, *all with respect to that same axis.* This is the method for computing the moment of any area which can be divided into simple parts.

The moment of inertia of an area which may be regarded as consisting of a larger area *minus* other areas, is computed by subtracting from the moment of inertia of the large area those of the "minus areas."

Examples. 1. Compute the moment of inertia of the built-up section represented in Fig. 30 (in part same as Fig. 25) with respect to a horizontal axis passing through the center of gravity, it being given that the moment of inertia of each channel section with respect to a horizontal axis through its center of gravity is 128.1 inches^4, and its area 6.03 square inches.

The center of gravity of the whole section was found in the example of Art. 49 to be 8.30 inches from the bottom of the section; hence the distances from the inertia-axis to the centers of gravity of the channel section and the plate are 2.30 and 3.95 inches respectively (see Fig. 30).

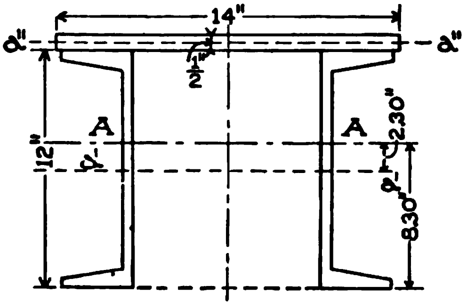


Fig. 30.

The moment of inertia of one channel section with respect to the axis AA (see equation 5, Art. 53) is:

$$128.1+6.03\times 2.30^2=160.00 \text{ inches}^4.$$

The moment of inertia of the plate section (rectangle) with respect to the line $a''a''$ (see Art. 52) is:

$$\tfrac{1}{12}\, ba^3=\tfrac{1}{12}[14\times (\tfrac{1}{2})^3]=0.15 \text{ inches}^4;$$

and with respect to the axis AA (the area being 7 square inches) it is:

$$0.15+7\times 3.95^2=109.37 \text{ inches}^4.$$

Therefore the moment of inertia of the whole section with respect to AA is:

$$2\times 160.00+109.37=429.37 \text{ inches}^4.$$

2. It is required to compute the moment of inertia of the "hollow rectangle" of Fig. 29 with respect to a line through the center of gravity and parallel to the short side.

The amount of inertia of the large rectangle with respect to the named axis (see Art. 52) is:

$$\tfrac{1}{12}(5 \times 10^3) = 416\tfrac{2}{3};$$

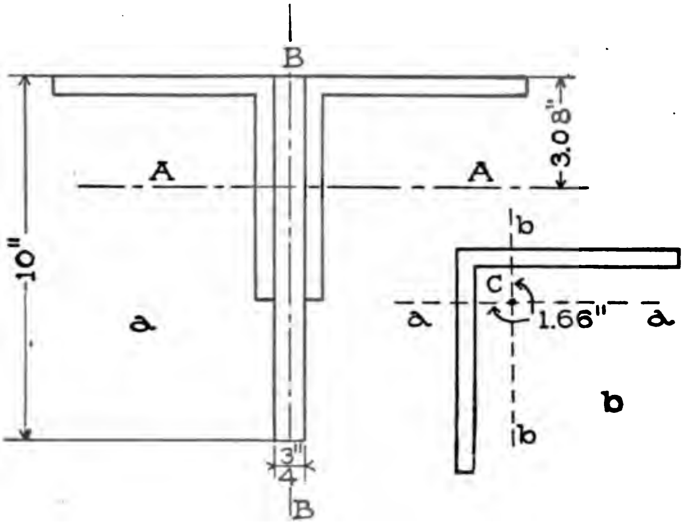


Fig. 31.

and the moment of inertia of the smaller one with respect to the same axis is:

$$\tfrac{1}{12}(4 \times 8^3) = 170\tfrac{2}{3};$$

hence the moment of inertia of the hollow section with respect to the axis is:

$$416\tfrac{2}{3} - 170\tfrac{2}{3} = 246 \text{ inches}^4.$$

EXAMPLES FOR PRACTICE.

1. Compute the moment of inertia of the section represented in Fig. 31, *a*, about the axis AA, it being 3.08 inches from the top. Given also that the area of one angle section is 5.06 square inches, its center of gravity C (Fig. 31, *b*) 1.66 inches from the top, and its moment of inertia with respect to the axis *aa* 17.68 inches^4. Ans. 145.8 inches^4.

2. Compute the moment of inertia of the section of Fig. 31, *a*,

with respect to the axis BB. Given that distance of the center of gravity of one angle from one side is 1.66 inches (see Fig. 31,*b*), and its moment of inertia with respect to *bb* 17.68 inches.

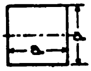
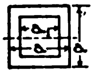
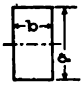
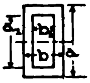
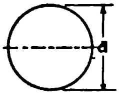
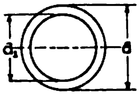
Ans. 77.5 inches^4.

55. Table of Centers of Gravity and Moments of Inertia. Column 2 in Table A below gives the formula for moment of inertia with respect to the horizontal line through the center of gravity. The numbers in the third column are explained in Art. 62; and those in the fourth, in Art. 80.

TABLE A.

Moments of Inertia, Section Moduli, and Radii of Gyration.

In each case the axis is horizontal and passes through the center of gravity.

Section.	Moment of Inertia.	Section Modulus.	Radius of Gyration.
	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}}$
	$\frac{a^4 - a_1^4}{12}$	$\frac{a^4 - a_1^4}{6a}$	$\sqrt{\frac{a^2 + a_1^2}{12}}$
	$\frac{ba^3}{12}$	$\frac{ba^2}{6}$	$\frac{a}{\sqrt{12}}$
	$\frac{ba^3 - b_1a_1^3}{12}$	$\frac{ba^3 - b_1a_1^3}{6a}$	$\sqrt{\frac{ba^3 - b_1a_1^3}{12\,(ba - b_1a_1)}}$
	$0.049d^4$	$0.098d^3$	$\frac{d}{4}$
	$0.049\,(d^4 - d_1^4)$	$0.098\frac{d^4 - d_1^4}{d}$	$\frac{\sqrt{d^2 + d_1^2}}{4}$

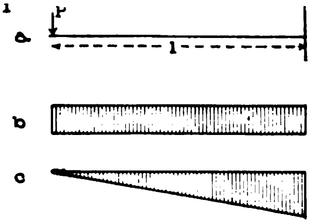
STRENGTH OF BEAMS.

56. Kinds of Loads Considered. The loads that are applied to a horizontal beam are usually vertical, but sometimes forces are applied otherwise than at right angles to beams. Forces acting on beams at right angles are called **transverse forces**; those applied

TABLE B.

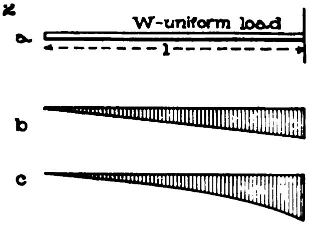
Shear Diagrams (b) and Moment Diagrams (c) for Eight Different Cases (a). Also Values of Maximum Shear (V), Bending Moment (M), and Deflection (d).

1



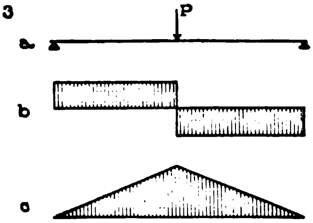
$V=P,\ M=Pl,\ d=Pl^3\div 3EI.$

2



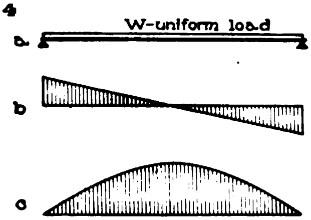
$V=W,\ M=\frac{1}{2}Wl,\ d=Wl^3\div 8EI.$

3



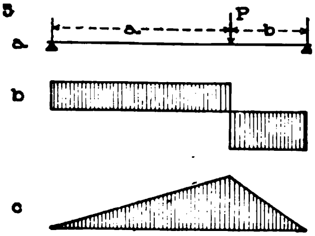
$V=\frac{1}{2}P,\ M=\frac{1}{4}Pl,\ d=Wl^3\div 48EI..$

4



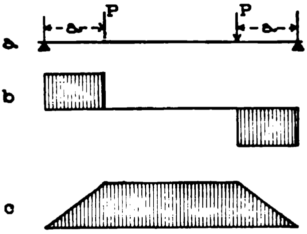
$V=\frac{1}{2}W,\ M=\frac{1}{8}Wl,\ d=5Wl^3\div 384EI.$

5



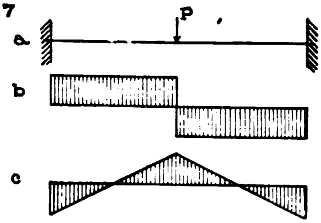
$V=Pa\div l,\quad M=Pab\div l.$

6



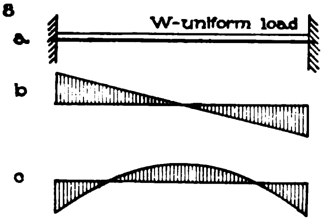
$V=P,\ M=Pa,\ d=Pa\ (3l^2-4a^2)\div 24EI.$

7



$V=\frac{1}{2}P,\quad M=\frac{1}{8}Pl,\quad d=Pl^3\div 192EI.$

8



$V=\frac{1}{2}W,l\ \ M=\frac{1}{12}Wl,\quad d=Wl^3\div 384EI.$

parallel to a beam are called **longitudinal forces**; and others are called **inclined forces.** For the present we deal only with beams subjected to transverse forces (loads and reactions).

57. Neutral Surface, Neutral Line, and Neutral Axis. When a beam is loaded it may be wholly convex up (concave down), as a cantilever; wholly convex down (concave up), as a simple beam on end supports; or partly convex up and partly convex down, as a simple beam with overhanging ends, a restrained beam, or a continuous beam. Two vertical parallel lines drawn close together on the side of a beam before it is loaded will not be parallel after it is loaded and bent. If they are on a convex-down portion of a beam, they will be closer at the top and farther apart below than when drawn (Fig. 32*a*), and if they are on a convex-up portion, they will be closer below and farther apart above than when drawn (Fig. 32*b*).

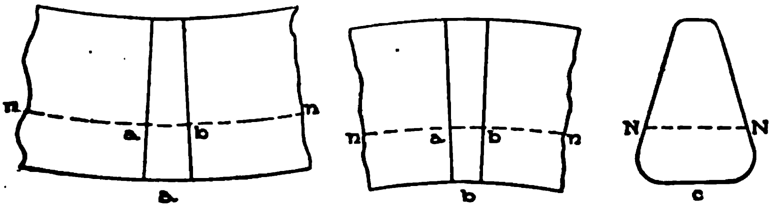


Fig. 32.

The "fibres" on the convex side of a beam are stretched and therefore under tension, while those on the concave side are shortened and therefore under compression. Obviously there must be some intermediate fibres which are neither stretched nor shortened, *i. e.*, under neither tension nor compression. These make up a sheet of fibres and define a surface in the beam, which surface is called the **neutral surface** of the beam. The intersection of the neutral surface with either side of the beam is called the **neutral line,** and its intersection with any cross-section of the beam is called the **neutral axis** of that section. Thus, if *ab* is a fibre that has been neither lengthened nor shortened with the bending of the beam, then *nn* is a portion of the neutral line of the beam; and, if Fig. 32*c* be taken to represent a cross-section of the beam, NN is the neutral axis of the section.

It can be proved that *the neutral axis of any cross-section of* *a loaded beam passes through the center of gravity of that section,* provided that all the forces applied to the beam are transverse, and that the tensile and compressive stresses at the cross-section are all within the elastic limit of the material of the beam.

58. Kinds of Stress at a Cross-section of a Beam. It has already been explained in the preceding article that there are tensile and compressive stresses in a beam, and that the tensions are on the convex side of the beam and the compressions on the concave (see Fig. 33). The forces T and C are exerted upon the portion of the beam represented by the adjoining portion to the right (not shown). These, the student is reminded, are often called **fibre stresses.**

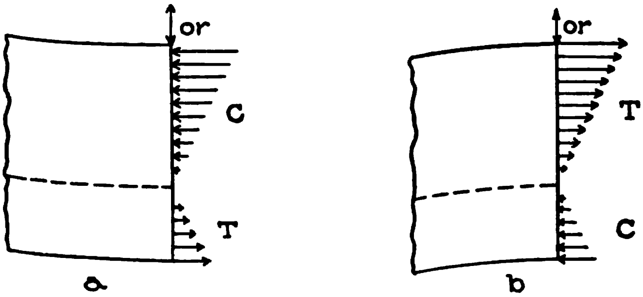


Fig. 33.

Besides the fibre stresses there is, in general, a shearing stress at every cross-section of a beam. This may be proved as follows:

Fig. 34 represents a simple beam on end supports which has actually been cut into two parts as shown. The two parts can maintain loads when in a horizontal position, if forces are applied at the cut ends equivalent to the forces that would act there if the beam were not cut. Evidently in the solid beam there are at the section a compression above and a tension below, and such forces can be applied in the cut beam by means of a short block C and a chain or cord T, as shown. The block furnishes the compressive forces and the chain the tensile forces. At first sight it appears as if the beam would stand up under its load after the block and chain have been put into place. Except in certain cases*, however, it would not remain in a horizontal position, as would the solid beam. This shows that the forces exerted by the block and chain (horizontal compression and tension) are not equivalent to the actual stresses in the solid beam. What is needed is a vertical force at each cut end.

---

* When the external shear for the section is zero.

Suppose that R_1 is less than $L_1 + L_2 +$ weight of A, *i. e.*, that the external shear for the section is negative; then, if vertical pulls be applied at the cut ends, upward on A and downward on B, the beam will stand under its load and in a horizontal position, just as a solid beam. These pulls can be supplied, in the model of the beam, by means of a cord S tied to two brackets fastened on A and B, as shown. In the solid beam the two parts act upon each other directly, and the vertical forces are shearing stresses, since they act in the plane of the surfaces to which they are applied.

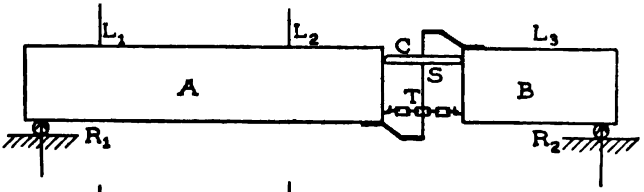


Fig. 34.

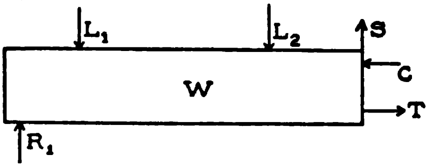


Fig. 35.

59. Relation Between the Stress at a Section and the Loads and Reactions on Either Side of It. Let Fig. 35 represent the portion of a beam on the left of a section; and let R_1 denote the left reaction; L_1 and L_2 the loads; W the weight of the left part; C, T, and S the compression, tension, and shear respectively which the right part exerts upon the left.

Since the part of the beam here represented is at rest, all the forces exerted upon it are balanced; and when a number of horizontal and vertical forces are balanced, then

1. The algebraic sum of the horizontal forces equals zero.
2. " " " " " vertical " " "
3. " " " " " moments of all the forces with respect to any point equals zero.

To satisfy condition 1, since the tension and compression are the only horizontal forces, *the tension must equal the compression.*

To satisfy condition 2, S (the internal shear) must equal the algebraic sum of all the other vertical forces on the portion, that is, must equal the external shear for the section; also, S must act up or down according as the external shear is negative or positive. In other words, briefly expressed, *the internal and external shears at a section are equal and opposite.*

To satisfy condition 3, the algebraic sum of the moments of the fibre stresses about the neutral axis must be equal to the sum of the moments of all the other forces acting on the portion about the same line, and the signs of those sums must be opposite. (The moment of the shear about the neutral axis is zero.) Now, the sum of the moments of the loads and reactions is called the bending moment at the section, and if we use the term **resisting moment** to signify the sum of the moments of the fibre stresses (tensions and compressions) about the neutral axis, then we may say briefly that *the resisting and the bending moments at a section are equal, and the two moments are opposite in sign.*

60. The Fibre Stress. As before stated, the fibre stress is not a uniform one, that is, it is not uniformly distributed over the section on which it acts. At any section, the compression is most "intense" (or the unit-compressive stress is greatest) on the concave side; the tension is most intense (or the unit-tensile stress is greatest) on the convex side; and the unit-compressive and unit-tensile stresses decrease toward the neutral axis, at which place the unit-fibre stress is zero.

If the fibre stresses are within the elastic limit, then the two straight lines on the side of a beam referred to in Art. 57 will still be straight after the beam is bent; hence the elongations and shortenings of the fibres vary directly as their distance from the neutral axis. Since the stresses (if within the elastic limit) and deformations in a given material are proportional, *the unit-fibre stress varies as the distance from the neutral axis.*

Let Fig. 36*a* represent a portion of a bent beam, 36*b* its cross-section, *nn* the neutral line, and NN the neutral axis. The way in which the unit-fibre stress on the section varies can be represented graphically as follows: Lay off *ac*, by some scale, to represent the unit-fibre stress in the top fibre, and join *c* and *n*, extending the line to the lower side of the beam; also make *bc′* equ to *bc″* and draw *nc′*. Then the arrows represent the unit-fibre stresses, for their lengths vary as their distances from the neutral axis.

61. Value of the Resisting Moment. If S denotes the unit-fibre stress in the fibre farthest from the neutral axis (the greatest unit-fibre stress on the cross-section), and c the distance from the neutral axis to the remotest fibre, while S_1, S_2, S_3, etc., denote the unit-fibre stresses at points whose distances from the neutral axis are, respectively, y_1, y_2, y_3, etc. (see Fig. 36*b*), then

$$S : S_1 :: c : y_1; \text{ or } S_1 = \frac{S}{c} y_1.$$

Also, $$S_2 = \frac{S}{c} y_2; \; S_3 = \frac{S}{c} y_3, \text{ etc.}$$

Let a_1, a_2, a_3, etc., be the areas of the cross-sections of the fibres

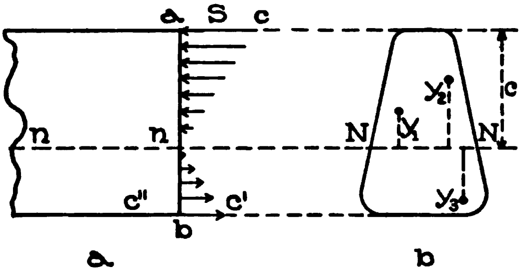


Fig. 36.

whose distances from the neutral axis are, respectively, y_1, y_2, y_3, etc. Then the stresses on those fibres are, respectively,

$$S_1 a_1, \; S_2 a_2, \; S_3 a_3, \text{ etc.};$$

or, $$\frac{S}{c} y_1 a_1, \; \frac{S}{c} y_2 a_2, \; \frac{S}{c} y_3 a_3, \text{ etc.}$$

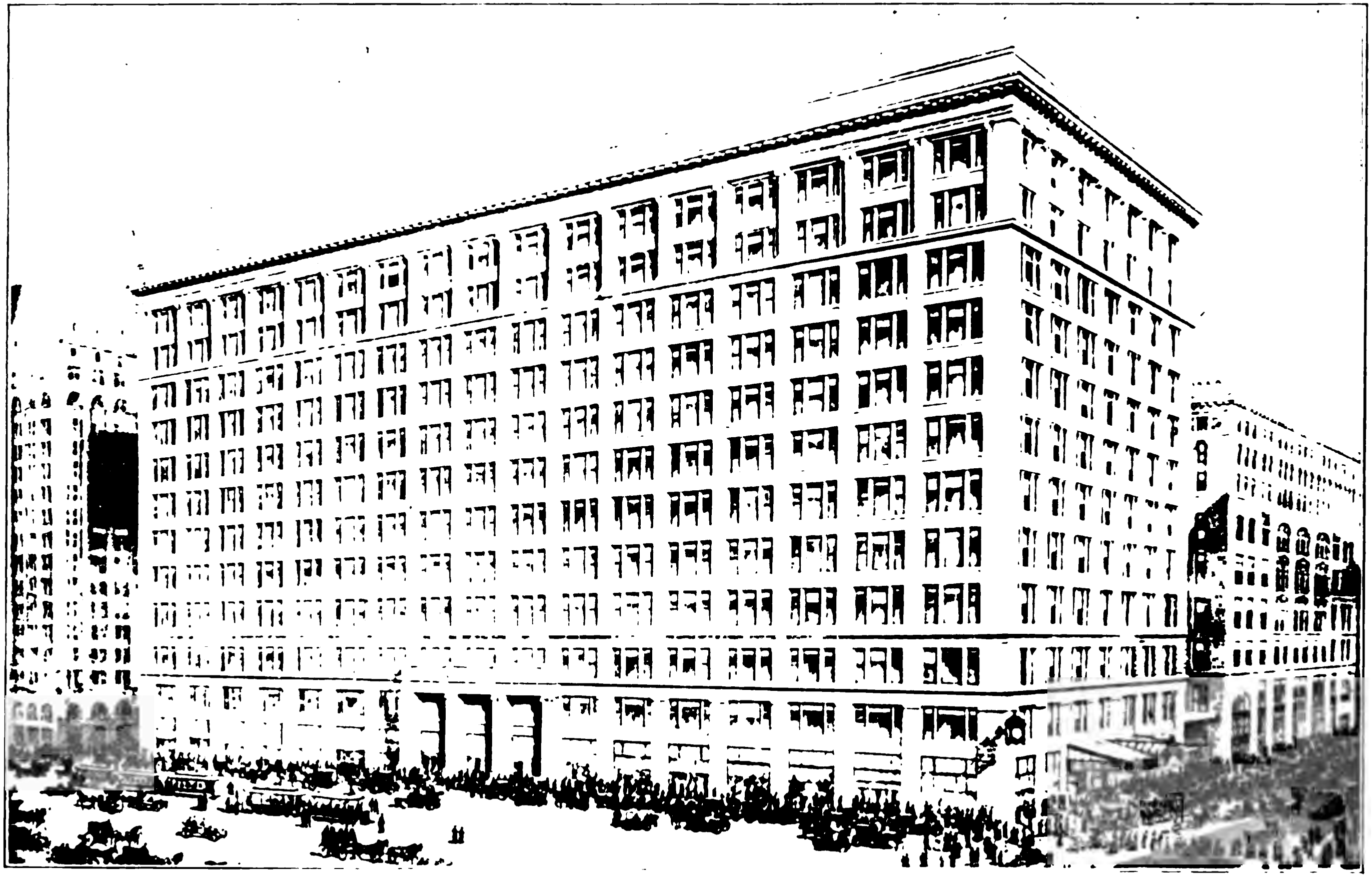
The arms of these forces or stresses with respect to the neutral axis are, respectively, y_1, y_2, y_3, etc.; hence their moments are

$$\frac{S}{c} a_1 y_1^2, \; \frac{S}{c} a_2 y_2^2, \; \frac{S}{c} a_3 y_3^2, \text{ etc.},$$

and the sum of the moments (that is, the resisting moment) is

$$\frac{S}{c} a_1 y_1^2 + \frac{S}{c} a_2 y_2^2 + \text{etc.} = \frac{S}{c} (a_1 y_1^2 + a_2 y_2^2 + \text{etc.})$$

Now $a_1 y_1^2 + a_2 y_2^2 + \text{etc.}$ is the sum of the products obtained by multiplying each infinitesimal part of the area of the cross-section by the square of its distance from the neutral axis; hence, it is the moment of inertia of the cross-section with respect to the neutral axis. If this moment is denoted by I, then the value of the resisting moment is $\frac{SI}{c}$.



RETAIL STORE OF MARSHALL FIELD & CO., CHICAGO, ILL.

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STRENGTH OF MATERIALS.

PART II.

STRENGTH OF BEAMS---(Concluded).

62. First Beam Formula. As shown in the preceding article, the resisting and bending moments for any section of a beam are equal; hence

$$\frac{SI}{c} = M, \qquad (6)$$

all the symbols referring to the same section. This is the most important formula relating to beams, and will be called the "first beam formula."

The ratio $I \div c$ is now quite generally called the **section modulus.** Observe that for a given beam it depends only on the dimensions of the cross-section, and not on the material or anything else. Since I is the product of four lengths (see article 51), $I \div c$ is the product of three; and hence a section modulus can be expressed in units of volume. The cubic inch is practically always used; and in this connection it is written thus, inches^3. See Table A, page 54, for values of the section moduli of a few simple sections.

63. Applications of the First Beam Formula. There are three principal applications of equation 6, which will now be explained and illustrated.

64. *First Application.* The dimensions of a beam and its manner of loading and support are given, and it is required to compute the greatest unit-tensile and compressive stresses in the beam.

This problem can be solved by means of equation 6, written in this form,

$$S = \frac{Mc}{I} \text{ or } \frac{M}{I \div c}. \qquad (6')$$

Unless otherwise stated, we assume that the beams are uniform in cross-section, as they usually are; then the section modulus $(I \div c)$ is the same for all sections, and S (the unit-fibre stress on the remotest fibre) varies just as M varies, and is therefore greatest where M is a maximum.* Hence, to compute the value of the greatest unit-fibre stress in a given case, *substitute the values of the section modulus and the maximum bending moment in the preceding equation, and reduce.*

If the neutral axis is equally distant from the highest and lowest fibres, then the greatest tensile and compressive unit-stresses are equal, and their value is S. If the neutral axis is unequally distant from the highest and lowest fibres, let c' denote its distance from the nearer of the two, and S′ the unit-fibre stress there. Then, since the unit-stresses in a cross-section are proportional to the distances from the neutral axis,

$$\frac{S'}{S} = \frac{c'}{c}, \text{ or } S' = \frac{c'}{c}S.$$

If the remotest fibre is on the convex side of the beam, S is tensile and S′ compressive; if the remotest fibre is on the concave side, S is compressive and S′ tensile.

Examples. 1. A beam 10 feet long is supported at its ends, and sustains a load of 4,000 pounds two feet from the left end (Fig. 37, *a*). If the beam is 4 × 12 inches in cross-section (the long side vertical as usual), compute the maximum tensile and compressive unit-stresses.

The section modulus of a rectangle whose base and altitude are b and a respectively (see Table A, page 54), is $\frac{1}{6}ba^2$; hence, for the beam under consideration, the modulus is

$$\frac{1}{6} \times 4 \times 12^2 = 96 \text{ inches}^3.$$

To compute the maximum bending moment, we have, first, to find the dangerous section. This section is where the shear changes sign (see article 45); hence, we have to construct the shear diagram, or as much thereof as is needed to find where the change of sign occurs. Therefore we need the values of the reaction. Neglecting the weight of the beam, the moment equation with origin at C (Fig. 37, *a*) is

$$R_1 \times 10 - 4{,}000 \times 8 = 0, \text{ or } R_1 = 3{,}200 \text{ pounds}$$

---

* NOTE. Because S is greatest in the section where M is maximum, this section is usually called the "dangerous section" of the beam.

Then, constructing the shear diagram, we see (Fig. 37, *b*) that the change of sign of the shear (also the dangerous section) is at the load. The value of the bending moment there is

$$3{,}200 \times 2 = 6{,}400 \text{ foot-pounds,}$$

or

$$6{,}400 \times 12 = 76{,}800 \text{ inch-pounds.}$$

Substituting in equation 6′, we find that

$$S = \frac{76{,}800}{96} = 800 \text{ pounds per square inch.}$$

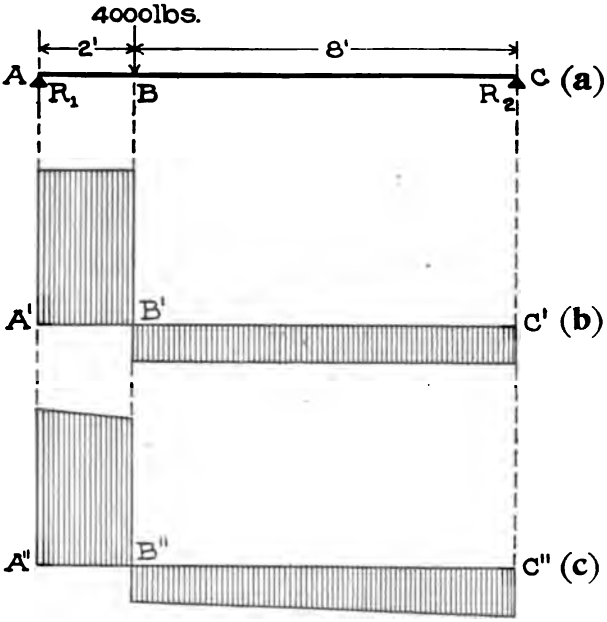


Fig. 37.

2. It is desired to take into account the weight of the beam in the preceding example, supposing the beam to be wooden.

The volume of the beam is

$$\frac{4 \times 12}{144} \times 10 = 3\tfrac{1}{3} \text{ cubic feet;}$$

and supposing the timber to weigh 45 pounds per cubic foot, the beam weighs 150 pounds (insignificant compared to the load). The left reaction, therefore, is

$$3{,}200 + (\frac{1}{2} \times 150) = 3{,}275;$$

and the shear diagram looks like Fig. 37, *c*, the shear changing sign at the load as before. The weight of the beam to the left of the dangerous section is 30 pounds; hence the maximum bending moment equals

$$3{,}275 \times 2 - 30 \times 1 = 6{,}520 \text{ foot-pounds,}$$

or

$$6{,}520 \times 12 = 78{,}240 \text{ inch-pounds.}$$

Substituting in equation (6′, we find that

$$S = \frac{78{,}240}{96} = 815 \text{ pounds per square inch.}$$

The weight of the beam therefore increases the unit-stress produced by the load at the dangerous section by 15 pounds per square inch.

3. A T-bar (see Fig. 38) 8 feet long and supported at each end, bears a uniform load of 1,200 pounds. The moment of inertia of its cross-section with respect to the neutral axis being 2.42 inches⁴, compute the maximum tensile and compressive unit-stresses in the beam

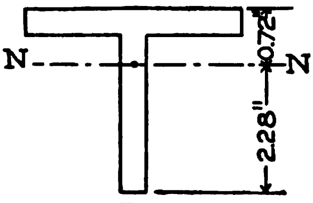


Fig. 38.

Evidently the dangerous section is in the middle, and the value of the maximum bending moment (see Table B, page 55, Part I) is $\frac{1}{8}$ W*l*, W and *l* denoting the load and length respectively. Here

$$\frac{1}{8} Wl = \frac{1}{8} \times 1{,}200 \times 8 = 1{,}200 \text{ foot-pounds,}$$

or

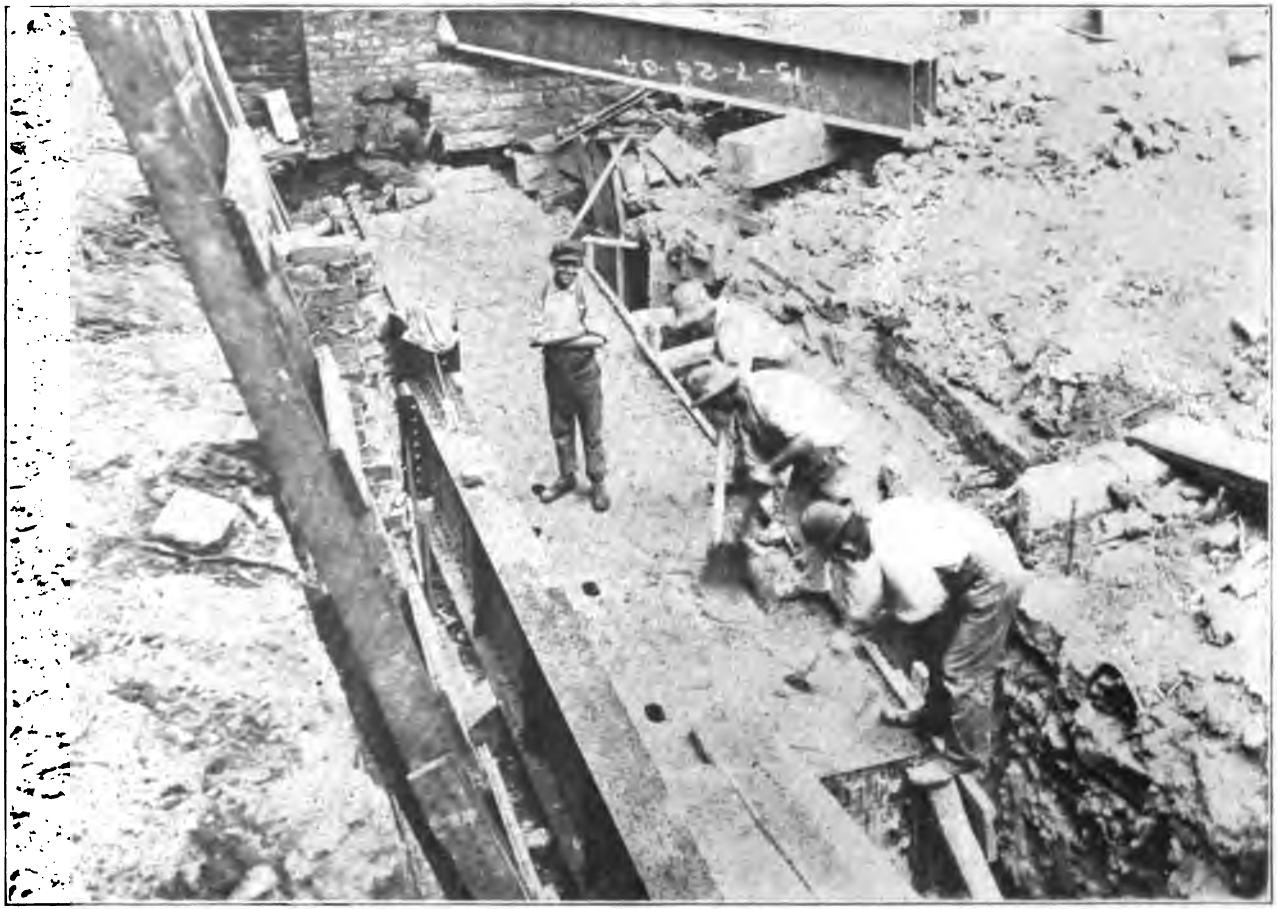
$$1{,}200 \times 12 = 14{,}400 \text{ inch-pounds.}$$

The section modulus equals 2.42 ÷ 2.28 = 1.06; hence

$$S = \frac{14{,}400}{1.06} = 13{,}585 \text{ pounds per square inch.}$$

This is the unit-fibre stress on the lowest fibre at the middle section, and hence is tensile. On the highest fibre at the middle section the unit-stress is compressive, and equals (see page 62):

$$S' = \frac{c'}{c} S = \frac{0.72}{2.28} \times 13{,}585 = 4{,}290 \text{ pounds per square inch.}$$



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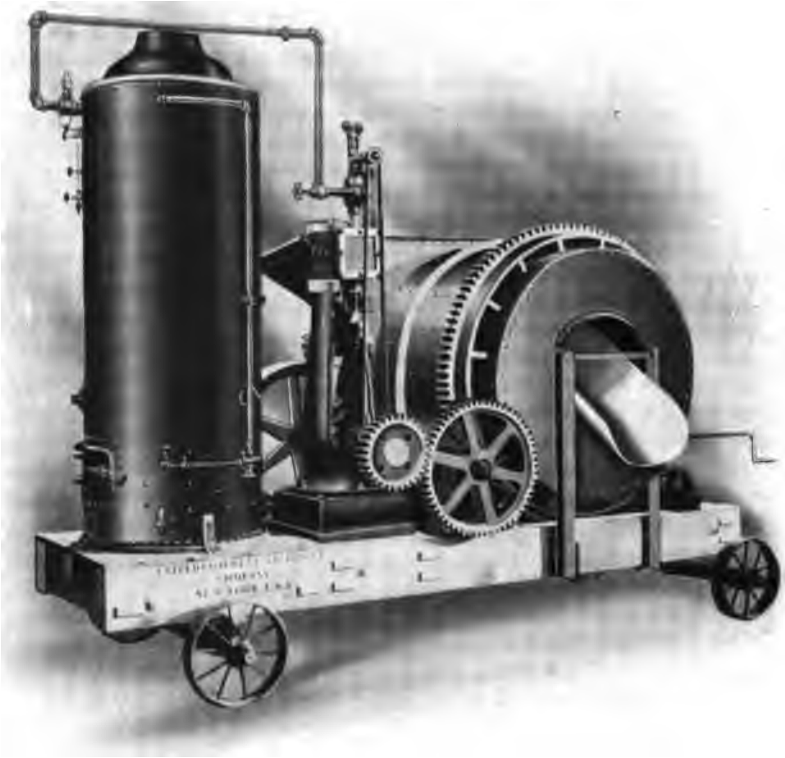
The 52,500-lb. Girder Shown in Vol. V, Page 262, being Grouted with Concrete. This is Done on Girders Put in the Ground, to Protect Same from Corrosion. The Outsides of the Girders are also Encased in Concrete for the Same Purpose.



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EXAMPLES FOR PRACTICE.

1. A beam 12 feet long and 6 × 12 inches in cross-section rests on end supports, and sustains a load of 3,000 pounds in the middle. Compute the greatest tensile and compressive unit-stresses in the beam, neglecting the weight of the beam.

Ans. 750 pounds per square inch.

2. Solve the preceding example taking into account the weight of the beam, 300 pounds

Ans. 787.5 pounds per square inch.

3. Suppose that a built-in cantilever projects 5 feet from the wall and sustains an end load of 250 pounds. The cross-section of the cantilever being represented in Fig. 38, compute the greatest tensile and compressive unit-stresses, and tell at what places they occur. (Neglect the weight.)

Ans. { Tensile, 4,469 pounds per square inch.
Compressive, 14,150 " " " "

4. Compute the greatest tensile and compressive unit-stresses in the beam of Fig. 18, *a*, due to the loads and the weight of beam (400 pounds). (A moment diagram is represented in Fig. 18, *b*; for description see example 2, Art. 44, p. 39.) The section of the beam is a rectangle 8 × 12 inches.

Ans. 580 pounds per square inch.

5. Compute the greatest tensile and compressive unit-stresses in the cantilever beam of Fig. 19, *a*, it being a steel I-beam whose section modulus is 20.4 inches^3. (A bending moment diagram for it is represented in Fig. 19, *b*; for description, see Ex. 3, Art. 44.)

Ans. 11,470 pounds per square inch.

6. Compute the greatest tensile and compressive unit-stresses in the beam of Fig. 10, neglecting its weight, the cross-sections being rectangular 6 × 12 inches. (See example for practice 1, Art. 43.)

Ans. 600 pounds per square inch.

65. *Second Application.* The dimensions and the working strengths of a beam are given, and it is required to determine its safe load (the manner of application being given).

This problem can be solved by means of equation 6 written in this form,

$$M = \frac{SI}{c} \qquad (6'')$$

We substitute for S the given working strength for the material of the beam, and for I and c their values as computed from the given dimensions of the cross-section; then reduce, thus obtaining the value of the safe resisting moment of the beam, which equals the greatest safe bending moment that the beam can stand. We next compute the value of the maximum bending moment in terms of the unknown load; equate this to the value of the resisting moment previously found; and solve for the unknown load.

In cast iron, the tensile and compressive strengths are very different; and the smaller (the tensile) should always be used if the neutral surface of the beam is midway between the top and bottom of the beam; but if it is unequally distant from the top and bottom, proceed as in example 4, following.

Examples. 1. A wooden beam 12 feet long and 6 × 12 inches in cross-section rests on end supports. If its working strength is 800 pounds per square inch, how large a load uniformly distributed can it sustain?

The section modulus is $\frac{1}{6}ba^2$, b and a denoting the base and altitude of the section (see Table A, page 54); and here

$$\frac{1}{6}ba^2 = \frac{1}{6} \times 6 \times 12^2. = 144 \text{ inches}^3.$$

Hence $$S\frac{I}{c} = 800 \times 144 = 115{,}200 \text{ inch-pounds.}$$

For a beam on end supports and sustaining a uniform load, the maximum bending moment equals $\frac{1}{8}Wl$ (see Table B, page 55), W denoting the sum of the load and weight of beam, and l the length. If W is expressed in pounds, then

$$\frac{1}{8}Wl = \frac{1}{8}W \times 12 \text{ foot-pounds} = \frac{1}{8}W \times 144 \text{ inch-pounds.}$$

Hence, equating the two values of maximum bending moment and the safe resisting moment, we get

$$\frac{1}{8}W \times 144 = 115{,}200;$$

or, $$W = \frac{115{,}200 \times 8}{144} = 6{,}400 \text{ pounds.}$$

The safe load for the beam is 6,400 pounds minus the weight of the beam.

2. A steel I-beam whose section modulus is 20.4 inches^3 rests on end supports 15 feet apart. Neglecting the weight of the beam, how large a load may be placed upon it 5 feet from one end, if the working strength is 16,000 pounds per square inch?

The safe resisting moment is

$$\frac{SI}{c} = 16{,}000 \times 20.4 = 326{,}400 \text{ inch-pounds};$$

hence the bending moment must not exceed that value. The dangerous section is under the load; and if P denotes the unknown value of the load in pounds, the maximum moment (see Table B, page 55, Part I) equals $\frac{2}{3}$ P × 5 foot-pounds, or $\frac{2}{3}$ P × 60 inch-pounds. Equating values of bending and resisting moments, we get

$$\frac{2}{3} P \times 60 = 326{,}400;$$

or,

$$P = \frac{326{,}400 \times 3}{2 \times 60} = 8{,}160 \text{ pounds.}$$

3. In the preceding example, it is required to take into account the weight of the beam, 375 pounds.

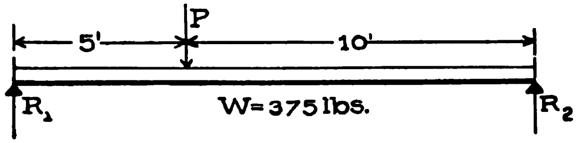


Fig. 39.

As we do not know the value of the safe load, we cannot construct the shear diagram and thus determine where the dangerous section is. But in cases like this, where the distributed load (the weight) is small compared with the concentrated load, the dangerous section is practically always where it is under the concentrated load alone; in this case, at the load. The reactions due to the weight equal $\frac{1}{2} \times 375 = 187.5$; and the reactions due to the load equal $\frac{1}{3}$ P and $\frac{2}{3}$ P, P denoting the value of the load. The larger reaction R_1 (Fig. 39) hence equals $\frac{2}{3}$ P + 187.5. Since the weight of the beam per foot is 375 ÷ 15 = 25 pounds, the maximum bending moment (at the load) equals

$$\left(\frac{2}{3}P + 187.5\right)5 - (25 \times 5)2\tfrac{1}{2} =$$

$$\frac{10}{3}P + 937.5 - 312.5 = \frac{10}{3}P + 625.$$

This is in foot-pounds if P is in pounds.

The safe resisting moment is the same as in the preceding illustration, 326,400 inch-pounds; hence

$$\left(\frac{10}{3}P + 625\right)12 = 326{,}400.$$

Solving for P, we have

$$\frac{10}{3}P + 625 = \frac{326{,}400}{12};$$

$$10\,P + 625 \times 3 = \frac{326{,}400 \times 3}{12} = 81{,}600;$$

$$10\,P = 79{,}725;$$

or, $$P = 7{,}972.5 \text{ pounds.}$$

It remains to test our assumption that the dangerous section is at the load. This can be done by computing R_1 (with P = 7,972.5), constructing the shear diagram, and noting where the shear changes sign. It will be found that the shear changes sign at the load, thus verifying the assumption.

4. A cast-iron built-in cantilever beam projects 8 feet from the wall. Its cross-section is represented in Fig. 40, and the moment of inertia with respect to the neutral axis is 50 inches^4; the working strengths in tension and compression are 2,000 and 9,000 pounds per square inch respectively. Compute the safe uniform load which the beam can sustain, neglecting the weight of the beam.

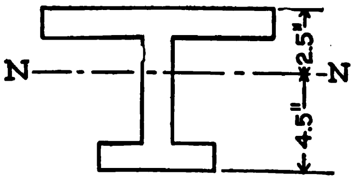


Fig. 40.

The beam being convex up, the upper fibres are in tension and the lower in compression. The resisting moment (SI ÷ *c*), as determined by the compressive strength, is

$$\frac{9{,}000 \times 50}{4.5} = 100{,}000 \text{ inch-pounds};$$

and the resisting moment, as determined by the tensile strength, is

$$\frac{2{,}000 \times 50}{2.5} = 40.000 \text{ inch-pounds}.$$

Hence the safe resisting moment is the lesser of these two, or 40,000 inch-pounds. The dangerous section is at the wall (see Table B, page 55), and the value of the maximum bending moment is $\frac{1}{2}$ Wl, W denoting the load and l the length. If W is in pounds, then

$$M = \tfrac{1}{2} W \times 8 \text{ foot-pounds} = \tfrac{1}{2} W \times 96 \text{ inch-pounds}.$$

Equating bending and resisting moments, we have

$$\frac{1}{2} W \times 96 = 40{,}000;$$

or, $$W = \frac{40{,}000 \times 2}{96} = 833 \text{ pounds}.$$

EXAMPLES FOR PRACTICE.

1. An 8 × 8-inch timber projects 8 feet from a wall. If its working strength is 1,000 pounds per square inch, how large an end load can it safely sustain?

Ans. 890 pounds.

2. A beam 12 feet long and 8 × 16 inches in cross-section, on end supports, sustains two loads P, each 3 feet from its ends respectively. The working strength being 1,000 pounds per square inch, compute P (see Table B, page 55).

Ans. 9,480 pounds.

3. An I-beam weighing 25 pounds per foot rests on end supports 20 feet apart. Its section modulus is 20.4 inches^3, and its working strength 16,000 pounds per square inch. Compute the safe uniform load which it can sustain.

Ans. 10,380 pounds.

66. *Third Application.* The loads, manner of support, and working strength of beam are given, and it is required to determine the size of cross-section necessary to sustain the load safely, that is, to "design the beam."

To solve this problem, we use the first beam formula (equation 6), written in this form,

$$\frac{I}{c} = \frac{M}{S}. \qquad (6''')$$

We first determine the maximum bending moment, and then substitute its value for M, and the working strength for S. Then we have the value of the section modulus ($I \div c$) of the required beam. Many cross-sections can be designed, all having a given section modulus. Which one is to be selected as most suitable will depend on the circumstances attending the use of the beam and on considerations of economy.

Examples. 1. A timber beam is to be used for sustaining a uniform load of 1,500 pounds, the distance between the supports being 20 feet. If the working strength of the timber is 1,000 pounds per square inch, what is the necessary size of cross-section ?

The dangerous section is at the middle of the beam; and the maximum bending moment (see Table B, page 55) is

$$\frac{1}{8}Wl = \frac{1}{8} \times 1{,}500 \times 20 = 3{,}750 \text{ foot-pounds,}$$

or $$3{,}750 \times 12 = 45{,}000 \text{ inch-pounds.}$$

Hence $$\frac{I}{c} = \frac{45{,}000}{1{,}000} = 45 \text{ inches}^3.$$

Now the section modulus of a rectangle is $\frac{1}{6}ba^2$ (see Table A, page 54, Part I); therefore, $\frac{1}{6}ba^2 = 45$, or $ba^2 = 270$.

Any wooden beam (safe strength 1,000 pounds per square inch) whose breadth times its depth square equals or exceeds 270, is strong enough to sustain the load specified, 1,500 pounds.

To determine a size, we may choose any value for b or a, and solve the last equation for the unknown dimension. It is best, however, to select a value of the breadth, as 1, 2, 3, or 4 inches, and solve for a. Thus, if we try $b = 1$ inch, we have

$$a^2 = 270, \text{ or } a = 16.43 \text{ inches.}$$

This would mean a board 1 × 18 inches, which, if used, would have to be supported sidewise so as to prevent it from tipping or "buckling." Ordinarily, this would not be a good size.

Next try $b = 2$ inches; we have

$$2 \times a^2 = 270; \text{ or } a = \sqrt{270 \div 2} = 11.62 \text{ inches.}$$

This would require a plank 2 × 12, a better proportion than the first. Trying $b = 3$ inches, we have

$$3 \times a^2 = 270; \text{ or } a = \sqrt{270 \div 3} = 9.49 \text{ inches.}$$

This would require a plank 3 × 10 inches; and a choice between a 2 × 12 and a 3 × 10 plank would be governed by circumstances in the case of an actual construction.

It will be noticed that we have neglected the weight of the beam. Since the dimensions of wooden beams are not fractional, and we have to select a commercial size next larger than the one computed (12 inches instead of 11.62 inches, for example), the additional depth is usually sufficient to provide strength for the weight of the beam. If there is any doubt in the matter, we can settle it by computing the maximum bending moment including the weight of the beam, and then computing the greatest unit-fibre stress due to load and weight. If this is less than the safe strength, the section is large enough; if greater, the section is too small.

Thus, let us determine whether the 2 × 12-inch plank is strong enough to sustain the load and its own weight. The plank will weigh about 120 pounds, making a total load of

$$1{,}500 + 120 = 1{,}620 \text{ pounds.}$$

Hence the maximum bending moment is

$$\frac{1}{8}\mathrm{W}l = \frac{1}{8}1{,}620 \times 20 \times 12 = 48{,}600 \text{ inch-pounds.}$$

Since $\frac{\mathrm{I}}{c} = \frac{1}{6}ba^2 = \frac{1}{6} \times 2 \times 12^2 = 48$, and $\mathrm{S} = \frac{\mathrm{M}}{\mathrm{I} \div c}$,

$$\mathrm{S} = \frac{48{,}600}{48} = 1{,}013 \text{ pounds per square inch.}$$

Strictly, therefore, the 2 × 12-inch plank is not large enough; but as the greatest unit-stress in it would be only 13 pounds per square inch too large, its use would be permissible.

2. What size of steel I-beam is needed to sustain safely the loading of Fig. 9 if the safe strength of the steel is 16,000 pounds per square inch?

The maximum bending moment due to the loads was found in example 1, Art. 43, to be 8,800 foot-pounds, or 8,800 × 12 = 105,600 inch-pounds.

Hence $\frac{\mathrm{I}}{c} = \frac{105{,}600}{16{,}000} = 6.6 \text{ inches}^3.$

That is, an I-beam is needed whose section modulus is a little larger than 6.6, to provide strength for its own weight.

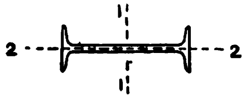
To select a size, we need a descriptive table of I-beams, such as is published in handbooks on structural steel.

Below is an abridged copy of such a table. (The last two columns contain information for use later.) The figure illustrates a cross-section of an I-beam, and shows the axes referred to in the table.

It will be noticed that two sizes are given for each depth; these are the lightest and heaviest of each size that are made, but intermediate sizes can be secured. In column 5 we find 7.3 as the next larger section modulus than the one required (6.6); and this corresponds to a 12¼-pound 6-inch I-beam, which is probably the proper size. To ascertain whether the excess (7.3 - 6.6 = 0.70) in the section modulus is sufficient to provide for the weight of the beam, we might proceed as in example 1. In this case, however, the excess is quite large, and the beam selected is doubtless safe.

TABLE C.

Properties of Standard I-Beams



Section of beam, showing axes 1-1 and 2-2.

1	2	3	4	5	6
Depth of Beam, in inches.	Weight per foot, in pounds.	Area of cross-section, in square inches.	Moment of inertia, axis 1—1.	Section modulus, axis 1—1.	Moment of inertia, axis 2—2.
3	5.50	1.63	2.5	1.7	0.46
3	7.50	2.21	2.9	1.9	.60
4	7.50	2.21	6.0	3.0	.77
4	10.50	3.09	7.1	3.6	1.01
5	9.75	2.87	12.1	4.8	1.23
5	14.75	4.34	15.1	6.1	1.70
6	12.25	3.61	21.8	7.3	1.85
6	17.25	5.07	26.2	8.7	2.36
7	15.00	4.42	36.2	10.4	2.67
7	20.00	5.88	42.2	12.1	3.24
8	18.00	5.33	56.9	14.2	3.78
8	25.25	7.43	68.0	17.0	4.71
9	21.00	6.31	84.9	18.9	5.16
9	35.00	10.29	111.8	24.8	7.31
10	25.00	7.37	122.1	24.4	6.89
10	40.00	11.76	158.7	31.7	9.50
12	31.50	9.26	215.8	36.0	9.50
12	40.00	11.76	245.9	41.0	10.95
15	42.00	12.48	441.8	58.9	14.62
15	60.00	17.65	538.6	71.8	18.17
18	55.00	15.93	795.6	88.4	21.19
18	70.00	20.59	921.2	102.4	24.62
20	65.00	19.08	1,169.5	117.0	27.86
20	75.00	22.06	1,268.8	126.9	30.25
24	80.00	23.32	2,087.2	173.9	42.86
24	100.00	29.41	2,379.6	198.3	48.55

EXAMPLES FOR PRACTICE.

1. Determine the size of a wooden beam which can safely sustain a middle load of 2,000 pounds, if the beam rests on end supports 16 feet apart, and its working strength is 1,000 pounds per square inch. Assume width 6 inches.

Ans. 6 × 10 inches.

2. What sized steel I-beam is needed to sustain safely a uniform load of 200,000 pounds, if it rests on end supports 10 feet apart, and its working strength is 16,000 pounds per square inch?

Ans. 100-pound 24-inch.

3. What sized steel I-beam is needed to sustain safely the loading of Fig. 10, if its working strength is 16,000 pounds per square inch?

Ans. 14.75-pound 5-inch.

67. Laws of Strength of Beams. The strength of a beam is measured by the bending moment that it can safely withstand; or, since bending and resisting moments are equal, by its safe resisting moment (SI ÷ c). Hence the **safe strength** of a beam varies (1) directly as the working fibre strength of its material, and (2) directly as the section modulus of its cross-section. For beams rectangular in cross-section (as wooden beams), the section modulus is $\frac{1}{6}ba^2$, b and a denoting the breadth and altitude of the rectangle. Hence the strength of such beams varies also directly as the breadth, and as the square of the depth. Thus, doubling the breadth of the section for a rectangular beam doubles the strength, but doubling the depth quadruples the strength.

The **safe load** that a beam can sustain varies directly as its resisting moment, and depends on the way in which the load is distributed and how the beam is supported. Thus, in the first four and last two cases of the table on page 55,

$$\begin{array}{llll}
M = Pl, & \text{hence} & P = & SI \div lc, \\
M = \frac{1}{2}Wl, & \text{"} & W = & 2SI \div lc, \\
M = \frac{1}{4}Pl, & \text{"} & P = & 4SI \div lc, \\
M = \frac{1}{8}Wl, & \text{"} & W = & 8SI \div lc, \\
M = \frac{1}{8}Pl, & \text{"} & P = & 8SI \div lc, \\
M = \frac{1}{12}Wl, & \text{"} & W = & 12SI \div lc,
\end{array}$$

Therefore the safe load in all cases varies inversely with the length; and for the different cases the safe loads are as 1, 2, 4, 8, 8, and 12 respectively.

Example. What is the ratio of the strengths of a plank 2 × 10 inches when placed edgewise and when placed flatwise on its supports?

When placed edgewise, the section modulus of the plank is $\frac{1}{6} \times 2 \times 10^2 = 33\frac{1}{3}$, and when placed flatwise it is $\frac{1}{6} \times 10 \times 2^2 = 6\frac{2}{3}$; hence its strengths in the two positions are as $33\frac{1}{3}$ to $6\frac{2}{3}$ respectively, or as 5 to 1.

EXAMPLE FOR PRACTICE.

What is the ratio of the safe loads for two beams of wood, one being 10 feet long, 3 × 12 inches in section, and having its load in the middle; and the other 8 feet long and 2 × 8 inches in section, with its load uniformly distributed.

Ans. As 135 to 100.

68. Modulus of Rupture. If a beam is loaded to destruction, and the value of the bending moment for the rupture stage is computed and substituted for M in the formula $SI \div c = M$, then the value of S computed from the equation is the **modulus of rupture** for the material of the beam. Many experiments have been performed to ascertain the moduli of rupture for different materials and for different grades of the same material. The following are fair values, all in pounds per square inch:

TABLE D.

Moduli of Rupture.

Timber:			
Spruce..........	4,000— 7,000,	average	5,000
Hemlock.........	3,500 7,000,	"	4,500
White pine.......	5,500 10,500,	"	8,000
Long-leaf pine....	10,000 16,000,	"	12,500
Short-leaf pine...	8,000 14,000,	"	10,000
Douglas spruce...	4,000 12,000,	"	8,000
White oak........	7,500 18,500,	"	13,000
Red oak..........	9,000 15,000,	"	11,500
Stone:			
Sandstone........	400— 1,200,		
Limestone........	400 1,000.		
Granite..........	800 1,400.		
Cast iron:	One and one-half to two and one-quarter times its ultimate tensile strength.		
Hard steel:	Varies from 100,000 to 150,000		

Wrought iron and structural steels have no modulus of rupture, as specimens of those materials will "bend double," but not break. The modulus of rupture of a material is used principally as a basis for determining its working strength. *The factor of safety of a loaded beam is computed by dividing the modulus of rupture of its material by the greatest unit-fibre stress in the beam.*

69. The Resisting Shear. The shearing stress on a cross-section of a loaded beam is not a uniform stress; that is, it is not uniformly distributed over the section. In fact the intensity or unit-stress is actually zero on the highest and lowest fibres of a cross-section, and is greatest, in such beams as are used in practice, on fibres at the neutral axis. In the following article we explain how to find the maximum value in two cases—cases which are practically important.

70. Second Beam Formula. Let S_s denote the average value of the unit-shearing stress on a cross-section of a loaded beam, and A the area of the cross-section. Then the value of the whole shearing stress on the section is :

$$\text{Resisting shear} = S_s\,A.$$

Since the resisting shear and the external shear at any section of a beam are equal (see Art. 59),

$$S_s\,A = V. \qquad (7)$$

This is called the "second beam formula" It is used to investigate and to design for shear in beams.

In beams uniform in cross-section, A is constant, and S_s is greatest in the section for which V is greatest. Hence the greatest unit-shearing stress in a loaded beam is at the neutral axis of the section at which the external shear is a maximum. There is a formula for computing this maximum value in any case, but it is not simple, and we give a simpler method for computing the value in the two practically important cases:

1. In wooden beams (rectangular or square in cross-section), the greatest unit-shearing stress in a section is 50 per cent larger than the average value S_s.
2. In I-beams, and in others with a thin vertical web, the greatest unit-shearing stress in a section practically equals S_s, as given by equation 7, if the area of the web is substituted for A.

Examples. 1. What is the greatest value of the unit-shearing stress in a wooden beam 12 feet long and 6 × 12 inches in cross-section when resting on end supports and sustaining a uniform load of 6,400 pounds? (This is the safe load as determined by working fibre stress; see example 1, Art. 65.)

The maximum external shear equals one-half the load (see Table B, page 55), and comes on the sections near the supports. Since

$$A = 6 \times 12 = 72 \text{ square inches};$$

$$S_s = \frac{3,200}{72} = 44 \text{ pounds per square inch},$$

and the greatest unit-shearing stress equals

$$\frac{3}{2} S_s = \frac{3}{2} 44 = 66 \text{ pounds per square inch}.$$

Apparently this is very insignificant; but it is not negligible, as is explained in the next article.

2. A steel I-beam resting on end supports 15 feet apart sustains a load of 8,000 pounds 5 feet from one end. The weight of the beam is 375 pounds, and the area of its web section is 3.2 square inches. (This is the beam and load described in examples 2 and 3, Art. 65.) What is the greatest unit-shearing stress?

The maximum external shear occurs near the support where the reaction is the greater, and its value equals that reaction. Calling that reaction R, and taking moments about the other end of the beam, we have

$$R \times 15 - 375 \times 7\frac{1}{2} - 8,000 \times 10 = 0;$$

therefore $15\,R = 80,000 + 2,812.5 = 82,812.5;$

or, $R = 5,520.8$ pounds.

Hence $S_s = \frac{5,520.8}{3.2} = 1,725$ pounds per square inch.

EXAMPLES FOR PRACTICE.

1. A wooden beam 10 feet long and 2 × 10 inches in cross-section sustains a middle load of 1,000 pounds. Neglecting the weight of the beam, compute the value of the greatest unit-shearing stress.

Ans. 37.5 pounds per square inch.

2. Solve the preceding example taking into account the weight of the beam, 60 pounds.

Ans. 40 pounds per square inch.

3. A wooden beam 12 feet long and 4 × 12 inches in cross-section sustains a load of 3,000 pounds 4 feet from one end. Neglecting the weight of the beam, compute the value of the greatest shearing unit-stress.

Ans. 62.5 pounds per square inch.

71. Horizontal Shear. It can be proved that there is a shearing stress on every horizontal section of a loaded beam. An experimental explanation will have to suffice here. Imagine a pile of six boards of equal length supported so that they do not bend. If the intermediate supports are removed, they will bend and their ends will not be flush but somewhat as represented in Fig. 41. This indicates that the boards slid over each other during the bending, and hence there was a rubbing and a frictional resistance exerted by the boards upon each other. Now, when a solid beam is being bent, there is an exactly similar tendency for the horizontal layers to slide over each other; and, instead of a frictional resistance, there exists shearing stress on all horizontal sections of the beam.

In the pile of boards the amount of slipping is different at different places between any two boards, being greatest near the supports and zero midway between them. Also, in any cross-section the slippage is least between the upper two and lower two boards, and is greatest between the middle two. These facts indicate that the shearing unit-stress on horizontal sections of a solid beam is greatest in the neutral surface at the supports.

It can be proved that at any place in a beam the shearing unit-stresses on a horizontal and on a vertical section are equal.

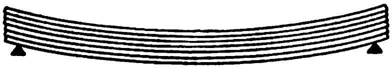


Fig. 41.

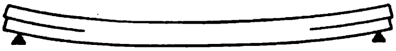


Fig. 42.

It follows that the horizontal shearing unit-stress is greatest at the neutral axis of the section for which the external shear (V) is a maximum. Wood being very weak in shear along the grain, timber beams sometimes fail under shear, the "rupture" being two horizontal cracks along the neutral surface somewhat as represented in Fig. 42. It is therefore necessary, when dealing with timber beams, to give due attention to their strength as determined by the working strength of the material in shear along the grain.

Example. A wooden beam 3 × 10 inches in cross-section rests on end supports and sustains a uniform load of 4,000 pounds Compute the greatest horizontal unit-stress in the beam.

The maximum shear equals one-half the load (see Table B, page 55), or 2,000 pounds. Hence, by equation 7, since $A = 3 \times 10 = 30$ square inches,

$$S_s = \frac{2{,}000}{30} = 66\frac{2}{3} \text{ pounds per square inch.}$$

This is the average shearing uni -stress on the cross-sections near the supports; and the greatest value equals

$$\frac{3}{2} \times 66\frac{2}{3} = 100 \text{ pounds per square inch.}$$

According to the foregoing, this is also the value of the greatest horizontal shearing unit-stress. (If of white pine, for example, the beam would not be regarded as safe, since the ultimate shearing strength along the grain of selected pine is only about 400 pounds per square inch.)

72. Design of Timber Beams. In any case we may proceed as follows:—(1) Determine the dimensions of the cross-section of the beam from a consideration of the fibre stresses as explained in Art. 66. (2) With dimensions thus determined, compute the value of the greatest shearing unit-stress from the formula,

$$\text{Greatest shearing unit-stress} = \frac{3}{2} V \div ab,$$

where V denotes the maximum external shear in the beam, and b and a the breadth and depth of the cross-section.

If the value of the greatest shearing unit-stress so computed does not exceed the working strength in shear along the grain, then the dimensions are large enough; but if it exceeds that value, then a or b, or both, should be increased until $\frac{3}{2} V \div ab$ is less than the working strength. Because timber beams are very often "season checked" (cracked) along the neutral surface, it is advisable to take the working strength of wooden beams, in shear along the grain, quite low. One-twentieth of the working fibre strength has been recommended* for all pine beams.

If the working strength in shear is taken equal to one-twentieth the working fibre strength, then it can be shown that,

1. For a beam on end supports loaded in the middle, the safe load depends on the shearing or fibre strength according as the ratio of length to depth ($l \div a$) is less or greater than 10.
2. For a beam on end supports uniformly loaded, the safe load depends on the shearing or fibre strength according as $l \div a$ is less or greater than 20.

Examples. 1. It is required to design a timber beam to sustain loads as represented in Fig. 11, the working fibre strength being 550 pounds and the working shearing strength 50 pounds per square inch.

The maximum bending moment (see example for practice 3, Art. 43; and example for practice 2, Art. 44) equals practically 7,000 foot-pounds or, $7,000 \times 12 = 84,000$ inch-pounds. Hence, according to equation 6‴,

$$\frac{I}{c} = \frac{84,000}{550} = 152.7 \text{ inches}^3.$$

Since for a rectangle

$$\frac{I}{c} = \frac{1}{6} ba^2,$$

$$\frac{1}{6} ba^2 = 152.7, \text{ or } ba^2 = 916.2.$$

Now, if we let $b = 4$, then $a^2 = 229$;

or, $a = 15.1$ (practically 16) inches.

If, again, we let $b = 6$, then $a^2 = 152.7$;

or $a = 12.4$ (practically 13) inches.

Either of these sizes will answer so far as fibre stress is concerned, but there is more "timber" in the second.

The maximum external shear in the beam equals 1,556 pounds, neglecting the weight of the beam (see example 3, Art. 37; and example 2, Art. 38). Therefore, for a 4×16-inch beam,

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* See "Materials of Construction."—Johnson. Page 55.

$$\text{Greatest shearing unit-stress} = \frac{3}{2} \times \frac{1{,}556}{4 \times 16}$$
$$= 36.5 \text{ pounds per square inch;}$$

and for a 6 × 14-inch beam, it equals

$$\frac{3}{2} \times \frac{1{,}556}{6 \times 14} = 27.7 \text{ pounds per square inch.}$$

Since these values are less than the working strength in shear, either size of beam is safe as regards shear.

If it is desired to allow for weight of beam, one of the sizes should be selected. First, its weight should be computed, then the new reactions, and then the unit-fibre stress may be computed as in Art. 64, and the greatest shearing unit-stress as in the foregoing. If these values are within the working values, then the size is large enough to sustain safely the load and the weight of the beam.

2. What is the safe load for a white pine beam 9 feet long and 2 × 12 inches in cross-section, if the beam rests on end supports and the load is at the middle of the beam, the working fibre strength being 1,000 pounds and the shearing strength 50 pounds per square inch.

The ratio of the length to the depth is less than 10; hence the safe load depends on the shearing strength of the material Calling the load P, the maximum external shear (see Table B, page 55) equals $\frac{1}{2}$ P, and the formula for greatest shearing unit stress becomes

$$50 = \frac{3}{2} \times \frac{\frac{1}{2} P}{2 \times 12}; \text{ or } P = 1{,}600 \text{ pounds.}$$

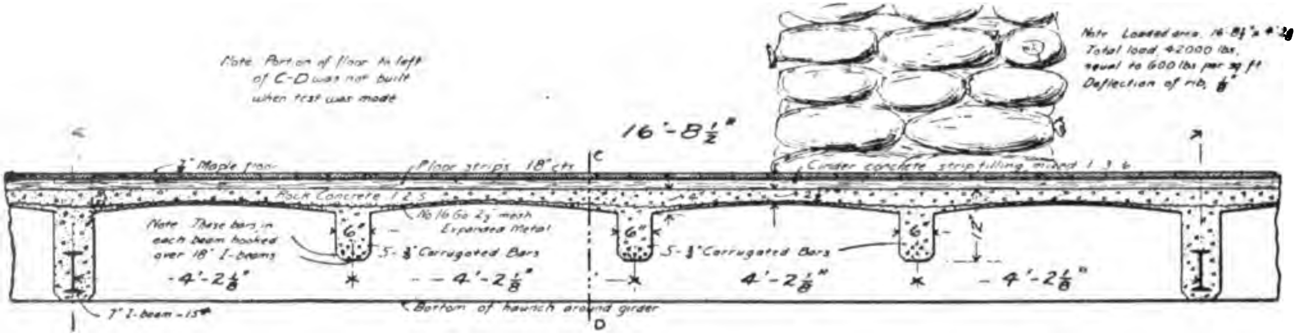
EXAMPLES FOR PRACTICE.

1. What size of wooden beam can safely sustain loads as in Fig. 12, with shearing and fibre working strength equal to 50 and 1,000 pounds per square inch respectively?

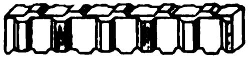
Ans. 6 × 12 inches

2. What is the safe load for a wooden beam 4 × 14 inches, and 18 feet long, if the beam rests on end supports and the load is uniformly distributed, with working strengths as in example 1?

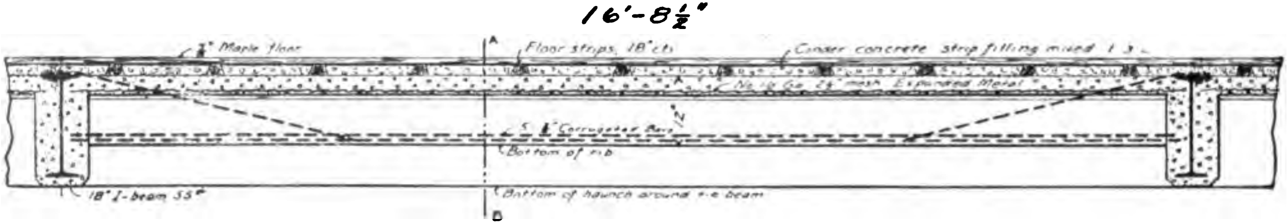
Ans. 3,730 pounds



TRANSVERSE SECTION A-B



DETAIL OF ⅜ CORRUGATED BAR


16'-8½"
⅞" Maple floor
Floor strips, 18" cts
Cinder concrete strip filling mixed 1 3 6
No 16 Ga 2¼" mesh Expanded Metal
A
B
12"
5-⅜" Corrugated Bars
Bottom of rib
18" I-beam 55#
Bottom of haunch around I-beam

LONGITUDINAL SECTION C-D

SECTIONS THROUGH REINFORCED CONCRETE FLOOR, SHOWING TEST OF FLOOR

Reproduced by Courtesy of the Expanded Metal & Corrugated Bar Co., St. Louis, Mo.

73. Kinds of Loads and Beams. We shall now discuss the strength of beams under **longitudinal** forces (acting parallel to the beam) and **transverse loads.** The longitudinal forces are supposed to be applied at the ends of the beams and along the axis* of the beam in each case. We consider only beams resting on end supports.

The transverse forces produce **bending** or **flexure,** and the longitudinal or end forces, if pulls, produce **tension** in the beam; if pushes, they produce **compression.** Hence the cases to be considered may be called "Combined Flexure and Tension" and "Combined Flexure and Compression."

74. Flexure and Tension. Let Fig. 43, *a*, represent a beam subjected to the transverse loads L_1, L_2 and L_3, and to two equal end pulls P and P. The reactions R_1 and R_2 are due to the transverse loads and can be computed by the methods of moments just as though there were no end pulls. To find the stresses at any cross-section, we determine those due to the transverse forces (L_1, L_2, L_3, R_1 and R_2) and those due to the longitudinal; then combine these stresses to get the total effect of all the applied forces.

The stress due to the transverse forces consists of a shearing stress and a fibre stress; it will be called the **flexural stress.** The fibre stress is compressive above and tensile below. Let M denote the value of the bending moment at the section considered; c_1 and c_2 the distances from the neutral axis to the highest and the lowest fibre in the section; and S_1 and S_2 the corresponding unit-fibre stresses due to the transverse loads. Then

$$S_1 = \frac{Mc_1}{I}; \text{ and } S_2 = \frac{Mc_2}{I}.$$

The stress due to the end pulls is a simple tension, and it equals P; this is sometimes called the **direct stress.** Let S_0 denote the unit-tension due to P, and A the area of the cross-section; then

$$S_0 = \frac{P}{A}.$$

Both systems of loads to the left of a section between L_1 and

---

* NOTE. By "axis of a beam" is meant the line through the centers of gravity of all the cross-sections.

L_2 are represented in Fig. 43, *b*; also the stresses caused by them at that section. Clearly the effect of the end pulls is to increase the tensile stress (on the lower fibres) and to decrease the compressive stress (on the upper fibres) due to the flexure. Let S_c denote the total (resultant) unit-stress on the upper fibre, and S_t that on the lower fibre, due to all the forces acting on the beam. In combining the stresses there are two cases to consider:

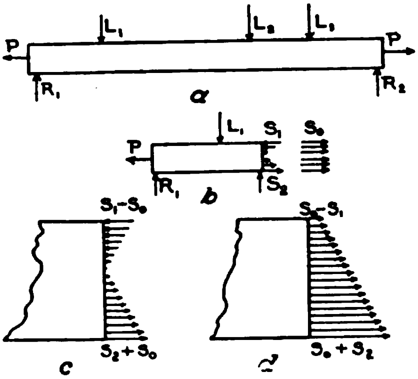


Fig. 43.

(1) The flexural compressive unit-stress on the upper fibre is greater than the direct unit-stress; that is, S_1 is greater than S_0. The resultant stress on the upper fibre is

$$S_c = S_1 - S_0 \text{ (compressive)};$$

and that on the lower fibre is

$$S_t = S_2 + S_0 \text{ (tensile)}.$$

The combined stress is as represented in Fig. 43, *c*, part tensile and part compressive.

(2) The flexural compressive unit-stress is less than the direct unit-stress; that is, S_1 is less than S_0. Then the combined unit-stress on the upper fibre is

$$S_c = S_0 - S_1 \quad \text{(tensile)};$$

and that on the lower fibre is

$$S_t = S_2 + S_0 \quad \text{(tensile)}.$$

The combined stress is represented by Fig. 43, *d*, and is all tensile.

Example. A steel bar 2 × 6 inches, and 12 feet long, is subjected to end pulls of 45,000 pounds. It is supported at each end, and sustains, as a beam, a uniform load of 6,000 pounds. It is required to compute the combined unit-fibre stresses.

Evidently the dangerous section is at the middle, and $M = \frac{1}{8} Wl$; that is,

$$M = \frac{1}{8} \times 6{,}000 \times 12 = 9{,}000 \text{ foot-pounds,}$$

or $$9{,}000 \times 12 = 108{,}000 \text{ inch-pounds.}$$

The bar being placed with the six-inch side vertical,

$$c_1 = c_2 = 3 \text{ inches, and}$$

$$I = \frac{1}{12} \times 2 \times 6^3 = 36 \text{ inches}^4. \quad \text{(See Art. 52.)}$$

Hence $$S_1 = S_2 = \frac{108{,}000 \times 3}{36} = 9{,}000 \text{ pounds per square inch.}$$

Since $$A = 2 \times 6 = 12 \text{ square inches,}$$

$$S_0 = \frac{45{,}000}{12} = 3{,}750 \text{ pounds per square inch.}$$

The greatest value of the combined compressive stress is
$S_1 - S_0 = 9{,}000 - 3{,}750 = 5{,}250$ pounds per square inch, and it occurs on the upper fibres of the middle section. The greatest value of the combined tensile stress is
$S_2 + S_0 = 9{,}000 + 3{,}750 = 12{,}750$ pounds per square inch, and it occurs on the lowest fibres of the middle section.

EXAMPLE FOR PRACTICE.

Change the load in the preceding illustration to one of 6,000 pounds placed in the middle, and then solve.

$$\text{Ans.} \begin{cases} S_c = 14{,}250 \text{ pounds per square inch.} \\ S_t = 21{,}750 \quad \text{" \quad " \quad " \quad "} \end{cases}$$

75. Flexure and Compression. Imagine the arrowheads on P reversed; then Fig. 43, *a*, will represent a beam under combined flexural and compressive stresses. The flexural unit-stresses are computed as in the preceding article. The direct stress is a compression equal to P, and the unit-stress due to P is computed as in the preceding article. Evidently the effect of P is to increase the compressive stress and decrease the tensile stress due to the flexure. In combining, we have two cases as before:

(1) The flexural tensile unit-stress is greater than the direct unit-stress; that is, S_2 is greater than S_0. Then the combined unit-stress on the lower fibre is

$$S_t = S_2 - S_0 \text{ (tensile)};$$

and that on the upper fibre is

$$S_c = S_1 + S_0 \text{ (compressive)}.$$

The combined fibre stress is represented by Fig. 44, *a*, and is part tensile and part compressive.

(2) The flexural unit-stress on the lower fibre is less than the direct unit-stress; that is, S_2 is less than S_0. Then the combined unit-stress on the lower fibre is

$$S_t = S_0 - S_2 \text{ (compressive)};$$

and that on the upper fibre is

$$S_c = S_0 + S_1 \text{ (compressive)}.$$

The combined fibre stress is represented by Fig. 44, *b*, and is all compressive.

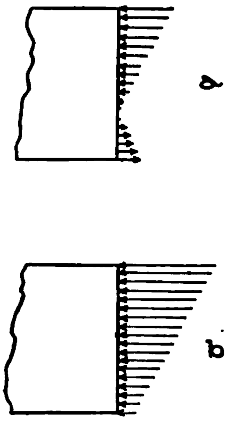


Fig. 44.

Example. A piece of timber 6 × 6 inches, and 10 feet long, is subjected to end pushes of 9,000 pounds. It is supported in a horizontal position at its ends, and sustains a middle load of 400 pounds. Compute the combined fibre stresses.

Evidently the dangerous section is at the middle, and $M = \frac{1}{4} Pl$; that is,

$$M = \frac{1}{4} \times 400 \times 10 = 1{,}000 \text{ foot-pounds},$$

or $$1{,}000 \times 12 = 12{,}000 \text{ inch-pounds}.$$

Since $$c_1 = c_2 = 3 \text{ inches, and}$$

$$I = \frac{1}{12} ba^3 = \frac{1}{12} \times 6 \times 6^3 = 108 \text{ inches}^4,$$

$$S_1 = S_2 = \frac{12{,}000 \times 3}{108} = 333\frac{1}{3} \text{ pounds per square inch.}$$

Since $$A = 6 \times 6 = 36 \text{ square inches,}$$

$$S_0 = \frac{9{,}000}{36} = 250 \text{ pounds per square inch.}$$

Hence the greatest value of the combined compressive stress is

$$S_0 + S_1 = 333\frac{1}{3} + 250 = 583\frac{1}{3} \text{ pounds per square inch.}$$

It occurs on the upper fibres of the middle section. The greatest value of the combined tensile stress is

$$S_2 - S_0 = 333\frac{1}{3} - 250 = 83\frac{1}{3} \text{ pounds per square inch.}$$

It occurs on the lowest fibres of the middle section.

EXAMPLE FOR PRACTICE.

Change the load of the preceding illustration to a uniform load and solve.

$$\text{Ans.} \begin{cases} S_c = 417 \text{ pounds per square inch.} \\ S_t = 83 \text{ " " " " (compression).} \end{cases}$$

76. Combined Flexural and Direct Stress by More Exact Formulas. The results in the preceding articles are only approximately correct. Imagine the beam represented in Fig. 45, *a*, to be first loaded with the transverse loads alone. They cause the beam to bend more or less, and produce certain flexural stresses at each section of the beam. Now, if end pulls are applied they tend to straighten the beam and hence diminish the flexural stresses. This effect of the end pulls was omitted in the discussion of Art. 74, and the results there given are therefore only approximate, the value of the greatest combined fibre unit-stress (S_t) being too large. On the other hand, if the end forces are pushes, they increase the bending, and therefore increase the flexural fibre stresses already caused by the transverse forces (see Fig. 45, *b*). The results indicated in Art. 75 must therefore in this case also be regarded as only approximate, the value of the greatest unit-fibre stress (S_c) being too small.

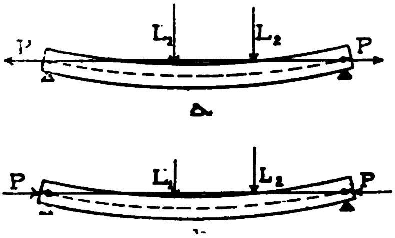


Fig. 45.

For beams loaded in the middle or with a uniform load, the following formulas, which take into account the flexural effect of the end forces, may be used:

M denotes bending moment at the middle section of the beam;

I denotes the moment of inertia of the middle section with respect to the neutral axis;

S_1, S_2, c_1 and c_2 have the same meanings as in Arts. 74 and 75, but refer always to the middle section;

l denotes length of the beam;

E is a number depending on the stiffness of the material, the average values of which are, for timber, 1,500,000; and for structural steel 30,000,000.*

$$S_1 = \frac{Mc_1}{I \pm \frac{Pl^2}{10E}}, \text{ and } S_2 = \frac{Mc_2}{I \pm \frac{Pl^2}{10E}}. \quad (8)$$

The plus sign is to be used when the end forces P are pulls, and the minus sign when they are pushes.

It must be remembered that S_1 and S_2 are flexural unit-stresses. The combination of these and the direct unit-stress is made exactly as in articles 74 and 75.

Examples. 1. It is required to apply the formulas of this article to the example of article 74.

As explained in the example referred to, M = 108,000 inch-pounds; $c_1 = c_2 = 3$ inches; and $I = 36$ inches^4.

Now, since $l = 12$ feet $= 144$ inches,

$$S_1 = S_2 = \frac{108{,}000 \times 3}{36 + \frac{45{,}000 \times 144^2}{10 \times 30{,}000{,}000}} = \frac{324{,}000}{36 + 3.11} = 8{,}284 \text{ pounds}$$

per square inch, as compared with 9,000 pounds per square inch, the result reached by the use of the approximate formula.

As before, $S_0 = 3{,}750$ pounds per square inch; hence

$S_c = 8{,}284 - 3{,}750 = 4{,}534$ pounds per square inch;

and $S_t = 8{,}284 + 3{,}750 = 12{,}034$ " " " "

2. It is required to apply the formulas of this article to the example of article 75.

As explained in that example,

$M = 12{,}000$ inch-pounds;

$c_1 = c_2 = 3$ inches, and $I = 108$ inches^4.

Now, since $l = 120$ inches,

$$S_1 = S_2 = \frac{12{,}000 \times 3}{108 - \frac{9{,}000 \times 120^2}{10 \times 1{,}500{,}000}} = \frac{36{,}000}{108 - 8.64} = 362 \text{ pounds}$$

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* NOTE. This quantity "E" is more fully explained in Article 95.

per square inch, as compared with 333⅓ pounds per square inch, the result reached by use of the approximate method.

As before, $S_o = 250$ pounds per square inch; hence

$S_c = 362 + 250 = 612$ pounds per square inch; and

$S_t = 362 - 250 = 112$ " " " " .

EXAMPLES FOR PRACTICE.

1. Solve the example for practice of Art. 74 by the formulas of this article.

Ans. $\begin{cases} S_c = 12{,}820 \text{ pounds per square inch.} \\ S_t = 20{,}320 \text{ " " " "} \end{cases}$

2. Solve the example for practice of Art. 75 by the formulas of this article.

Ans. $\begin{cases} S_c = 430 \text{ pounds per square inch.} \\ S_t = 70 \text{ " " " " (compression).} \end{cases}$

STRENGTH OF COLUMNS.

A stick of timber, a bar of iron, etc., when used to sustain end loads which act lengthwise of the pieces, are called **columns, posts,** or **struts** if they are so long that they would bend before breaking. When they are so short that they would not bend before breaking, they are called **short blocks,** and their compressive strengths are computed by means of equation 1. The strengths of columns cannot, however, be so simply determined, and we now proceed to explain the method of computing them.

77. End Conditions. The strength of a column depends in part on the way in which its ends bear, or are joined to other parts of a structure, that is, on its "end conditions." There are practically but three kinds of end conditions, namely:

1. "Hinge" or "pin" ends,
2. "Flat" or "square" ends, and
3. "Fixed" ends.

(1) When a column is fastened to its support at one end by means of a pin about which the column could rotate if the other end were free, it is said to be "hinged" or "pinned" at the former end. Bridge posts or columns are often hinged at the ends.

(2) A column either end of which is flat and perpendicular to its axis and bears on other parts of the structure at that surface, is said to be "flat" or "square" at that end.

(3) Columns are sometimes riveted near their ends directly to other parts of the structure and do not bear directly on their ends; such are called "fixed ended." A column which bears on its flat ends is often fastened near the ends to other parts of the structure, and such an end is also said to be "fixed." The fixing of an end of a column stiffens and therefore strengthens it more or less, but the strength of a column with fixed ends is computed as though its ends were flat. Accordingly we have, so far as strength is concerned, the following classes of columns:

78. Classes of Columns. (1) Both ends hinged or pinned; (2) one end hinged and one flat; (3) both ends flat.

Other things being the same, columns of these three classes are unequal in strength. Columns of the first class are the weakest, and those of the third class are the strongest.

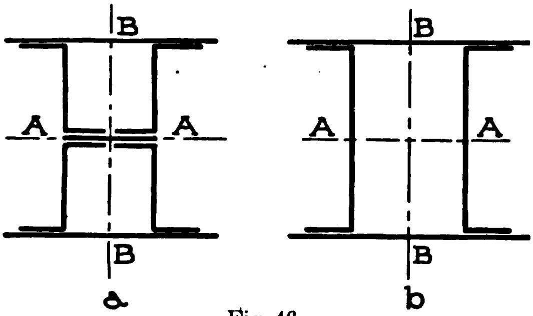


Fig. 46.

79. Cross-sections of Columns. Wooden columns are usually solid, square, rectangular, or round in section; but sometimes they are "built up" hollow. Cast-iron columns are practically always made hollow, and rectangular or round in section. Steel columns are made of single rolled shapes—angles, zees, channels, etc.; but the larger ones are usually "built up" of several shapes. Fig. 46, *a*, for example, represents a cross-section of a "Z-bar" column; and Fig. 46, *b*, that of a "channel" column.

80. Radius of Gyration. There is a quantity appearing in almost all formulas for the strength of columns, which is called "radius of gyration." It depends on the form and extent of the cross-section of the column, and may be defined as follows:

The radius of gyration of any plane figure (as the section of a column) with respect to any line, is such a length that the square of this length multiplied by the area of the figure equals the moment of inertia of the figure with respect to the given line.

Thus, if A denotes the area of a figure; I, its moment of inertia with respect to some line; and r, the radius of gyration with respect to that line; then

$$r^2\text{A} = \text{I}; \text{ or } r = \sqrt{\text{I} \div \text{A}}. \qquad (9)$$

In the column formulas, the radius of gyration always refers to an axis through the center of gravity of the cross-section, and usually to that axis with respect to which the radius of gyration (and moment of inertia) is least. (For an exception, see example 3, Art. 83.) Hence the radius of gyration in this connection is often called for brevity the "least radius of gyration," or simply the "least radius."

Examples. 1. Show that the value of the radius of gyration given for the square in Table A, page 54, is correct.

The moment of inertia of the square with respect to the axis is $\frac{1}{12}a^4$. Since $\text{A} = a^2$, then, by formula 9 above,

$$r = \sqrt{\frac{1}{12}a^4 \div a^2} = \sqrt{\frac{1}{12}a^2} = a\sqrt{\frac{1}{12}}.$$

2. Prove that the value of the radius of gyration given for the hollow square in Table A, page 54, is correct.

The value of the moment of inertia of the square with respect to the axis is $\frac{1}{12}(a^4 - a_1^4)$. Since $\text{A} = a^2 - a_1^2$,

$$r = \sqrt{\frac{\frac{1}{12}(a^4 - a_1^4)}{a^2 - a_1^2}} = \sqrt{\frac{1}{12}(a^2 + a_1^2)}.$$

EXAMPLE FOR PRACTICE.

Prove that the values of the radii of gyration of the other figures given in Table A, page 54, are correct. The axis in each case is indicated by the line through the center of gravity.

81. Radius of Gyration of Built-up Sections. The radius of gyration of a built-up section is computed similarly to that of any other figure. First, we have to compute the moment of inertia of the section, as explained in Art. 54; and then we use formula 9, as in the examples of the preceding article.

Example. It is required to compute the radius of gyration of the section represented in Fig. 30 (page 52) with respect to the axis AA.

In example 1, Art. 54, it is shown that the moment of inertia of the section with respect to the axis AA is 429 inches^4. The area of the whole section is

$$2 \times 6.03 + 7 = 19.06;$$

hence the radius of gyration r is

$$r = \sqrt{\frac{429}{19.06}} = 4.74 \text{ inches.}$$

EXAMPLE FOR PRACTICE.

Compute the radii of gyration of the section represented in Fig. 31, *a*, with respect to the axes AA and BB. (See examples for practice 1 and 2, Art. 54.)

Ans. { 2.87 inches.
2.09 "

82. Kinds of Column Loads. When the loads applied to a column are such that their resultant acts through the center of gravity of the top section and along the axis of the column, the column is said to be **centrally loaded.** When the resultant of the loads does not act through the center of gravity of the top section, the column is said to be **eccentrically loaded.** All the following formulas refer to columns centrally loaded.

83. Rankine's Column Formula. When a perfectly straight column is centrally loaded, then, if the column does not bend and if it is homogeneous, the stress on every cross-section is a uniform compression. If P denotes the load and A the area of the cross-section, the value of the unit-compression is P ÷ A.

On account of lack of straightness or lack of uniformity in material, or failure to secure exact central application of the load, the load P has what is known as an "arm" or "leverage" and bends the column more or less. There is therefore in such a column a bending or flexural stress in addition to the direct compressive stress above mentioned; this bending stress is compressive on the concave side and tensile on the convex. The value of the stress per unit-area (unit-stress) on the fibre at the concave side, according to equation 6, is $Mc \div I$, where M denotes the bending moment at the section (due to the load on the column), c the distance from the neutral axis to the concave side, and I the moment of inertia of the cross-section with respect to the neutral axis. (Notice that this axis is perpendicular to the plane in which the column bends.)

The upper set of arrows (Fig. 47) represents the direct compressive stress; and the second set the bending stress if the load is not excessive, so that the stresses are within the elastic limit of the material. The third set represents the combined stress that actually exists on the cross-section. The greatest combined unit-stress evidently occurs on the fibre at the concave side and where the deflection of the column is greatest. The stress is compressive, and its value S per unit-area is given by the formula,

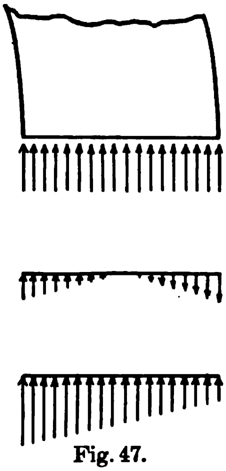


Fig. 47.

$$S = \frac{P}{A} + \frac{Mc}{I}.$$

Now, the bending moment at the place of greatest deflection equals the product of the load P and its arm (that is, the deflection). Calling the deflection d, we have $M = Pd$; and this value of M, substituted in the last equation, gives

$$S = \frac{P}{A} + \frac{Pdc}{I}.$$

Let r denote the radius of gyration of the cross-section with respec to the neutral axis. Then $I = Ar^2$ (see equation 9); and this value, substituted in the last equation, gives

$$S = \frac{P}{A} + \frac{Pdc}{Ar^2} = \frac{P}{A}\left(1 + \frac{dc}{r^2}\right).$$

According to the theory of the stiffness of beams on end supports, deflections vary directly as the square of the length l, and inversely as the distance c from the neutral axis to the remotest fibre of the cross-section. Assuming that the deflections of columns follow the same laws, we may write $d = k\ (l^2 \div c)$, where k is some constant depending on the material of the column and on the end conditions. Substituting this value for d in the last equation, we find that

$$\left.\begin{aligned} S &= \frac{P}{A}\left(1 + k\frac{l^2}{r^2}\right); \\ \frac{P}{A} &= \frac{S}{1 + k\dfrac{l^2}{r^2}}; \\ \text{and} \qquad P &= \frac{SA}{1 + k\dfrac{l^2}{r^2}}. \end{aligned}\right\} \qquad (10)$$

Each of these (usually the last) is known as "Rankine's formula."

For *mild-steel* columns a certain large steel company uses $S = 50{,}000$ pounds per square inch, and the following values of k:

1. Columns with two pin ends, $k = 1 \div 18{,}000$.
2. " " one flat and one pin end, $k = 1 \div 24{,}000$.
3. " " both ends flat, $k = 1 \div 36{,}000$.

With these values of S and k, P of the formula means the **ultimate load,** that is, the load causing failure. The safe load equals P divided by the selected factor of safety—a factor of 4 for steady loads, and 5 for moving loads, being recommended by the company referred to. The same unit is to be used for l and r.

Cast-iron columns are practically always made hollow with comparatively thin walls, and are usually circular or rectangular in cross-section. The following modifications of Rankine's formula are sometimes used:

$$\left.\begin{aligned} &\text{For circular sections,} \qquad && \frac{P}{A} = \frac{80{,}000}{1 + \dfrac{l^2}{800\,d^2}} \\ &\text{For rectangular sections,} \qquad && \frac{P}{A} = \frac{80{,}000}{1 + \dfrac{l^2}{1{,}000\,d^2}} \end{aligned}\right\} \qquad (10')$$

In these formulas d denotes the outside diameter of the circular sections or the length of the lesser side of the rectangular sections. The same unit is to be used for l and d.

Examples. 1. A 40-pound 10-inch steel I-beam 8 feet long is used as a flat-ended column. Its load being 100,000 pounds, what is its factor of safety?

Obviously the column tends to bend in a plane perpendicular to its web. Hence the radius of gyration to be used is the one with respect to that central axis of the cross-section which is in the web, that is, axis 2 – 2 (see figure accompanying table, page 72). The moment of inertia of the section with respect to that axis, according to the table, is 9.50 inches^4; and since the area of the section is 11.76 square inches,

$$r^2 = \frac{9.50}{11.76} = 0.81.$$

Now, $l = 8$ feet $= 96$ inches; and since $k = 1 \div 36{,}000$, and $S = 50{,}000$, the breaking load for this column, according to Rankine's formula, is

$$P = \frac{50{,}000 \times 11.76}{1 + \frac{96^2}{36{,}000 \times 0.81}} = 446{,}790 \text{ pounds.}$$

Since the factor of safety equals the ratio of the breaking load to the actual load on the column, the factor of safety in this case is

$$\frac{446{,}790}{100{,}000} = 4.5 \text{ (approx.).}$$

2. What is the safe load for a cast-iron column 10 feet long with square ends and a hollow rectangular section, the outside dimensions being 5 × 8 inches; the inner, 4 × 7 inches; and the factor of safety, 6?

In this case $l = 10$ feet $= 120$ inches; $A = 5 \times 8 - 4 \times 7 = 12$ square inches; and $d = 5$ inches. Hence, according to formula 10′, for rectangular sections, the breaking load is

$$P = \frac{80{,}000 \times 12}{1 + \frac{120^2}{1{,}000 \times 5^2}} = 610{,}000 \text{ pounds.}$$

Since the safe load equals the breaking load divided by the factor of safety, in this case the safe load equals

$$\frac{610{,}000}{6} = 101{,}700 \text{ pounds.}$$

3. A channel column (see Fig. 46, *b*) is pin-ended, the pins being perpendicular to the webs of the channel (represented by AA in the figure), and its length is 16 feet (distance between axes

of the pins). If the sectional area is 23.5 square inches, and the moment of inertia with respect to AA is 386 inches^4 and with respect to BB 214 inches^4, what is the safe load with a factor of safety of 4?

The column is liable to bend in one of two ways, namely, in the plane perpendicular to the axes of the two pins, or in the plane containing those axes.

(1) For bending in the first plane, the strength of the column is to be computed from the formula for a pin-ended column. Hence, for this case, $r^2 = 386 \div 23.5 = 16$; and the breaking load is

$$P = \frac{50{,}000 \times 23.5}{1 + \frac{(16 \times 12)^2}{18{,}000 \times 16}} = 1{,}041{,}600 \text{ pounds.}$$

The safe load for this case equals $\frac{1{,}041{,}600}{4} = 260{,}400$ pounds.

(2) If the supports of the pins are rigid, then the pins stiffen the column as to bending in the plane of their axes, and the strength of the column for bending in that plane should be computed from the formula for the strength of columns with flat ends. Hence, $r^2 = 214 \div 23.5 = 9.11$, and thebreaking load is

$$P = \frac{50{,}000 \times 23.5}{1 + \frac{(16 \times 12)^2}{36{,}000 \times 9.11}} = 1{,}056{,}000 \text{ pounds.}$$

The safe load for this case equals $\frac{1{,}056{,}000}{4} = 264{,}000$ pounds.

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a column with flat ends sustaining a load of 100,000 pounds. What is its factor of safety?

Ans. 4.1

2. A cast-iron column 15 feet long sustains a load of 150,000 pounds. Its section being a hollow circle, 9 inches outside and 7 inches inside diameter, what is the factor of safety?

Ans. 8.9

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is

3.1 inches; and the area of the cross-section is 24.5 square inches. What is the safe load for the column with a factor of safety of 4?

Ans. 247,000 pounds.

4. A cast-iron column 13 feet long has a hollow circular cross-section 7 inches outside and $5\frac{1}{2}$ inches inside diameter. What is its safe load with a factor of safety of 6?

Ans. 121,142 pounds.

5. Compute the safe load for a 40-pound 12-inch steel I-beam used as a column with flat ends, its length being 17 feet. Use a factor of safety of 5.

Ans. 52,470 pounds.

84. Graphical Representation of Column Formulas. Column (and most other engineering) formulas can be represented graphically. To represent Rankine's formula for flat-ended mild-steel columns,

$$\frac{\mathrm{P}}{\mathrm{A}} = \frac{50{,}000}{1 + \frac{(l \div r)^2}{36{,}000}},$$

we first substitute different values of $l \div r$ in the formula, and solve for P ÷ A. Thus we find, when

$l \div r = 40$, $\mathrm{P} \div \mathrm{A} = 47{,}900$;
$l \div r = 80$, $\mathrm{P} \div \mathrm{A} = 42{,}500$;
$l \div r = 120$, $\mathrm{P} \div \mathrm{A} = 35{,}750$;
etc., etc.

Now, if these values of $l \div r$ be laid off by some scale on a line from O, Fig. 48, and the corresponding values of P ÷ A be laid off vertically from the points on the line, we get a series of points as *a*, *b*, *c*, etc.; and a smooth curve through the points *a*, *b*, *c*,

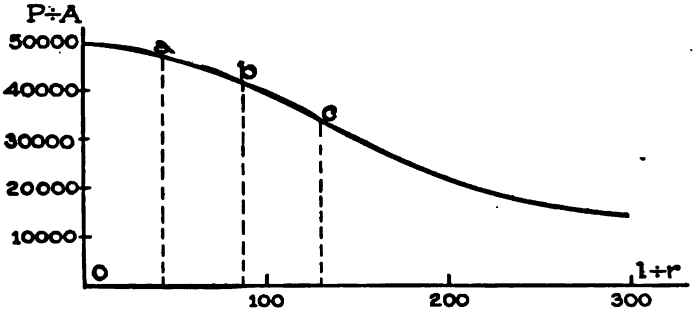


Fig. 48.

etc., represents the formula. Such a curve, besides representing the formula to one's eye, can be used for finding the value of P ÷ A for any value of $l \div r$; or the value of $l \div r$ for any value of P ÷ A. The use herein made is in explaining other column formulas in succeeding articles.

85. Combination Column Formulas. Many columns have been tested to destruction in order to discover in a practical way the laws relating to the strength of columns of different kinds. The results of such tests can be most satisfactorily represented graphically by plotting a point in a diagram for each test. Thus, suppose that a column whose $l \div r$ was 80 failed under a load of 276,000 pounds, and that the area of its cross-section was 7.12 square inches. This test would be represented by laying off O*a*, Fig. 49, equal to 80, according to some scale; and then *ab* equal to 276,000 ÷ 7.12 (P ÷ A), according to some other convenient scale. The point *b* would then represent the result of this particular test. All the dots in the figure represent the way in which the results of a series of tests appear when plotted.

It will be observed at once that the dots do not fall upon any one curve, as the curve of Rankine's formula. Straight lines and

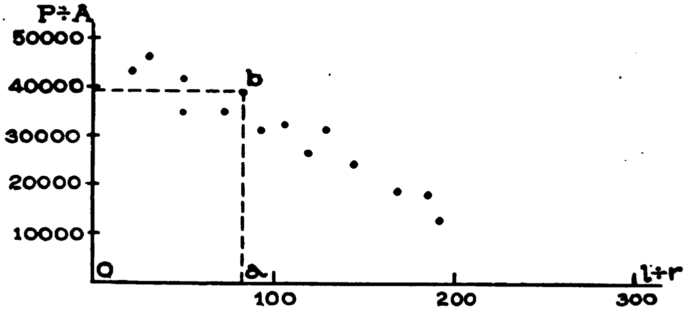
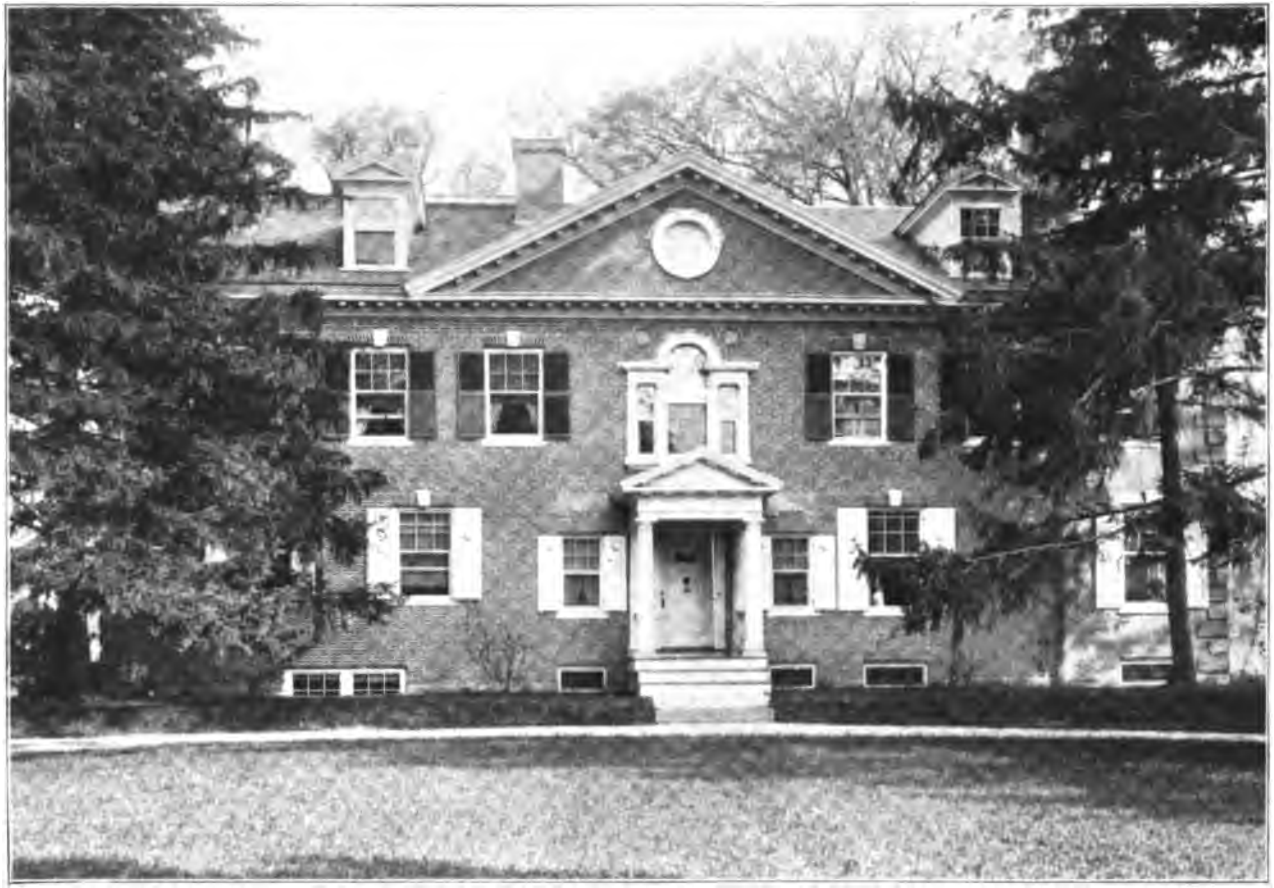


Fig. 49.

curves simpler than the curve of Rankine's formula have been fitted to represent the average positions of the dots as determined by actual tests, and the formulas corresponding to such lines have been deduced as column formulas. These are explained in the following articles.

86. Straight-Line and Euler Formulas. It occurred to Mr. T. H. Johnson that most of the dots corresponding to ordinary



STREET FRONT OF RESIDENCE AT DEDHAM, MASS.
Frank Chouteau Brown, Architect, Boston, Mass.
For Garden Front, See Page 10; for Plans, See Page 122.

lengths of columns agree with a straight line just as well as with a curve. He therefore, in 1886, made a number of such plats or diagrams as Fig. 49, fitted straight lines to them, and deduced the formula corresponding to each line. These have become known as "straight-line formulas," and their general form is as follows:

$$\frac{P}{A} = S - m\frac{l}{r}, \qquad (11)$$

P, A, l, and r having meanings as in Rankine's formula (Art. 83), and S and m being constants whose values according to Johnson are given in Table E below.

For the slender columns, another formula (Euler's, long since deduced) was used by Johnson. Its general form is—

$$\frac{P}{A} = \frac{n}{(l \div r)^2}, \qquad (12)$$

n being a constant whose values, according to Johnson, are given in the following table:

TABLE E.

Data for Mild-Steel Columns.

	S	m	Limit ($l \div r$)	n
Hinged ends......	52,500	220	160	444,000,000
Flat ends.........	52,500	180	195	666,000,000

The numbers in the fourth column of the table mark the point of division between columns of ordinary length and slender columns. For the former kind, the straight-line formula applies; and for the second, Euler's. That is, if the ratio $l \div r$ for a steel column with hinged end, for example, is less than 160, we must use the straight-line formula to compute its safe load, factor of safety, etc.; but if the ratio is greater than 160, we must use Euler's formula.

For *cast-iron columns* with flat ends, $S = 34{,}000$, and $m = 88$; and since they should never be used "slender," there is no use of Euler's formula for cast-iron columns.

The line AB, Fig. 50, represents Johnson's straight-line formula; and BC, Euler's formula. It will be noticed that the two lines are tangent; the point of tangency corresponds to the "limiting value" $l \div r$, as indicated in the table.

Examples. 1. A 40-pound 10-inch steel I-beam column 8 feet long sustains a load of 100,000 pounds, and the ends are flat. Compute its factor of safety according to the methods of this article.

The first thing to do is to compute the ratio $l \div r$ for the column, to ascertain whether the straight-line formula or Euler's formula should be used. From Table C, on page 72, we find that the moment of inertia of the column about the neutral axis of its cross-section is 9.50 inches^4, and the area of the section is 11.76 square inches. Hence

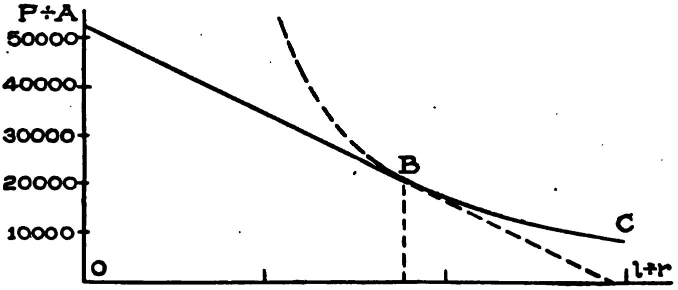


Fig. 50.

$$r^2 = \frac{9.50}{11.76} = 0.81; \text{ or } r = 0.9 \text{ inch.}$$

Since $l = 8$ feet $= 96$ inches,

$$\frac{l}{r} = \frac{96}{0.9} = 106\frac{2}{3}$$

This value of $l \div r$ is less than the limiting value (195) indicated by the table for steel columns with flat ends (Table E, p. 97), and we should therefore use the straight-line formula; hence

$$\frac{P}{11.76} = 52{,}500 - 180 \times 106\frac{2}{3};$$

or, $$P = 11.76\left(52{,}500 - 180 \times 106\frac{2}{3}\right) = 391{,}600 \text{ pounds.}$$

This is the breaking load for the column according to the straight-line formula; hence the factor of safety is

$$\frac{391{,}600}{100{,}000} = 3.9$$

2. Suppose that the length of the column described in the preceding example were 16 feet. What would its factor of safety be?

Since l = 16 feet = 192 inches; and, as before, r = 0.9 inch, $l \div r = 213\frac{1}{3}$. This value is greater than the limiting value (195) indicated by Table E (p. 97) for flat-ended steel columns; hence Euler's formula is to be used. Thus

$$\frac{P}{11.76} = \frac{666,000,000}{(213\frac{1}{3})^2};$$

or, $$P = \frac{11.76 \times 666,000,000}{(213\frac{1}{3})^2} = 172,100 \text{ pounds.}$$

This is the breaking load; hence the factor of safety is

$$\frac{172,100}{100,000} = 1.7$$

3. What is the safe load for a cast-iron column 10 feet long with square ends and hollow rectangular section, the outside dimensions being 5×8 inches and the inside 4×7 inches, with a factor of safety of 6?

Substituting in the formula for the radius of gyration given in Table A, page 54, we get

$$r = \sqrt{\frac{8 \times 5^3 - 7 \times 4^3}{12\,(8 \times 5 - 7 \times 4)}} = 1.96 \text{ inches.}$$

Since l = 10 feet = 120 inches,

$$\frac{l}{r} = \frac{120}{1.96} = 61.22$$

According to the straight-line formula for cast iron, A being equal to 12 square inches,

$$\frac{P}{12} = 34,000 - 88 \times 61.22;$$

or, $$P = 12\,(34,000 - 88 \times 61.22) = 343,360 \text{ pounds.}$$

This being the breaking load, the safe load is

$$\frac{343,360}{6} = 57,227 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a flat-ended column. Its load being 100,000 pounds, compute the factor of safety by the formulas of this article.

Ans. 3.5

2. A cast-iron column 15 feet long sustains a load of 150,000 pounds. Its section being a hollow circle of 9 inches outside and 7 inches inside diameter, compute the factor of safety by the straight-line formula.

Ans. 4.8

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches; and the area of the cross-section is 24.5 square inches. Compute the safe load for the column by the formulas of this article, using a factor of safety of 4.

Ans. 219,000 pounds.

4. A hollow cast-iron column 13 feet long has a circular cross-section, and is 7 inches outside and $5\frac{1}{2}$ inches inside in diameter. Compute its safe load by the formulas of this article, using a factor of safety of 6.

Ans. 68,500 pounds

5. Compute by the methods of this article the safe load for a 40-pound 12-inch steel I-beam used as a column with flat ends, if the length is 17 feet and the factor of safety 5.

Ans. 35,100 pounds.

87. Parabola-Euler Formulas. As better fitting the results of tests of the strength of columns of "ordinary lengths," Prof. J. B. Johnson proposed (1892) to use parabolas instead of straight lines. The general form of the "parabola formula" is

$$\frac{P}{A} = S - m\left(\frac{l}{r}\right)^2, \qquad (13)$$

P, A, l and r having the same meanings as in Rankine's formula, Art. 83; and S and m denoting constants whose values, according to Professor Johnson, are given in Table F below.

Like the straight-line formula, the parabola formula should not be used for slender columns, but the following (Euler's) is applicable:

$$\frac{P}{A}=\frac{n}{(l \div r)^2}, \qquad (14)$$

the values of n (Johnson) being given in the following table:

TABLE F.

Data for Mild Steel Columns.

	S	m	Limit $(l + r)$	n
Hinged ends......	42,000	0.97	150	456,000,000
Flat ends..	42,000	0.62	190	712,000,000

The point of division between columns of ordinary length and slender columns is given in the fourth column of the table. That is, if the ratio $l \div r$ for a column with hinged ends, for example, is less than 150, the parabola formula should be used to compute the safe load, factor of safety, etc.; but if the ratio is greater than 150, then Euler's formula should be used.

The line AB, Fig. 51, represents the parabola formula; and the line BC, Euler's formula. The two lines are tangent, and the point of tangency corresponds to the "limiting value" $l \div r$ of the table.

For *wooden columns* square in cross-section, it is convenient to replace r by d, the latter denoting the length of the sides of the square. The formula becomes

$$\frac{P}{A}=S - m\left(\frac{l}{d}\right)^2,$$

S and m for flat-ended columns of various kinds of wood having the following values according to Professor Johnson:

For White pine,	S=2,500,	$m = 0.6$;
" Short-leaf yellow pine,	S=3,300,	$m = 0.7$;
" Long-leaf yellow pine,	S=4,000,	$m = 0.8$;
" White oak,	S=3,500,	$m = 0.8$.

The preceding formula applies to any wooden column whose ratio, $l \div d$, is less than 60, within which limit columns of practice are included.

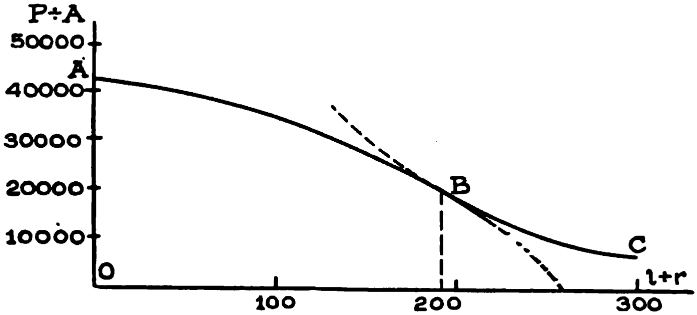


Fig. 51.

Examples. 1. A 40-pound 10-inch steel I-beam column

8 feet long sustains a load of 100,000 pounds, and its ends are flat. Compute its factor of safety according to the methods of this article.

The first thing to do is to compute the ratio $l \div r$ for the column, to ascertain whether the parabola formula or Euler's formula should be used. As shown in example 1 of the preceding article, $l \div r = 106\frac{2}{3}$. This ratio being less than the limiting value, 190, of the table, we should use the parabola formula. Hence, since the area of the cross-section is 11.76 square inches (see Table C, page 72),

$$\frac{P}{11.76} = 42{,}000 - 0.62\ (106\tfrac{2}{3})^2;$$

or, $$P = 11.76\ [42{,}000 - 0.62\ (106\tfrac{2}{3})^2] = 410{,}970 \text{ pounds.}$$

This is the breaking load according to the parabola formula; hence the factor of safety is

$$\frac{410{,}970}{100{,}000} = 4.1$$

2. A white pine column 10 × 10 inches in cross-section and 18 feet long sustains a load of 40,000 pounds. What is its factor of safety?

The length is 18 feet or 216 inches; hence the ratio $l \div d = 21.6$, and the parabola formula is to be applied.
Now, since $A = 10 \times 10 = 100$ square inches,

$$\frac{P}{100} = 2{,}500 - 0.6 \times 21.6^2;$$

or, $$P = 100\ (2{,}500 - 0.6 \times 21.6^2) = 222{,}000 \text{ pounds.}$$

This being the breaking load according to the parabola formula, the factor of safety is

$$\frac{222{,}000}{40{,}000} = 5.5$$

3. What is the safe load for a long-leaf yellow pine column 12 × 12 inches square and 30 feet long, the factor of safety being 5?

The length being 30 feet or 360 inches,

$$\frac{l}{d} = \frac{360}{12} = 30;$$

hence the parabola formula should be used. Since A = 12 × 12 = 144 square inches,

$$\frac{P}{144} = 4{,}000 - 0.8 \times 30^2;$$

or, $$P = 144\ (4{,}000 - 0.8 \times 30^2) = 472{,}320 \text{ pounds.}$$

This being the breaking load according to the parabola formula, the safe load is

$$\frac{472{,}320}{5} = 94{,}465 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a flat-ended column. Its load being 100,000 pounds, compute its factor of safety by the formulas of this article.

Ans. 3.8

2. A white oak column 15 feet long sustains a load of 30,000 pounds. Its section being 8 × 8 inches, compute the factor of safety by the parabola formula.

Ans. 6.6

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches; and the area of its cross-section is 24.5 square inches. Compute the safe load for the column by the formulas of this article, using a factor of safety of 4.

Ans. 224,500 pounds.

4. A short-leaf yellow pine column 14 × 14 inches in section is 20 feet long. What load can it sustain, with a factor of safety of 6?

Ans. 101,000 pounds.

88. "Broken Straight-Line" Formula. A large steel company computes the strength of its flat-ended steel columns by two formulas represented by two straight lines AB and BC, Fig. 52. The formulas are

$$\frac{P}{A} = 48{,}000,$$

and $$\frac{P}{A} = 68{,}400 - 228\ \frac{l}{r},$$

P, A, l, and r having the same meanings as in Art. 83.

The point B corresponds very nearly to the ratio $l \div r = 90$. Hence, for columns for which the ratio $l \div r$ is less than 90, the first formula applies; and for columns for which the ratio is greater than 90, the second one applies. The point C corresponds to the ratio $l \div r = 200$, and the second formula does not apply to a column for which $l \div r$ is greater than that limit.

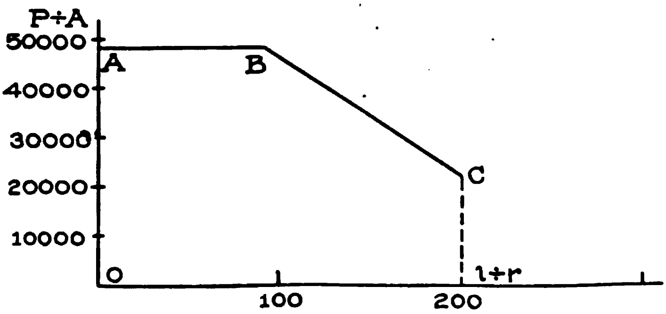


Fig. 52.

The ratio $l \div r$ for steel columns of practice rarely exceeds 150, and is usually less than 100.

Fig. 53 is a combination of Figs. 49, 50, 51 and 52, and represents graphically a comparison of the Rankine, straight-line, Euler, parabola-Euler, and broken straight-line formulas for flat-ended mild-steel columns, It well illustrates the fact that our knowledge of the strength of columns is not so exact as that, for example, of the strength of beams.

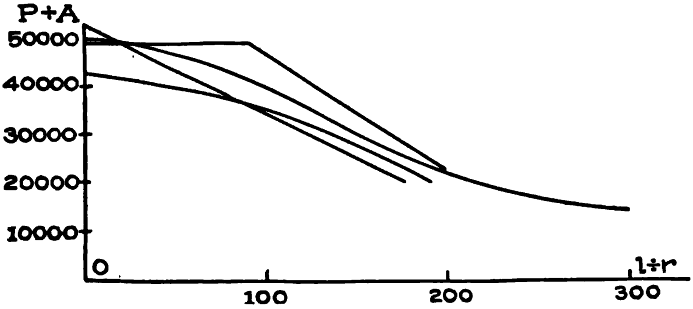


Fig. 53.

89. Design of Columns. All the preceding examples relating to columns were on either (1) computing the factor of safety of a given loaded column, or (2) computing the safe load for a given column. A more important problem is to design a column to sustain a given load under given conditions. A complete discussion of this problem is given in a later paper on design. We show here merely how to compute the *dimensions* of the cross-section of the column after the *form* of the cross-section has been decided upon.

In only a few cases can the dimensions be computed directly (see example 1 following), but usually, when a column formula is applied to a certain case, there will be two unknown quantities in it, A and r or d. Such cases can best be solved by trial (see examples 2 and 3 below).

Example. 1. What is the proper size of white pine column to sustain a load of 80,000 pounds with a factor of safety of 5, when the length of the column is 22 feet?

We use the parabola formula (equation 13). Since the safe load is 80,000 pounds and the factor of safety is 5, the breaking load P is

$$80{,}000 \times 5 = 400{,}000 \text{ pounds.}$$

The unknown side of the (square) cross-section being denoted by d, the area A is d^2. Hence, substituting in the formula, since $l = 22$ feet $= 264$ inches, we have

$$\frac{400{,}000}{d^2} = 2{,}500 - 0.6\,\frac{264^2}{d^2}.$$

Multiplying both sides by d^2 gives

$$400{,}000 = 2{,}500\,d^2 - 0.6 \times 264^2,$$

or $$2{,}500\,d^2 = 400{,}000 + 0.6 \times 264^2 = 441{,}817.6.$$

Hence $$d^2 = 176.73, \text{ or } d = 13.3 \text{ inches.}$$

2. What size of cast-iron column is needed to sustain a load of 100,000 pounds with a factor of safety of 10, the length of the column being 14 feet?

We shall suppose that it has been decided to make the cross-section circular, and shall compute by Rankine's formula modified for cast-iron columns (equation 10′). The breaking load for the column would be

$$100{,}000 \times 10 = 1{,}000{,}000 \text{ pounds.}$$

The length is 14 feet or 168 inches; hence the formula becomes

$$\frac{1{,}000{,}000}{A} = \frac{80{,}000}{1 + \frac{168^2}{800d^2}};$$

or, reducing by dividing both sides of the equation by 10,000, and then clearing of fractions, we have

$$100\left[1 + \frac{168^2}{800d^2}\right] = 8A.$$

There are two unknown quantities in this equation, d and A, and we cannot solve directly for them. Probably the best way to proceed is to assume or guess at a practical value of d, then solve for A, and finally compute the thickness or inner diameter. Thus, let us try d equal to 7 inches, first solving the equation for A as far as possible. Dividing both sides by 8 we have

$$A = \frac{100}{8}\left[1 + \frac{168^2}{800d^2}\right],$$

and, combining,

$$A = 12.5 + \frac{441}{d^2}.$$

Now, substituting 7 for d, we have

$$A = 12.5 + \frac{441}{49} = 21.5 \text{ square inches.}$$

The area of a hollow circle whose outer and inner diameters are d and d_1 respectively, is $0.7854\ (d^2 - d_1^2)$. Hence, to find the inner diameter of the column, we substitute 7 for d in the last expression, equate it to the value of A just found, and solve for d_1. Thus,

$$0.7854\ (49 - d_1^2) = 21.5;$$

hence

$$49 - d_1^2 = \frac{21.5}{0.7854} = 27.37;$$

and $$d_1^2 = 49 - 27.37 = 21.63 \text{ or } d_1 = 4.65.$$

This value of d makes the thickness equal to

$$\tfrac{1}{2}\ (7 - 4.65) = 1.175 \text{ inches,}$$

which is safe. It might be advisable in an actual case to try d equal to 8 repeating the computation.*

EXAMPLE FOR PRACTICE.

1. What size of white oak column is needed to sustain a load of 45,000 pounds with a factor of safety of 6, the length of the column being 12 feet.

Ans. $d = 8\frac{1}{2}$, practically a 10×10-inch section

STRENGTH OF SHAFTS.

A **shaft** is a part of a machine or system of machines, and is used to transmit power by virtue of its torsional strength, or resistance to twisting. Shafts are almost always made of metal and are usually circular in cross-section, being sometimes made hollow.

90. Twisting Moment. Let AF, Fig. 54, represent a shaft with four pulleys on it. Suppose that D is the driving pulley and that B, C and E are pulleys from which power is taken off to drive machines. The portions of the shafts between the pulleys are twisted when it is transmitting power; and by the twisting moment at any cross-section of the shaft is meant the algebraic sum of the moments of all the forces acting on the shaft on either

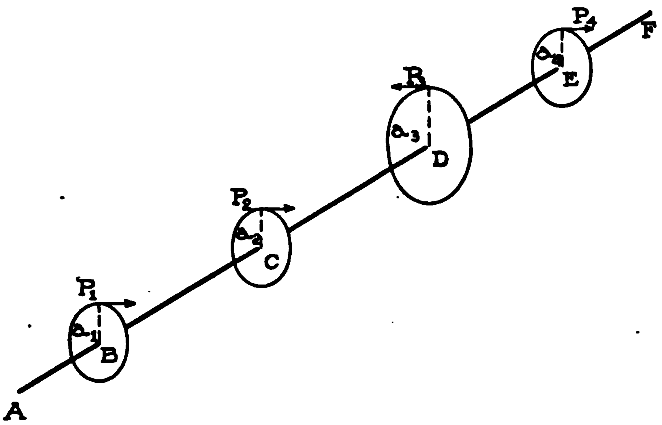


Fig. 54.

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*Note. The structural steel handbooks contain extensive tables by means of which the design of columns of steel or cast iron is much facilitated. The difficulties encountered in the use of formulæ are well illustrated in this example.

side of the section, the moments being taken with respect to the axis of the shaft. Thus, if the forces acting on the shaft (at the pulleys) are P_1, P_2, P_3, and P_4 as shown, and if the arms of the forces or radii of the pulleys are a_1, a_2, a_3, and a_4 respectively, then the twisting moment at any section in

$$\begin{aligned} &\text{BC is } P_1 a_1, \\ &\text{CD is } P_1 a_1 + P_2 a_2, \\ &\text{DE is } P_1 a_1 + P_2 a_2 - P_3 a_3. \end{aligned}$$

Like bending moments, twisting moments are usually expressed in inch-pounds.

Example. Let $a_1 = a_2 = a_4 = 15$ inches, $a_3 = 30$ inches, $P_1 = 400$ pounds, $P_2 = 500$ pounds, $P_3 = 750$ pounds, and $P_4 = 600$ pounds.* What is the value of the greatest twisting moment in the shaft?

At any section between the first and second pulleys, the twisting moment is

$$400 \times 15 = 6{,}000 \text{ inch-pounds;}$$

at any section between the second and third it is

$$400 \times 15 + 500 \times 15 = 13{,}500 \text{ inch-pounds; and}$$

at any section between the third and fourth it is

$$400 \times 15 + 500 \times 15 - 750 \times 30 = -9{,}000 \text{ inch-pounds.}$$

Hence the greatest value is 13,500 inch-pounds.

91. Torsional Stress. The stresses in a twisted shaft are called "torsional" stresses. The torsional stress on a cross-section of a shaft is a shearing stress, as in the case illustrated by Fig. 55, which represents a flange coupling in a shaft. Were it not for the bolts, one flange would slip over the other when either part of the shaft is turned; but the bolts prevent the slipping. Obviously there is a tendency to shear the bolts off unless they are screwed up very tight; that is, the material of the bolts is subjected to shearing stress.

Just so, at any section of the solid shaft there is a tendency for one part to slip past the other, and to prevent the slipping or shearing of the shaft, there arise shearing stresses at all parts of the cross-section. The shearing stress on the cross-section of a shaft is not a uniform stress, its value per unit-area being zero at the center of the section, and increasing toward the circumference. In circular sections, solid or hollow, the shearing stress per unit-area (unit-stress) varies directly as the distance from the center of the section, provided the elastic limit is not exceeded. Thus, if the shearing unit-stress at the circumference of a section is 1,000 pounds per square inch, and the diameter of the shaft is 2 inches, then, at ½ inch from the center, the unit-stress is 500 pounds per square inch; and at ¼ inch from the center it is 250 pounds per square inch. In Fig. 55 the arrows indicate the values and the directions of the shearing stresses on very small portions of the cross-section of a shaft there represented.

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* Note. These numbers were so chosen that the moment of P (driving moment) equals the sum of the moments of the other forces. This is always the case in a shaft rotating at constant speed; that is, the power given the shaft equals the power taken off.

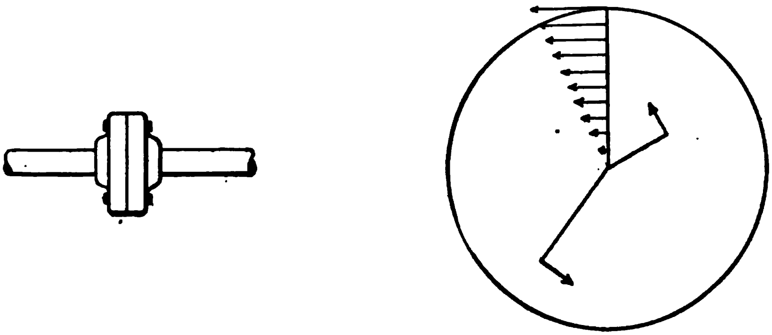


Fig. 55.

92. Resisting Moment. By "resisting moment" at a section of a shaft is meant the sum of the moments of the shearing stresses on the cross-section about the axis of the shaft.

Let S_s denote the value of the shearing stress per unit-area (unit-stress) at the outer points of a section of a shaft; d the diameter of the section (outside diameter if the shaft is hollow); and d_1 the inside diameter. Then it can be shown that the resisting moment is:

$$\text{For a solid section,} \quad 0.1963\, S_s\, d^3;$$

$$\text{For a hollow section,} \quad \frac{0.1963\, S_s\, (d^4 - d_1^4)}{d}.$$

93. Formula for the Strength of a Shaft. As in the case of beams, the resisting moment equals the twisting moment at any section. If T be used to denote twisting moment, then we have the formulas:

$$\left.\begin{array}{ll}\text{For solid circular shafts,} & 0.1963\ S_s\, d^3 = T; \\ \text{For hollow circular shafts,} & \dfrac{0.1963\ S_s\,(d^4 - d_1^4)}{d} = T.\end{array}\right\} \quad \mathbf{(15)}$$

In any portion of a shaft of constant diameter, the unit-shearing stress S_s is greatest where the twisting moment is greatest. Hence, to compute the greatest unit-shearing stress in a shaft, we first determine the value of the greatest twisting moment, substitute its value in the first or second equation above, as the case may be, and solve for S_s. It is customary to express T in inch-pounds and the diameter in inches, S_s then being in pounds per square inch.

Examples. 1. Compute the value of the greatest shearing unit-stress in the portion of the shaft between the first and second pulleys represented in Fig. 54, assuming values of the forces and pulley radii as given in the example of article 90. Suppose also that the shaft is solid, its diameter being 2 inches.

The twisting moment T at any section of the portion between the first and second pulleys is 6,000 inch-pounds, as shown in the example referred to. Hence, substituting in the first of the two formulas 15 above, we have

$$0.1963\ S_s \times 2^3 = 6,000;$$

or, $$S_s = \frac{6,000}{0.1963 \times 8} = 3,820 \text{ pounds per square inch.}$$

This is the value of the unit-stress at the outside portions of all sections between the first and second pulleys.

2. A hollow shaft is circular in cross-section, and its outer and inner diameters are 16 and 8 inches respectively. If the working strength of the material in shear is 10,000 pounds per square inch, what twisting moment can the shaft safely sustain?

The problem requires that we merely substitute the values of S_s, d, and d_1 in the second of the above formulas 15, and solve for T. Thus,

$$T = \frac{0.1963 \times 10,000\ (16^4 - 8^4)}{16} = 7,537,920 \text{ inch-pounds.}$$

EXAMPLES FOR PRACTICE.

1. Compute the greatest value of the shearing unit-stress in the shaft represented in Fig. 54, using the values of the forces and pulley radii given in the example of article 90, the diameter of the shaft being 2 inches.

Ans. 8,595 pounds per square inch

2. A solid shaft is circular in cross-section and is 9.6 inches in diameter. If the working strength of the material in shear is 10,000 pounds per square inch, how large a twisting moment can the shaft safely sustain? (The area of the cross-section is practically the same as that of the hollow shaft of example 2 preceding.)

Ans. 1,736,736 inch-pounds.

94. Formula for the Power Which a Shaft Can Transmit. The power that a shaft can safely transmit depends on the shearing working strength of the material of the shaft, on the size of the cross-section, and on the speed at which the shaft rotates.

Let H denote the amount of horse-power; S_s the shearing working strength in pounds per square inch; d the diameter (outside diameter if the shaft is hollow) in inches; d_1 the inside diameter in inches if the shaft is hollow; and n the number of revolutions of the shaft per minute. Then the relation between power transmitted, unit-stress, etc., is:

$$\left.\begin{array}{l} \text{For solid shafts, } H = \dfrac{S_s d^3 n}{321{,}000}; \\[2ex] \text{For hollow shafts, } H = \dfrac{S_s (d^4 - d_1^4)\, n}{321{,}000\, d}. \end{array}\right\} \quad (16)$$

Examples. 1. What horse-power can a hollow shaft 16 inches and 8 inches in diameter safely transmit at 50 revolutions per minute, if the shearing working strength of the material is 10,000 pounds per square inch?

We have merely to substitute in the second of the two formulas 16 above, and reduce. Thus,

$$H = \frac{10{,}000\,(16^4 - 8^4)\,50}{321{,}000 \times 16} = 6{,}000 \text{ horse-power (nearly).}$$

2. What size of solid shaft is needed to transmit 6,000 horse-power at 50 revolutions per minute if the shearing working strength of the material is 10,000 pounds per square inch?

We have merely to substitute in the first of the two formulas 16, and solve for d. Thus,

$$6{,}000 = \frac{10{,}000 \times d^3 \times 50}{321{,}000};$$

therefore $$d^3 = \frac{6{,}000 \times 321{,}000}{10{,}000 \times 50} = 3{,}852;$$

or, $$d = \sqrt[3]{3{,}852} = 15.68 \text{ inches.}$$

(A solid shaft of this diameter contains over 25% more material than the hollow shaft of example 1 preceding. There is therefore considerable saving of material in the hollow shaft.)

3. A solid shaft 4 inches in diameter transmits 200 horsepower while rotating at 200 revolutions per minute. What is the greatest shearing unit-stress in the shaft?

We have merely to substitute in the first of the equations 16, and solve for S_s. Thus,

$$200 = \frac{S_s \times 4^3 \times 200}{321{,}000};$$

or, $$S = \frac{200 \times 321{,}000}{4^3 \times 200} = 5{,}016 \text{ pounds per square inch.}$$

EXAMPLES FOR PRACTICE.

1. What horse-power can a solid shaft 9.6 inches in diameter safely transmit at 50 revolutions per minute, if its shearing working strength is 10,000 pounds per square inch?

Ans. 1,378 horse-power.

2. What size of solid shaft is required to transmit 500 horsepower at 150 revolutions per minute, the shearing working strength of the material being 8,000 pounds per square inch.

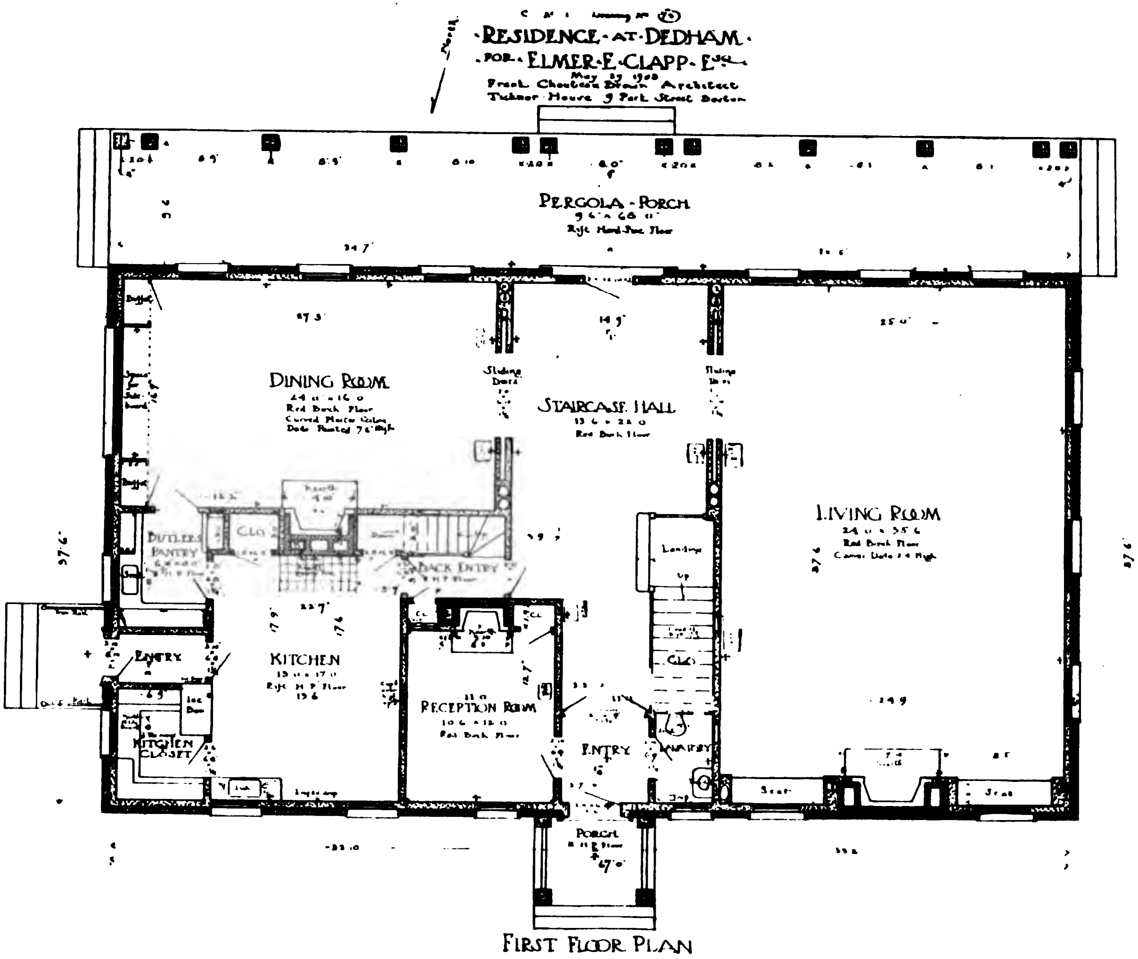
Ans. 5.1 inches.

3. A hollow shaft whose outer diameter is 14 and inner 6.7 inches transmits 5,000 horse-power at 60 revolutions per minute. What is the value of the greatest shearing unit-stress in the shaft?

Ans. 10,273 pounds per square inch.

STIFFNESS OF RODS, BEAMS, AND SHAFTS.

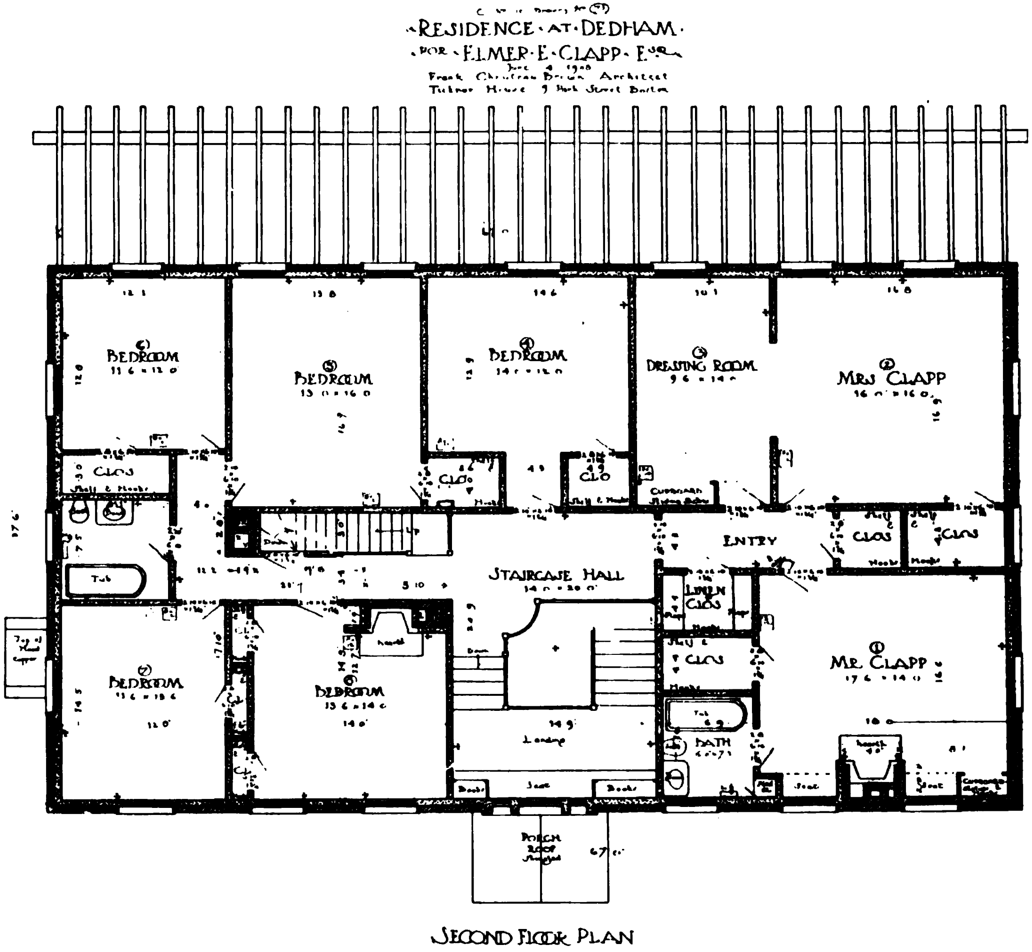
The preceding discussions have related to the *strength* of



FIRST-FLOOR PLAN OF RESIDENCE AT DEDHAM, MASS.

Frank Chouteau Brown, Architect, Boston, Mass.

For Exteriors, See Page 10 and Page 106. Second-Floor Plan Shown on Opposite Page.



SECOND-FLOOR PLAN OF RESIDENCE AT DEDHAM, MASS.
Frank Chouteau Brown, Architect, Boston, Mass.
First-Floor Plan Shown on Opposite Page.

materials. We shall now consider principally the *elongation of rods*, *deflection of beams*, and *twist of shafts*.

95. Coefficient of Elasticity. According to Hooke's Law (Art. 9, p. 7), the elongations of a rod subjected to an increasing pull are proportional to the pull, provided that the stresses due to the pull do not exceed the elastic limit of the material. Within the elastic limit, then, the ratio of the pull and the elongation is constant; hence the ratio of the unit-stress (due to the pull) to the unit-elongation is also constant. This last-named ratio is called "coefficient of elasticity." If E denotes this coefficient, S the unit-stress, and s the unit-deformation, then

$$E = \frac{S}{s}. \qquad (17)$$

Coefficients of elasticity are usually expressed in pounds per square inch.

The preceding remarks, definition, and formula apply also to a case of compression, provided that the material being compressed does not bend, but simply shortens in the direction of the compressing forces. The following table gives the *average* values of the coefficient of elasticity for various materials of construction:

TABLE G.

Coefficients of Elasticity.

Material.	Average Coefficient of Elasticity.
Steel.............	30,000,000 pounds per square inch.
Wrought iron.....	27,500,000 " " " "
Cast iron.........	15,000,000 " " " "
Timber..........	1,800,000 " " " "

The coefficients of elasticity for steel and wrought iron, for different grades of those materials, are remarkably constant; but for different grades of cast iron the coefficients range from about 10,000,000 to 30,000,000 pounds per square inch. Naturally the coefficient has not the same value for the different kinds of wood; for the principal woods it ranges from 1,600,000 (for spruce) to 2,100,000 (for white oak).

Formula 17 can be put in a form more convenient for use, as follows:

Let P denote the force producing the deformation; A the area of the cross-section of the piece on which P acts; l the length of the piece; and D the deformation (elongation or shortening).

Then

$$S = P \div A \text{ (see equation 1)},$$

and

$$s = D \div l \text{ (see equation 2)}.$$

Hence, substituting these values in equation 17, we have

$$E = \frac{Pl}{AD}; \text{ or } D = \frac{Pl}{AE}. \qquad (17')$$

The first of these two equations is used for computing the value of the coefficient of elasticity from measurements of a "test," and the second for computing the elongation or shortening of a given rod or bar for which the coefficient is known.

Examples. 1. It is required to compute the coefficient of elasticity of the material the record of a test of which is given on page 9.

Since the unit-stress S and unit-elongation s are already computed in that table, we can use equation 17 instead of the first of equations 17′. The elastic limit being between 40,000 and 45,000 pounds per square inch, we may use any value of the unit-stress less than that, and the corresponding unit-elongation.

Thus, with the first values given,

$$E = \frac{5{,}000}{0.00017} = 29{,}400{,}000.$$

With the second,

$$E = \frac{10{,}000}{0.00035} = 28{,}600{,}000.$$

This lack of constancy in the value of E as computed from different loads in a test of a given material, is in part due to errors in measuring the deformation, a measurement difficult to make. The value of the coefficient adopted from such a test, is the average of all the values of E which can be computed from the record.

2. How much will a pull of 5,000 pounds stretch a round steel rod 10 feet long and 1 inch in diameter?

We use the second of the two formulas 17′. Since $A = 0.7854 \times 1^2 = 0.7854$ square inches, $l = 120$ inches, and $E = 30{,}000{,}000$ pounds per square inch, the stretch is:

$$D = \frac{5{,}000 \times 120}{0.7854 \times 30{,}000{,}000} = 0.0254 \text{ inch}.$$

EXAMPLES FOR PRACTICE.

1. What is the coefficient of elasticity of a material if a pull of 20,000 pounds will stretch a rod 1 inch in diameter and 4 feet long 0.045 inch?

Ans. 27,000,000 pounds per square inch.

2. How much will a pull of 15,000 pounds elongate a round cast-iron rod 10 feet long and 1 inch in diameter?

Ans. 0.152 inch.

96. Temperature Stresses. In the case of most materials, when a bar or rod is heated, it lengthens; and when cooled, it shortens if it is free to do so. The **coefficient of linear expansion** of a material is the ratio which the elongation caused in a rod or bar of the material by a change of one degree in temperature bears to the length of the rod or bar. Its values for Fahrenheit degrees are about as follows:

For Steel,	0.0000065.
For Wrought iron,	.0000067.
For Cast iron,	.0000062.

Let K be used to denote this coefficient; t a change of temperature, in degrees Fahrenheit; l the length of a rod or bar; and D the change in length due to the change of temperature. Then

$$D = K\,t\,l. \qquad (18)$$

D and l are expressed in the same unit.

If a rod or bar is confined or restrained so that it cannot change its length when it is heated or cooled, then any change in its temperature produces a stress in the rod; such are called **temperature stresses.**

Examples. 1. A steel rod connects two solid walls and is screwed up so that the unit-stress in it is 10,000 pounds per square inch. Its temperature falls 10 degrees, and it is observed that the walls have not been drawn together. What is the temperature stress produced by the change of temperature, and what is the actual unit-stress in the rod at the new temperature?

Let l denote the length of the rod. Then the change in length which would occur if the rod were free, is given by formula 18, above, thus:

$$D = 0.0000065 \times 10 \times l = 0.000065\,l.$$

Now, since the rod could not shorten, it has a greater than normal length at the new temperature; that is, the fall in temperature has produced an effect equivalent to an elongation in the rod amounting to D, and hence a tensile stress. This tensile stress can be computed from the elongation D by means of formula 17. Thus,

$$S = E s;$$

and since s, the unit-elongation, equals

$$\frac{D}{l} = \frac{0.000065\ l}{l} = 0.000065,$$

$S = 30{,}000{,}000 \times 0.000065 = 1{,}950$ pounds per square inch. This is the value of the temperature stress; and the new unit-stress equals

$$10{,}000 + 1{,}950 = 11{,}950 \text{ pounds per square inch.}$$

Notice that the unit temperature stresses are independent of the length of the rod and the area of its cross-section.

2. Suppose that the change of temperature in the preceding example is a rise instead of a fall. What are the values of the temperature stress due to the change, and of the new unit-stress in the rod?

The temperature stress is the same as in example 1, that is, 1,950 pounds per square inch; but the rise in temperature releases, as it were, the stress in the rod due to its being screwed up, and the final unit stress is

$$10{,}000 - 1{,}950 = 8{,}050 \text{ pounds per square inch.}$$

EXAMPLE FOR PRACTICE.

1. The ends of a wrought-iron rod 1 inch in diameter are fastened to two heavy bodies which are to be drawn together, the temperature of the rod being 200 degrees when fastened to the objects. A fall of 120 degrees is observed not to move them. What is the temperature stress, and what is the pull exerted by the rod on each object?

Ans. { Temperature stress, 22,000 pounds per square inch.
Pull, 17,280 pounds.

97. Deflection of Beams. Sometimes it is desirable to know how much a given beam will deflect under a given load, or to design a beam which will not deflect more than a certain amount under a given load. In Table B, page 55, Part I, are given formulas for deflection in certain cases of beams and different kinds of loading.

In those formulas, *d* denotes deflection; I the moment of inertia of the cross-section of the beam with respect to the neutral axis, as in equation 6; and E the coefficient of elasticity of the material of the beam (for values, see Art. 95).

In each case, the load should be expressed in pounds, the length in inches, and the moment of inertia in biquadratic inches; then the deflection will be in inches.

According to the formulas for d, the deflection of a beam varies inversely as the coefficient of its material (E) and the moment of inertia of its cross-section (I); also, in the first four and last two cases of the table, the deflection varies directly as the cube of the length (l^3).

Example. What deflection is caused by a uniform load of 6,400 pounds (including weight of the beam) in a wooden beam on end supports, which is 12 feet long and 6 × 12 inches in cross-section? (This is the safe load for the beam; see example 1, Art. 65.)

The formula for this case (see Table B, page 55) is

$$d = \frac{5\ Wl^3}{384\ EI}.$$

Here $W = 6{,}400$ pounds; $l = 144$ inches; $E = 1{,}800{,}000$ pounds per square inch; and

$$I = \frac{1}{12}\,ba^3 = \frac{1}{12}\,6 \times 12^3 = 864 \text{ inches}^4.$$

Hence the deflection is

$$d = \frac{5 \times 6{,}400 \times 144^3}{384 \times 1{,}800{,}000 \times 864} = 0.16 \text{ inch}.$$

EXAMPLES FOR PRACTICE.

1. Compute the deflection of a timber built-in cantilever 8 × 8 inches which projects 8 feet from the wall and bears an end load of 900 pounds. (This is the safe load for the cantilever, see example 1, Art. 65.)

Ans. 0.43 inch.

2. Compute the deflection caused by a uniform load of 40,000

pounds on a 42-pound 15-inch steel I-beam which is 16 feet long and rests on end supports.

Ans. 0.28 inch.

98. Twist of Shafts. Let Fig. 57 represent a portion of a shaft, and suppose that the part represented lies wholly between two adjacent pulleys on a shaft to which twisting forces are applied (see Fig. 54). Imagine two radii *ma* and *nb* in the ends of the portion, they being parallel as shown when the shaft is not twisted. After the shaft is twisted they will not be parallel, *ma* having moved to *ma'*, and *nb* to *nb'*. The angle between the two lines in their twisted positions (*ma'* and *nb'*) is called the **angle of twist,** or **angle of torsion,** for the length *l*. If *a'a''* is parallel to *ab*, then the angle *a''nb'* equals the angle of torsion.

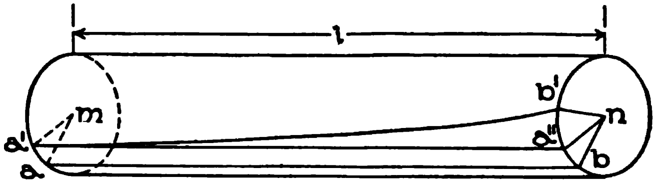


Fig. 57.

If the stresses in the portion of the shaft considered do not exceed the elastic limit, and if the twisting moment is the same for all sections of the portion, then the angle of torsion *a* (in degrees) can be computed from the following:

For solid circular shafts,

$$a = \frac{584\ \mathrm{T}l}{\mathrm{E}^1 d^4} = \frac{36{,}800{,}000\ \mathrm{H}l}{\mathrm{E}^1 d^4 n}.$$

For hollow circular shafts, **(19)**

$$a = \frac{584\ \mathrm{T}ld}{\mathrm{E}^1 (d^4 - d_1^4)} = \frac{36{,}800{,}000\ \mathrm{H}l}{\mathrm{E}^1 (d^4 - d_1^4)\, n}.$$

Here T, l, d, d_1, H, and n have the same meanings as in Arts. 93 and 94, and should be expressed in the units there used. The letter E^1 stands for a quantity called **coefficient of elasticity for shear;** it is analogous to the coefficient of elasticity for tension and compression (E), Art. 95. The values of E^1 for a few materials average about as follows (roughly $\mathrm{E}^1 = \frac{2}{5}\mathrm{E}$):

For Steel,	11,000,000	pounds per square inch.
For Wrought iron,	10,000,000	" " " "
For Cast iron,	6,000,000	" " " "

Example. What is the value of the angle of torsion of a steel shaft 60 feet long when transmitting 6,000 horse-power at 50 revolutions per minute, if the shaft is hollow and its outer and inner diameters are 16 and 8 inches respectively?

Here $l = 720$ inches; hence, substituting in the appropriate formula (19), we find that

$$a = \frac{36,800,000 \times 6,000 \times 720}{11,000,000 \times (16^4 - 8^4)\ 50} = 4.7 \text{ degrees.}$$

EXAMPLE FOR PRACTICE.

Suppose that the first two pulleys in Fig. 54 are 12 feet apart; that the diameter of the shaft is 2 inches; and that $P_1 = 400$ pounds, and $a_1 = 15$ inches. If the shaft is of wrought iron, what is the value of the angle of torsion for the portion between the first two pulleys?

Ans. 3.15 degrees.

99. Non-elastic Deformation. The preceding formulas for elongation, deflection, and twist hold only so long as the greatest unit-stress does not exceed the elastic limit. There is no theory, and no formula, for non-elastic deformations, those corresponding to stresses which exceed the elastic limit. It is well known, however, that non-elastic deformations are not proportional to the forces producing them, but increase much faster than the loads. The value of the ultimate elongation of a rod or bar (that is, the amount of elongation at rupture), is quite well known for many materials. This elongation, for eight-inch specimens of various materials (see Art. 16), is:

For Cast iron,	about 1 per cent.
For Wrought iron (plates),	12 - 15 per cent.
For " " (bars),	20 - 25 " " .
For Structural steel,	22 - 26 " " .

Specimens of ductile materials (such as wrought iron and structural steel), when pulled to destruction, **neck down**, that is, diminish very considerably in cross-section at some place along the length of the specimen. The decrease in cross-sectional area is known as **reduction of area,** and its value for wrought iron and steel may be as much as 50 per cent.

RIVETED JOINTS.

100. Kinds of Joints. A **lap joint** is one in which the plates or bars joined overlap each other, as in Fig. 58, *a*. A **butt joint** is one in which the plates or bars that are joined butt against each other, as in Fig. 58, *b*. The thin side plates on butt joints are called **cover-plates;** the thickness of each is always made not less than one-half the thickness of the **main plates,** that is, the plates or bars that are joined. Sometimes butt joints are made with only one cover-plate; in such a case the thickness of the cover-plate is made not less than that of the main plate.

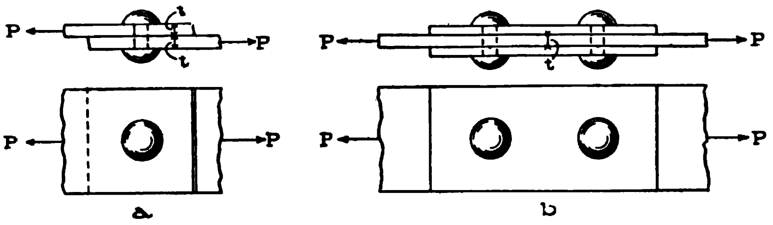


Fig. 58.

When wide bars or plates are riveted together, the rivets are placed in rows, always parallel to the "seam" and sometimes also perpendicular to the seam; but when we speak of a row of rivets, we mean a row parallel to the seam. A lap joint with a single row of rivets is said to be **single-riveted;** and one with two rows of rivets is said to be **double-riveted.** A butt joint with two rows of rivets (one on each side of the joint) is called "single-riveted," and one with four rows (two on each side) is said to be "double-riveted."

The distance between the centers of consecutive holes in a row of rivets is called **pitch.**

101. Shearing Strength, or Shearing Value, of a Rivet. When a lap joint is subjected to tension (that is, when P, Fig. 58, *a*, is a pull), and when the joint is subjected to compression (when P is a push), there is a tendency to cut or shear each rivet along the surface between the two plates. In butt joints with two cover-plates, there is a tendency to cut or shear each rivet on two surfaces (see Fig. 58, *b*). Therefore the rivets in the lap joint are said to be in **single shear**; and those in the butt joint (two covers) are said to be in **double shear.**

The "shearing value" of a rivet means the resistance which it can safely offer to forces tending to shear it on its cross-section. This value depends on the area of the cross-section and on the working strength of the material. Let d denote the diameter of the cross-section, and S_s the shearing working strength. Then, since the area of the cross-section equals $0.7854\, d^2$, the shearing strength of one rivet is:

For single shear, $0.7854\, d^2\, S_s$.
For double shear, $1.5708\, d^2\, S_s$.

102. Bearing Strength, or Bearing Value, of a Plate. When a joint is subjected to tension or compression, each rivet presses against a part of the sides of the holes through which it passes. By "bearing value" of a plate (in this connection) is meant the pressure, exerted by a rivet against the side of a hole in the plate, which the plate can safely stand. This value depends on the thickness of the plate, on the diameter of the rivet, and on the compressive working strength of the plate. Exactly how it depends on these three qualities is not known; but the bearing value is always computed from the expression $t\, d\, S_c$, wherein t denotes the thickness of the plate; d, the diameter of the rivet or hole; and S_c, the working strength of the plate.

103. Frictional Strength of a Joint. When a joint is subjected to tension or compression, there is a tendency to slippage between the faces of the plates of the joint. This tendency is overcome wholly or in part by frictional resistance between the plates. The frictional resistance in a well-made joint may be very large, for rivets are put into a joint hot, and are **headed** or **capped** before being cooled. In cooling they contract, drawing the plates of the joint tightly against each other, and producing a great pressure between them, which gives the joint a correspondingly large frictional strength. It is the opinion of some that all well-made joints perform their service by means of their frictional strength; that is to say, the rivets act only by pressing the plates together and are not under shearing stress, nor are the plates under compression at the sides of their holes. The "frictional strength" of a joint, however, is usually regarded as uncertain, and generally no allowance is made for friction in computations on the strength of riveted joints.

104. Tensile and Compressive Strength of Riveted Plates. The holes punched or drilled in a plate or bar weaken its tensile strength, and to compute that strength it is necessary to allow for the holes. By **net section,** in this connection, is meant the smallest cross-section of the plate or bar; this is always a section along a line of rivet holes.

If, as in the foregoing article, t denotes the thickness of the plates joined; d, the diameter of the holes; n_1, the number of rivets in a row; and w, the width of the plate or bar; then the net section $= (w - n_1 d)\, t$.

Let S_t denote the tensile working strength of the plate; then the strength of the unriveted plate is wtS_t, and the reduced tensile strength is $(w - n_1 d)\, t\, S_t$.

The compressive strength of a plate is also lessened by the presence of holes; but when they are again filled up, as in a joint, the metal is replaced, as it were, and the compressive strength of the plate is restored. No allowance is therefore made for holes in figuring the compressive strength of a plate.

105. Computation of the Strength of a Joint. The strength of a joint is determined by either (1) the shearing value of the rivets; (2) the bearing value of the plate; or (3) the tensile strength of the riveted plate if the joint is in tension. Let P_s denote the strength of the joint as computed from the shearing values of the rivets; P_c, that computed from the bearing value of the plates; and P_t, the tensile strength of the riveted plates. Then, as before explained,

$$\left.\begin{aligned} P_t &= (w - n_1 d)\, tS_t; \\ P_s &= n_2\, 0.7854\, d^2 S_s;\ \text{and} \\ P_c &= n_3 t d S_c\,; \end{aligned}\right\} \qquad (20)$$

n_2 denoting the total number of rivets in the joint; and n_3 denoting the total number of rivets in a lap joint, and one-half the number of rivets in a butt joint.

Examples. 1. Two half-inch plates 7½ inches wide are connected by a single lap joint double-riveted, six rivets in two rows. If the diameter of the rivets is $\frac{3}{4}$ inch, and the working strengths are as follows : $S_t = 12{,}000$, $S_s = 7{,}500$, and $S_c = 15{,}000$ pounds per square inch, what is the safe tension which the joint can transmit ?

Here $n_1 = 3$, $n_2 = 6$, and $n_3 = 6$; hence

$$P_t = (7\frac{1}{2} - 3 \times \frac{3}{4}) \times \frac{1}{2} \times 12{,}000 = 31{,}500 \text{ pounds};$$

$$P_s = 6 \times 0.7854 \times (\frac{3}{4})^2 \times 7{,}500 = 19{,}880 \text{ pounds};$$

$$P_c = 6 \times \frac{1}{2} \times \frac{3}{4} \times 15{,}000 = 33{,}750 \text{ pounds}.$$

Since P_s is the least of these three values, the strength of the joint depends on the shearing value of its rivets, and it equals 19,880 pounds.

2. Suppose that the plates described in the preceding example are joined by means of a butt joint (two cover-plates), and 12 rivets are used, being spaced as before. What is the safe tension which the joint can bear ?

Here $n_1 = 3$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_t = 31{,}500; \text{ and } P_c = 33{,}750 \text{ pounds; but}$$

$$P_s = 12 \times 0.7854 \times (\frac{3}{4})^2 \times 7{,}500 = 39{,}760 \text{ pounds}.$$

The strength equals 31,500 pounds, and the joint is stronger than the first.

3. Suppose that in the preceding example the rivets are arranged in rows of two. What is the tensile strength of the joint ?

Here $n_1 = 2$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_s = 39{,}760; \text{ and } P_c = 33{,}750 \text{ pounds; but}$$

$$P_t = (7\frac{1}{2} - 2 \times \frac{3}{4}) \frac{1}{2} \times 12{,}000 = 36{,}000 \text{ pounds}.$$

The strength equals 33,750 pounds, and this joint is stronger than either of the first two.

EXAMPLES FOR PRACTICE.

Note. Use working strengths as in example 1, above.

S_t = 12,000, S_s = 7,500, and S_c = 15,000 pounds per square inch.

1. Two half-inch plates 5 inches wide are connected by a lap joint, with two ¾-inch rivets in a row. What is the safe strength of the joint?

Ans. 6,625 pounds.

2. Solve the preceding example supposing that four ¾-inch rivets are used, in two rows.

Ans. 13,250 pounds.

3. Solve example 1 supposing that three 1-inch rivets are used, placed in a row lengthwise of the joint.

Ans. 17,670 pounds.

4. Two half-inch plates 5 inches wide are connected by a butt joint (two cover-plates), and four ¾-inch rivets are used, in two rows. What is the strength of the joint?

Ans. 11,250 pounds.

106. Efficiency of a Joint. The ratio of the strength of a joint to that of the solid plate is called the "efficiency of the joint." If ultimate strengths are used in computing the ratio, then the efficiency is called **ultimate efficiency;** and if working strengths are used, then it is called **working efficiency.** In the following, we refer to the latter. An efficiency is sometimes expressed as a per cent. To express it thus, multiply the ratio *strength of joint ÷ strength of solid plate*, by 100.

Example. It is required to compute the efficiencies of the joints described in the examples worked out in the preceding article.

In each case the plate is ½ inch thick and 7½ inches wide; hence the tensile working strength of the solid plate is

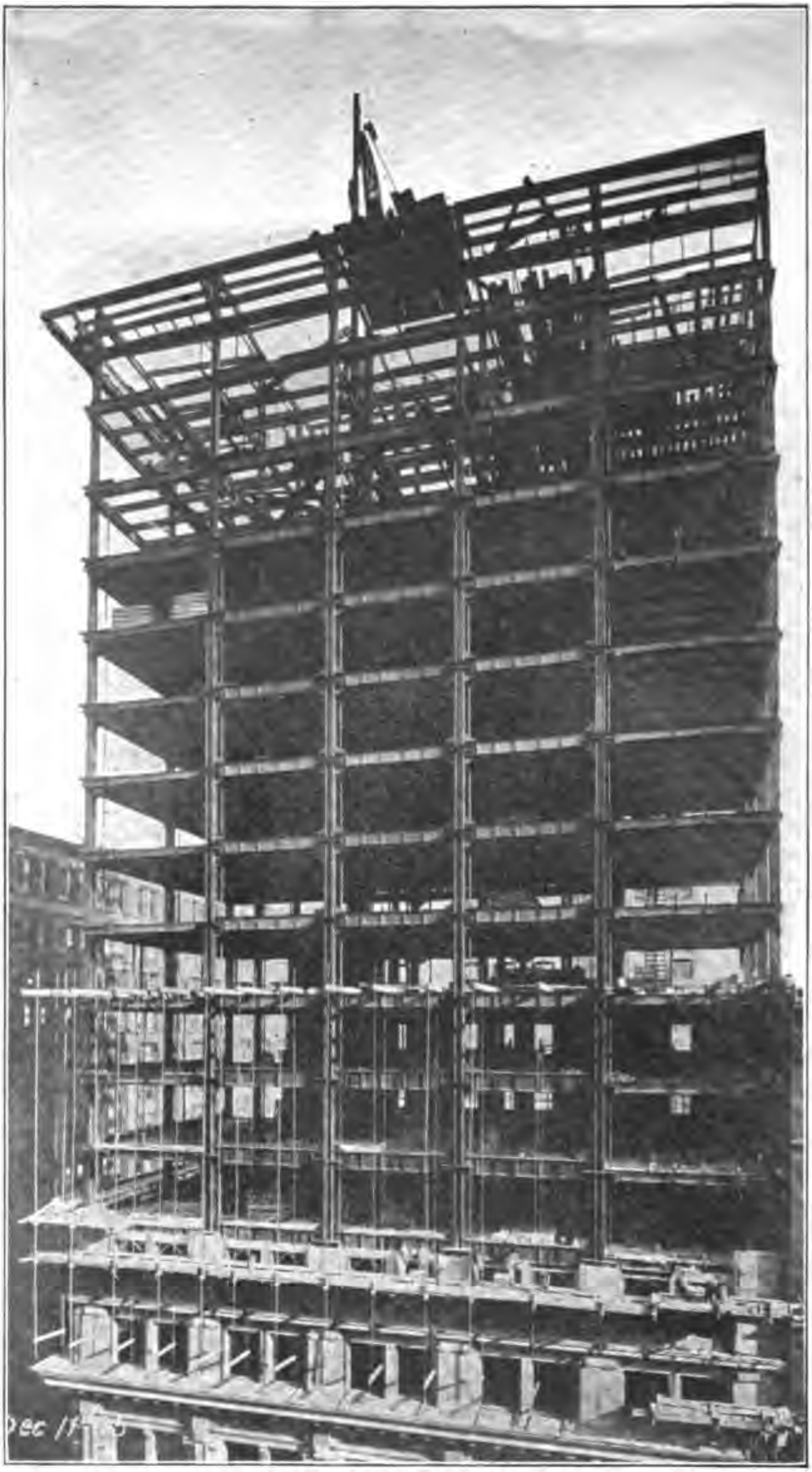
$$7\frac{1}{2} \times \frac{1}{2} \times 12{,}000 = 45{,}000 \text{ pounds.}$$

Therefore the efficiencies of the joints are:

$$(1) \quad \frac{19{,}880}{45{,}000} = 0.44, \text{ or } 44 \text{ per cent;}$$

$$(2) \quad \frac{31{,}500}{45{,}000} = 0.70, \text{ or } 70 \text{ per cent;}$$

$$(3) \quad \frac{33{,}750}{45{,}000} = 0.75, \text{ or } 75 \text{ per cent.}$$



BORLAND BUILDING, CHICAGO, ILL.
Shepley, Rutan & Coolidge, Architects. E. C. & R. M. Shankland, Engineers.
Stone and brick exterior. View taken six months after building was begun.

STATICS.

This subject, called Statics, is a branch of Mechanics. It deals with principles relating especially to forces which act upon bodies at rest, and with their useful applications.

There are two quite different methods of carrying on the discussions and computations. In one, the quantities under consideration are represented by lines and the discussion is wholly by means of geometrical figures, and computations are carried out by means of figures drawn to scale; this is called the *graphical method.* In the other, the quantities under consideration are represented by symbols as in ordinary Algebra and Arithmetic, and the discussions and computations are carried on by the methods of those branches and Trigonometry; this is called the *algebraic method.* In this paper, both methods are employed, and generally, in a given case, the more suitable of the two.

I. PRELIMINARY.

1. Force. The student, no doubt, has a reasonably clear idea as to what is meant by force, yet it may be well to repeat here a few definitions relative to it. By force is meant simply a *push* or *pull.* Every force has **magnitude,** and to express the magnitude of a given force we state how many times greater it is than some standard force. Convenient standards are those of weight and these are almost always used in this connection. Thus when we speak of a force of 100 pounds we mean a force equal to the weight of 100 pounds.

We say that a force has **direction,** and we mean by this the direction in which the force would move the body upon which it acts if it acted alone. Thus, Fig. 1 represents a body being pulled to the right by means of a cord; the direction of the force exerted upon the body is horizontal and to the right. The direction may be indicated by any line drawn in the figure parallel to the cord with an arrow on it pointing to the right.

We say also that a force has a **place of application,** and we mean by that the part or place on the body to which the force is applied. When the place of application is small so that it may be regarded as a point, it is called the "point of application." Thus the place of application of the pressure (push or force) which a locomotive wheel exerts on the rail is the part of the surface of the rail in contact with the wheel. For practically all purposes this pressure may be considered as applied at a point (the center of the surface of contact), and it is called the point of application of the force exerted by the wheel on the rail.

A force which has a point of application is said to have a **line of action**, and by this term is meant the line through the point of application of the force parallel to its direction. Thus, in the Fig. 1, the line of action of the force exerted on the body is the line representing the string. Notice clearly the distinction between the direction and line of action of the force; the direction of the force in the illustration could be represented by any horizontal line in the figure with an arrowhead upon it pointing toward the right, but the line of action can be represented only by the line representing the string, indefinite as to length, but definite in position.

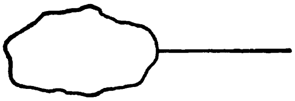


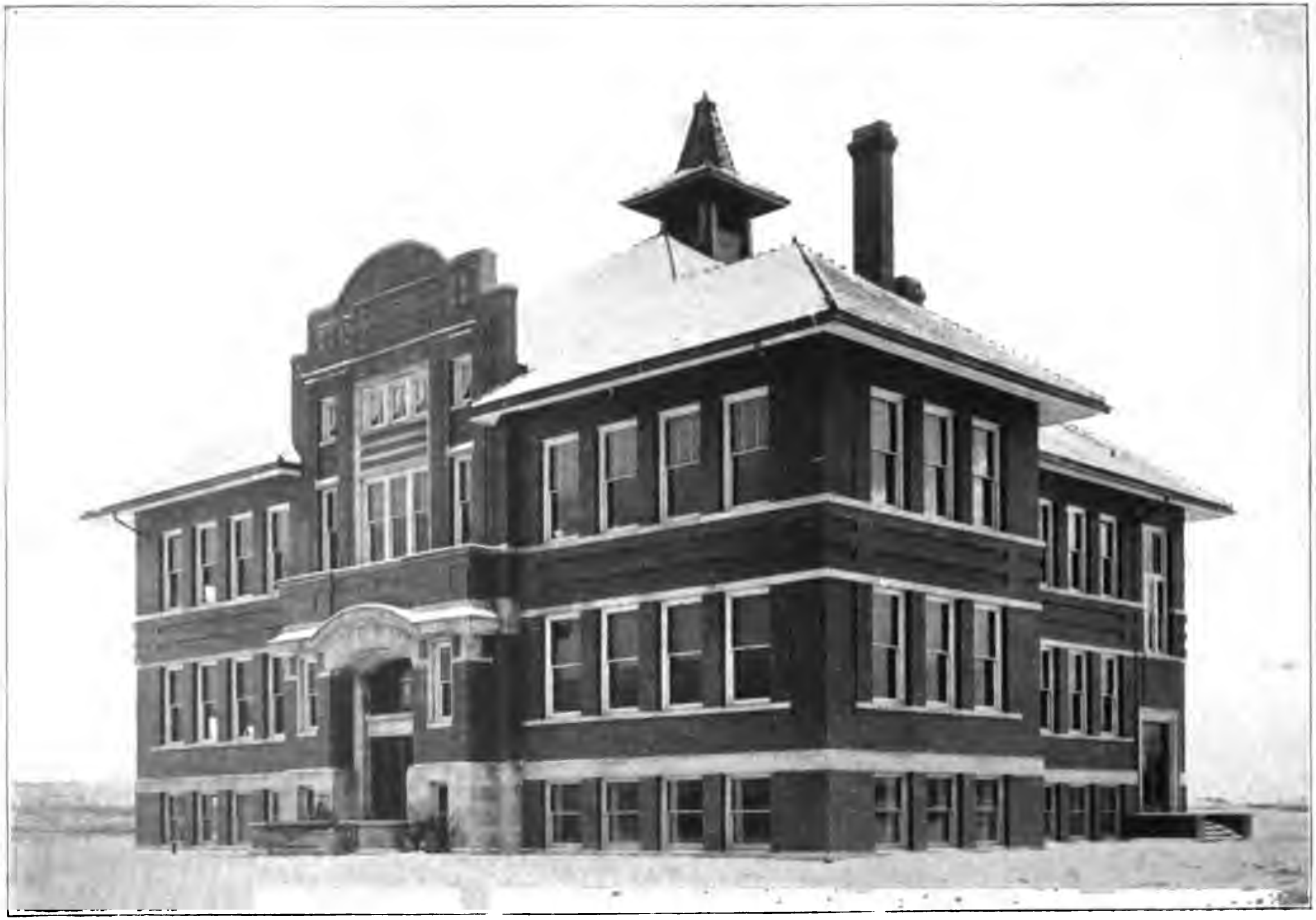
Fig. 1.

That part of the direction of a force which is indicated by means of the arrowhead on a line is called the **sense** of the force. Thus the sense of the force of the preceding illustration is toward the right and not toward the left.

2. Specification and Graphic Representation of a Force. For the purposes of statics, a force is completely specified or described if its

(1) magnitude, (2) line of action, and (3) sense are known or given.

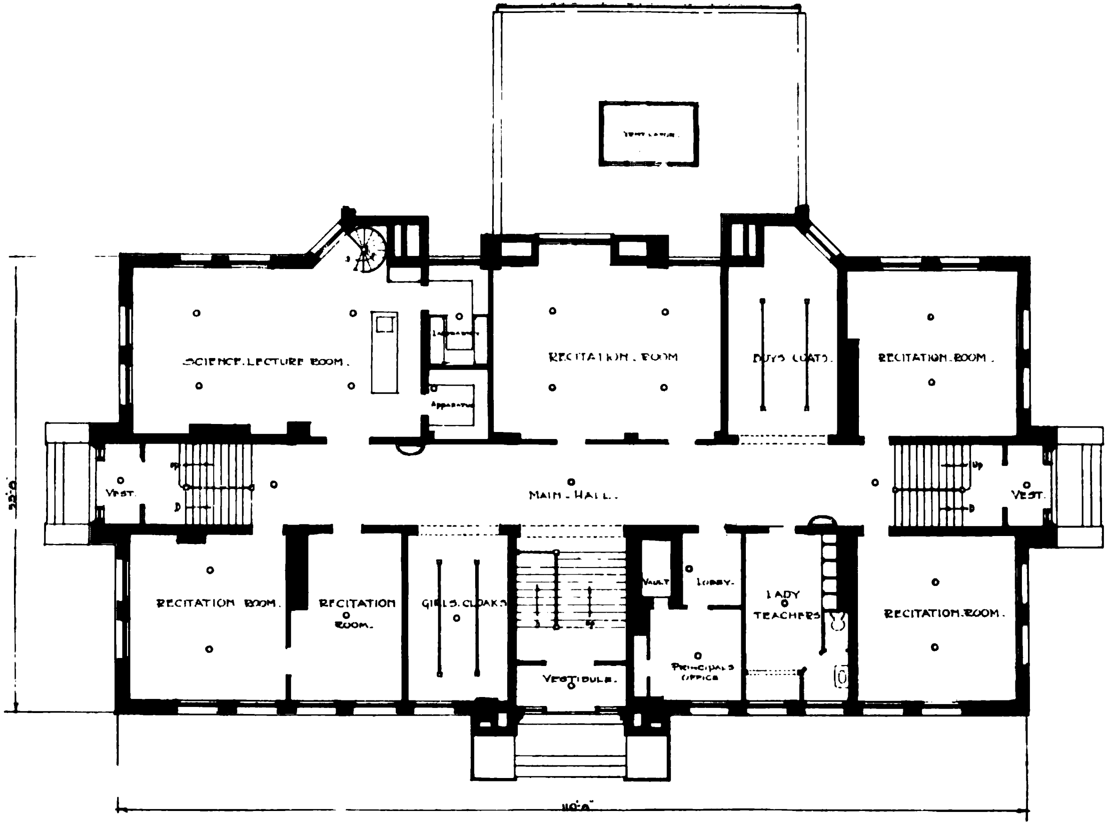
These three elements of a force can be represented graphically, that is by a drawing. Thus, as already explained, the straight line (Fig. 1) represents the line of action of the force exerted upon the body; an arrowhead placed on the line pointing toward the right gives the sense of the force; and a definite length marked off on the line represents to some scale the magnitude of the force. For example, if the magnitude is 50 pounds, then to a scale of 100 pounds to the inch, one-half of an inch represents the magnitude of the force.



HIGH SCHOOL AT THREE RIVERS, MICH.

J. C. Llewellyn, Architect, Chicago, Ill.

Paving Brick Walls and Buff Bedford Trimmings: Tile Roof. Built in 1905.



PLAN. OF. FIRST. FLOOR.

HIGH SCHOOL AT THREE RIVERS, MICH.

J. C. Llewellyn, Architect, Chicago, Ill.

Provision is Made in the Rear for a Future Auditorium. For Plans of Basement and Second Story, See Page 154.

It is often convenient, especially when many forces are concerned in a single problem, to use two lines instead of one to represent a force—one to represent the magnitude and one the line of action, the arrowhead being placed on either. Thus Fig. 2 also represents the force of the preceding example, AB (one-half inch long) representing the magnitude of the force and *ab* its line of action. The line AB might have been drawn anywhere in the figure, but its length is definite, being fixed by the scale.

The part of a drawing in which the body upon which forces act is represented, and in which the lines of action of the forces are drawn, is called the **space diagram** (Fig. 2*a*). If the body were drawn to scale, the scale would be a certain number of inches or feet to the inch. The part of a drawing in which the force magnitudes are laid off (Fig. 2*b*) is called by various names; let us call it the **force diagram.** The scale of a force diagram is always a certain number of pounds or tons to the inch.

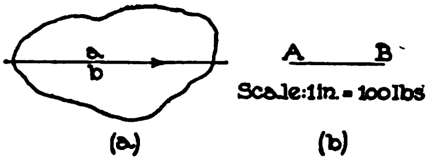


Fig. 2.

3. **Notation.** When forces are represented in two separate diagrams, it is convenient to use a special notation, namely: a capital letter at each end of the line representing the magnitude of the force, and the same small letters on opposite sides of the line representing the action line of the force (see Fig. 2). When we wish to refer to a force, we shall state the capital letters used in the notation of that force; thus "force AB" means the force whose magnitude, action line, and sense are represented by the lines AB and *ab*.

In the algebraic work we shall usually denote a force by the letter F.

4. **Scales.** In this subject, scales will always be expressed in feet or pounds to an inch, or thus, 1 inch = 10 feet, 1 inch = 100 pounds, etc. The number of feet or pounds represented by one inch on the drawing is called the *scale number.*

To find the length of the line to represent a certain distance or force, divide the distance or force by the scale number; the quotient is the length to be laid off in the drawing. To find the *magnitude of a distance or a force represented by a certain line in a drawing, multiply the length of the line by the scale number; the product is the magnitude of the distance or force, as the case may be.*

The scale to be used in making drawings depends, of course, upon how large the drawing is to be, and upon the size of the quantities which must be represented. In any case, it is convenient to select the scale number so that the quotients obtained by dividing the quantities to be represented may be easily laid off by means of the divided scale which is at hand.

Examples. 1. If one has a scale divided into 32nds, what is the convenient scale for representing 40 pounds, 32 pounds, 56 pounds, and 70 pounds?

According to the scale, 1 inch = 32 pounds, the lengths representing the forces are respectively:

$$\frac{40}{32} = 1\tfrac{1}{4};\ \frac{32}{32} = 1;\ \frac{56}{32} = 1\tfrac{3}{4};\ \frac{70}{32} = 2\tfrac{3}{16}\text{ inches.}$$

Since all of these distances can be easily laid off by means of the "sixteenths scale," 1 inch = 32 pounds is convenient.

2. What are the forces represented by three lines, 1.20, 2.11, and 0.75 inches long, the scale being 1 inch = 200 pounds?

According to the rule given in the foregoing, we multiply each of the lengths by 200, thus:

$$1.20 \times 200 = 240\text{ pounds.}$$
$$2.11 \times 200 = 422\text{ pounds.}$$
$$0.75 \times 200 = 150\text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. To a scale of 1 inch = 500 pounds, how long are the lines to represent forces of 1,250, 675, and 900 pounds?

Ans. 2.5, 1.35, and 1.8 inches

2. To a scale of 1 inch = 80 pounds, how large are the forces represented by $1\frac{1}{4}$ and 1.6 inches?

Ans. 100 and 128 pounds.

5. Concurrent and Non-concurrent Forces. If the lines of action of several forces intersect in a point they are called concurrent forces, or a concurrent system, and the point of intersection is called the *point of concurrence* of the forces. If the lines of action of several forces do not intersect in the same point, they are called non-concurrent, or a non-concurrent system.

We shall deal only with forces whose lines of action lie in the same plane. It is true that one meets with problems in which there are forces whose lines of action do not lie in a plane, but such problems can usually be solved by means of the principles herein explained.

6. Equilibrium and Equilibrant. When a number of forces act upon a body which is at rest, each tends to move it; but the effects of all the forces acting upon that body may counteract or neutralize one another, and the forces are said to be *balanced* or *in equilibrium*. Any one of the forces of a system in equilibrium balances all the others. A single force which balances a number of forces is called the *equilibrant* of those forces.

7. Resultant and Composition. Any force which would produce the same effect (so far as balancing other forces is concerned) as that of any system, is called the *resultant* of that system. Evidently the resultant and the equilibrant of a system of forces must be equal in magnitude, opposite in sense, and act along the same line.

The process of determining the resultant of a system of forces is called composition.

8. Components and Resolution. Any number of forces whose combined effect is the same as that of a single force are called *components* of that force. The process of determining the components of a force is called *resolution.* The most important case of this is the resolution of a force into two components.

II. CONCURRENT FORCES; COMPOSITION AND RESOLUTION.

9. Graphical Composition of Two Concurrent Forces. *If two forces are represented in magnitude and direction by AB and BC (Fig. 3), the magnitude and direction of their resultant is represented by AC.* This is known as the "triangle law."

The line of action of the resultant is parallel to AC and passes through the point of concurrence of the two given forces; thus the line of action of the resultant is *ac*.

The law can be proved experimentally by means of two spring balances, a drawing board, and a few cords arranged as shown in

Fig. 4. The drawing board (not shown) is set up vertically, then from two nails in it the spring balances are hung, and these in turn support by means of two cords a small ring A from which a heavy body (not shown) is suspended. The ring A is in equilibrium under the action of three forces, a downward force equal to the weight of the suspended body, and two forces exerted by the upper cords whose values or magnitudes can be read from the spring balances. The first force is the equilibrant of the other two. Knowing the weight of the suspended body and the readings of the balances, lay off AB equal to the pull of the right-hand upper string according to some convenient scale, and BC parallel to the left-hand upper string and equal to the force exerted by it. It will then be found that the line joining A and C is vertical, and equals (by scale) the weight of the suspended body. Hence AC, with arrowhead pointing down, represents the equilibrant of the two upward pulls on the ring; and with arrowhead pointing up, it represents the resultant of those two forces.

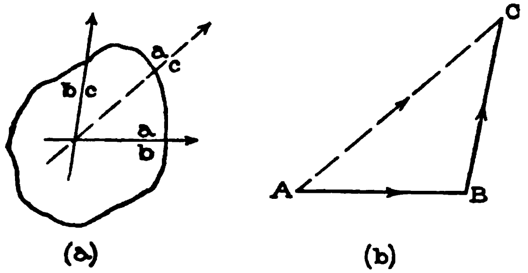


Fig. 3.

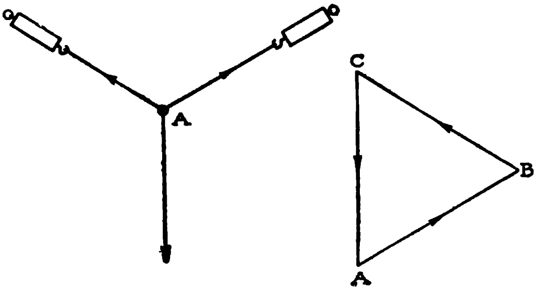


Fig. 4.

Notice especially how the arrowheads are related in the triangle (Fig. 3), and be certain that you understand this law before proceeding far, as it is the basis of most of this subject.

Examples. Fig. 5 represents a board 3 feet square to which forces are applied as shown. It is required to compound or find the resultant of the 100- and 80-pound forces.

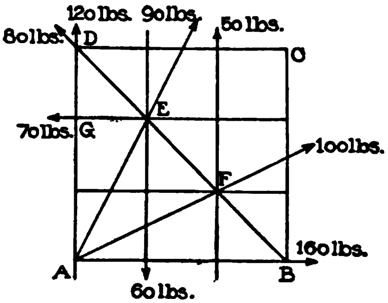
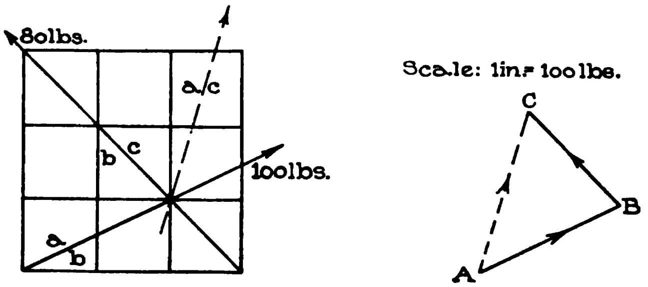


Fig. 5.



(a.) (b)

Fig. 6.

First we make a drawing of the board and mark upon it the lines of action of the two forces whose resultant is to be found, as in Fig. 6. Then by some convenient scale, as 100 pounds to the inch, lay off from any convenient point A, a line AB in the direction of the 100-pound force, and make AB one inch long, representing 100 pounds by the scale. Then from B lay off a line BC in the direction of the second force and make BC, 0.8 of an inch long, representing 80 pounds by the scale. Then the line AC, with the arrow pointing *from A to C*, represents the magnitude and direction of the resultant. Since AC equals 1.06 inch, the resultant equals

$$1.06 \times 100 = 106 \text{ pounds.}$$

The line of action of the resultant is *ac*, parallel to AC and passing through the intersection of the lines of action (the point of concurrence) of the given forces. To complete the notation, we mark these lines of action *ab* and *bc* as in the figure.

EXAMPLES FOR PRACTICE.*

1. Determine the resultant of the 100- and the 120-pound forces represented in Fig. 5.

Ans. { The magnitude is 194 pounds; the force acts upward through A and a point 1.62 feet to the right of D.

2. Determine the resultant of the 120- and the 160-pound forces represented in Fig. 5.

Ans. { The magnitude is 200 pounds; the force acts upward through A and a point 9 inches below C.

10. Algebraic Composition of Two Concurrent Forces. If the angle between the lines of action of the two forces is not 90 degrees, the algebraic method is not simple, and the graphical is usually preferable. If the angle is 90 degrees, the algebraic method is usually the shorter, and this is the only case herein explained.

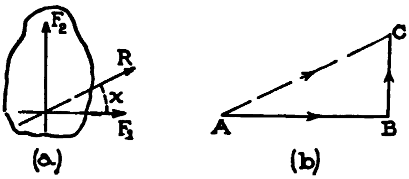


Fig. 7.

Let F_1 and F_2 be two forces acting through some point of a body as represented in Fig. 7*a*. AB and BC represent the magnitudes and direction of F_1 and F_2 respectively; then, according to the triangle law (Art. 9), AC represents the magnitude and direction of the resultant of F_1 and F_2, and the line marked R (parallel to AC) is the line of action of that resultant. Since ABC is a right triangle,

$$(AC)^2 = (AB)^2 + (BC)^2.$$

and,

$$\tan CAB = \frac{BC}{AB}.$$

---

* Use sheets of paper not smaller than large letter size, and devote a full sheet to each example. In reading the answers to these examples, remember that the board on which the forces act was stated to be 3 feet square.

Now let R denote the resultant. Since AC, AB, and BC represent R, F_1, and F_2 respectively, and angle CAB $= x$,

$$R^2 = F_1^2 + F_2^2; \text{ or } R = \sqrt{F_1^2 + F_2^2};$$

and,

$$\tan x = F_2 \div F_1.$$

By the help of these two equations we compute the magnitude of the resultant and inclination of its line of action to the force F_1.

Example. It is required to determine the resultant of the 120- and the 160-pound forces represented in Fig. 5.

Let us call the 160-pound force F_1; then,

$$R = \sqrt{160^2 + 120^2} = \sqrt{25{,}600 + 14{,}400}$$
$$= \sqrt{40{,}000} = 200 \text{ pounds};$$

and,

$$\tan x = \frac{120}{160} = \tfrac{3}{4}; \text{ hence } x = 36^\circ\ 52'.$$

The resultant therefore is 200 pounds in magnitude, acts through A (Fig. 5) upward and to the right, making an angle of 36° 52′ with the horizontal.

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the 50- and 70-pound forces represented in Fig. 5.

Ans. $\begin{cases} R = 86 \text{ pounds}; \\ \text{angle between R and 70-pound force} = 35^\circ\ 32'. \end{cases}$

2. Determine the resultant of the 60- and 70-pound forces represented in Fig 5.

Ans. $\begin{cases} R = 92.2 \text{ pounds}; \\ \text{angle between R and 70-pound force} = 40^\circ\ 36'. \end{cases}$

11. Force Polygon. If lines representing the magnitudes and directions of any number of forces be drawn continuous and so that the arrowheads on the lines point the same way around on the series of lines, the figure so formed is called the *force polygon* for the forces. Thus ABCD (Fig. 8) is a force polygon for the 80-, 90-, and 100-pound forces of Fig. 5, for AB, BC, and CD represent the magnitudes and directions of those forces respectively, and the arrowheads point in the same way around, from A to D.

A number of force polygons can be drawn for any system of forces, no two alike. Thus A_1 B_1 C_1 D_1 and A_2 B_2 C_2 D_2 are other force polygons for the same three forces, 80, 90, and 100 pounds. Notice that A_3 B_3 C_3 D_3 is not a force polygon for the three forces although the lines represent the three forces in magnitude and direction. The reason why it is not a force polygon is that the arrowheads do not all point the same way around.

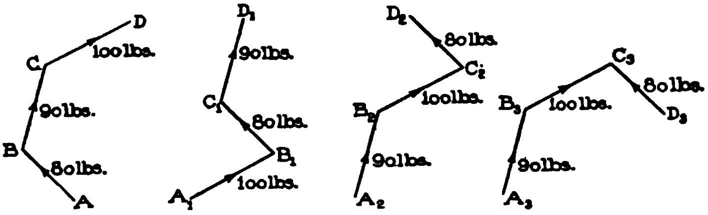


Fig. 8.

A force polygon is not necessarily a closed figure. If a force polygon closes for a system of concurrent forces, then evidently the resultant equals zero.

EXAMPLE FOR PRACTICE.

Draw to the same scale as many different force polygons as you can for the 100-, 120- and 160-pound forces of Fig. 5. Bear in mind that the arrowheads on a force polygon point the same way around.

12. Composition of More Than Two Concurrent Forces. The graphical is much the simpler method; therefore the algebraic one will not be explained. The following is a rule for performing the composition graphically:

(1). Draw a force polygon for the given forces.

(2). Join the two ends of the polygon and place an arrowhead on the joining line pointing from the beginning to the end of the polygon. That line then represents the magnitude and direction of the resultant.

(3). Draw a line through the point of concurrence of the given forces parallel to the line drawn as directed in (2). This line represents the action line of the resultant.

Example. It is required to determine the resultant of the four forces acting through the point E (Fig. 5).

First, make a drawing of the board and indicate the lines of action of the forces as shown in Fig. 9, but without lettering. Then to construct a force polygon, draw from any convenient point A, a line in the direction of one of the forces (the 70-pound force), and make AB equal to 70 pounds according to the scale ($70 \div 100 = 0.7$ inch). Then from B draw a line in the direction of the next force (80-pound), and make BC equal to 0.8 inch, representing 80 pounds. Next draw a line from C in the direction of the third force (90-pound), and make CD equal to 0.9 inch, representing 90 pounds. Finally draw a line from D in the direction of the last force, and make DE equal to 0.6 inch, representing 60 pounds. The force polygon is ABCDE, beginning at A and ending at E.

The second step is to connect A and E and place an arrowhead on the line pointing from A to E. This represents the magnitude and direction of the resultant. Since AE = 1.16 inches, the resultant is a force of

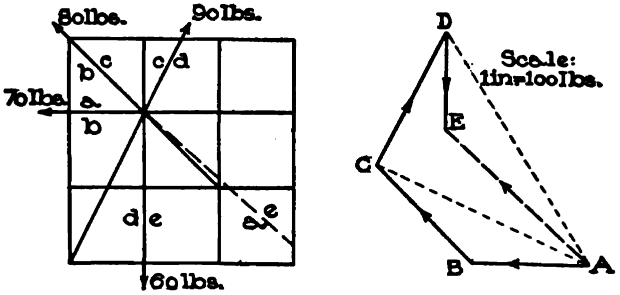


Fig. 9.

$$1.16 \times 100 = 116 \text{ pounds.}$$

The third step is to draw a line *ae* through the point of concurrence and parallel to AE. This is the line of action of the resultant. (To complete the notation the lines of action of the 70-, 80-, 90- and 60-pound forces should be marked *ab*, *bc*, *cd*, and *de* respectively.)

That the rule for composition is correct can easily be proved. According to the triangle law, AC (Fig. 9), with arrowhead pointing from A to C, represents the magnitude and direction of the resultant of the 70- and 80-pound forces. According to the law, AD, with arrowhead pointing from A to D, represents the magnitude and direction of the resultant of AC and the 90-pound force, hence also of the 70-, 80-, and 90-pound forces. According to the law, AE with arrowhead pointing from A to E, represents the magnitude and direction of the resultant of AD and the 60-pound force. Thus we see that the foregoing rule and the triangle law lead to the same result, but the application of the rule is shorter as in it we do not need the lines AC and AD.

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the four forces acting through the point A (Fig. 5).

Ans. { 380 pounds acting upward through A and a point 0.45 feet below C.

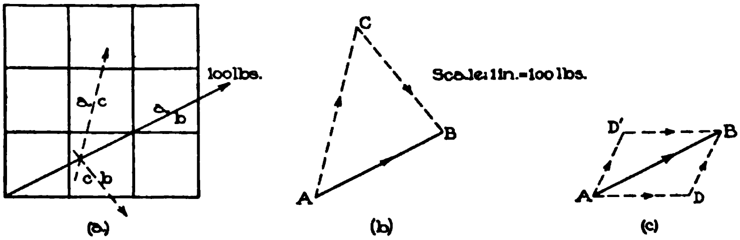


Fig. 10.

2. Determine the resultant of the three forces acting at the point F (Fig. 5).

Ans. { 175 pounds acting upward through F and a point 0.57 feet to left of C.

13. Graphical Resolution of Force into Two Concurrent Components. This is performed by applying the triangle law inversely. Thus, if it is required to resolve the 100-pound force of Fig. 5 into two components, we draw first Fig. 10 (a) to show the line of action of the force, and then AB, Fig. 10 (b), to represent the magnitude and direction. Then draw from A and B any two lines which intersect, mark their intersection C, and place arrowheads on AC and CB, pointing from A to C and from C to B. Also draw two lines in the space diagram parallel to AC and CB and so that they intersect on the line of action of the 100-pound force, *ab*.

The test of the correctness of a solution like this is to take the two components as found, and find their resultant; if the resultant thus found agrees in magnitude, direction, and sense with the given force (originally resolved), the solution is correct.

Notice that the solution above given is not definite, for the lines drawn from A and B were drawn at random. A force may therefore be resolved into two components in many ways. If, however, the components have to satisfy conditions, there may be but one solution. In the most important case of resolution, the lines of action of the components are given; this case is definite, there being but one solution, as is shown in the following example.

Example. It is required to resolve the 100-pound force (Fig. 5) into two components acting in the lines AE and AB.

Using the space diagram of Fig. 10, draw a line AB in Fig. 10 (c) to represent the magnitude and direction of the 100-pound force, and then a line from A parallel to the line of action of either of the components, and a line from B parallel to the other, thus locating D (or D′). Then AD and DB (or AD′ and D′B) represent the magnitudes and directions of the required components.

EXAMPLES FOR PRACTICE.

1. Resolve the 160-pound force of Fig. 5 into components which act in AF and AE.

Ans. { The first component equals 238½ pounds, and its sense is from A to F; the second component equals 119½ pounds, and its sense is from E to A.

2. Resolve the 50-pound force of Fig. 5 into two components, acting in FA and FB.

Ans. { The first component equals 37.3 pounds, and its sense is from A to F; the second component equals 47.0 pounds, and its sense is from B to F.

14. Algebraic Resolution of a Force Into Two Components. If the angle between the lines of action of the two components is not 90 degrees, the algebraic method is not simple and the graphical method is usually preferable. When the angle is 90 degrees, the algebraic method is usually the shorter, and this is the only case herein explained.

Let F (Fig. 11) be the force to be resolved into two components acting in the lines OX and OY. If AB is drawn to represent the magnitude and direction of F, and lines be drawn from A and B parallel to OX and OY, thus locating C, then AC and BC with arrowheads as shown represent the magnitudes and directions of the required components.

Now if F' and F'' represent the components acting in OX and OY, and x and y denote the angles between F and F', and F and F'' respectively, then AC and BC represent F' and F'', and the angles BAC and ABC equal x and y respectively. From the right triangle ABC it follows that

and, $F' = F \cos x$, and $F'' = F \cos y$.

If a force is resolved into two components whose lines of action are at right angles to each other, each is called a *rectangular component* of that force. Thus F' and F'' are rectangular components of F.

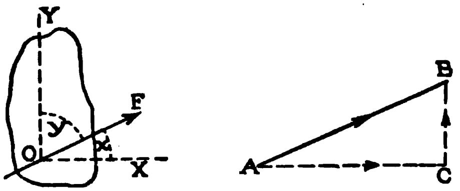


Fig. 11.

The foregoing equations show that *the rectangular component of a force along any line equals the product of the force and the cosine of the angle between the force and the line.* They show also that *the rectangular component of a force along its own line of action equals the force, and its rectangular component at right angles to the line of action equals zero.*

Examples. 1. A force of 120 pounds makes an angle of 22 degrees with the horizontal. What is the value of its component along the horizontal?*

Since $\cos 22° = 0.927$, the value of the component equals $120 \times 0.927 = 111.24$ pounds.

2. What is the value of the component of the 90-pound force of Fig. 5 along the vertical?

First we must find the value of the angle which the 90-pound force of Fig. 5 makes with the vertical.

---

* When nothing is stated herein as to whether a component is rectangular or not, then rectangular component is meant.

Since $\tan \text{EAG} = \frac{\text{EG}}{\text{AG}} = \frac{1}{2}$, angle $\text{EAG} = 26^\circ\ 34'$.

Hence the value of the desired component equals

$$90 \times \cos 26^\circ\ 34' = 90 \times 0.8944 = 80.50 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. Compute the horizontal and vertical components of a force of 80 pounds whose angle with the horizontal is 60 degrees

Ans. { 40 pounds.
69.28 pounds.

2. Compute the horizontal and vertical components of the 100-pound force in Fig. 5. What are their senses?

Ans. { 89.44 pounds to the right.
44.72 pounds upwards.

3. Compute the component of the 70-pound force in Fig. 5 along the line EA. What is the sense of the component?

Ans. 31. 29 pounds ; E to A.

III. CONCURRENT FORCES IN EQUILIBRIUM.

15. Condition of Equilibrium Defined. By condition of equilibrium of a system of forces is meant a relation which they must fulfill in order that they may be in equilibrium or a relation which they fulfill when they are in equilibrium.

In order that any system may be in equilibrium, or be balanced, their equilibrant, and hence their resultant, must be zero, and this is a condition of equilibrium. If a system is known to be in equilibrium, then, since the forces balance among themselves, their equilibrant and hence their resultant also equals zero. This (the necessity of a zero resultant) is known as the general condition of equilibrium for it pertains to all kinds of force systems. For special kinds of systems there are special conditions, some of which are explained in the following.

16. Graphical Condition of Equilibrium. *The "graphical condition of equilibrium" for a system of concurrent forces is that the polygon for the forces must close.* For if the polygon closes, then the resultant equals zero as was pointed out in Art 11.

By means of this condition we can solve problems relating to concurrent forces which are known to be in equilibrium. The most common and practically important of these is the following:

The forces of a concurrent system in equilibrium are all known except two, but the lines of action of these two are known; it is required to determine their magnitudes and directions. This problem arises again and again in the "analysis of trusses" (Arts. 23 to 26) but will be illustrated first in simpler cases.

Example. 1. Fig. 12 represents a body resting on an inclined plane being prevented from slipping down by a rope fastened to it as shown. It is required to determine the pull or tension on the rope and the pressure of the plane if the body weighs 120 pounds and the surface of the plane is perfectly smooth.*

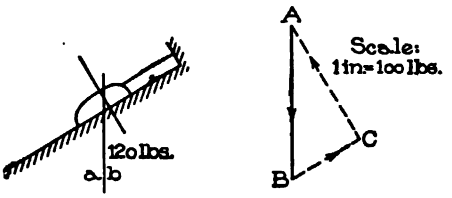


Fig. 12.

There are three forces acting upon the body, namely, its weight directly downwards, the pull of the rope and the reaction or pressure of the plane which, as explained in the footnote, is perpendicular to the plane. We now draw the polygon for these forces making it close; thus draw AB (1.2 inches long) to represent the magnitude and direction of the weight, 120 pounds, then from A a line parallel to either one of the other forces, from B a line parallel to the third, and mark the intersection of these two lines C; then ABCA is the polygon. Since the arrowhead on AB must point down and since the arrowheads in any force polygon must point the same way around, those on BC and CA must point as shown.

Hence BC (0.6 inch, or 60 pounds) represents the magnitude and direction of the pull of the rope and CA (1.04 inches, or 104 pounds) represents the magnitude and direction of the pressure of the plane on the body.

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* By "a perfectly smooth" surface is meant one which offers no resistance to the sliding of a body upon it. Strictly, there are no such surfaces, as all real surfaces exert more or less frictional resistance. But there are surfaces which are practically perfectly smooth. We use perfectly smooth surfaces in some of our illustrations and examples for the sake of simplicity, for we thus avoid the force of friction, and the reaction or force exerted by such a surface on a body resting upon it is perpendicular to the surface.

2. A body weighing 200 pounds is suspended from a small ring which is supported by means of two ropes as shown in Fig. 13. It is required to determine the pulls on the two ropes.

There are three forces acting on the ring, namely the downward force equal to the weight of the body and the pulls of the two ropes. Since the ring is at rest, the three forces balance or are in equilibrium, and hence their force polygon must close. We proceed to draw the polygon and in making it close, we shall determine the values of the unknown pulls. Thus, first draw AB (1 inch long) to represent the magnitude and direction of the known force, 200 pounds; the arrowhead on it must point down. Then from A a line parallel to one of the ropes and from B a line parallel to the other and mark their intersection C. ABCA is the polygon for the three forces, and since in any force polygon the arrows point the same way around, we place arrowheads on BC and CA as shown. Then BC and CA represent the magnitudes and directions of the pulls exerted on the ring by the right- and left-hand ropes respectively.

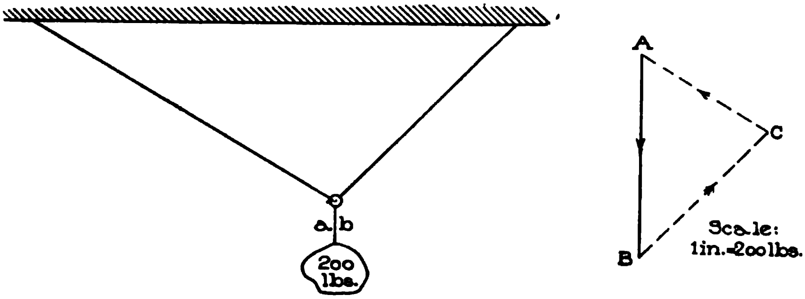


Fig. 13.

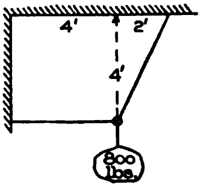


Fig. 14.

BC = 0.895 inches and represents 179 pounds.
CA = 0.725 inches and represents 145 pounds.

The directions of the pulls are evident in this case and the arrowheads are superfluous, but they are mentioned to show how to place them and what they mean so that they may be used when necessary. To complete the notation, the rope at the right should be marked *bc* and the other *ca*.

EXAMPLES FOR PRACTICE.

1. Fig. 14 represents a body weighing 800 pounds suspended from a ring which is supported by two ropes as shown. Compute the pulls on the ropes.

Ans. { Pull in the horizontal rope = 400 pounds.
Pull in the inclined rope = 894 pounds.

2. Suppose that in Fig. 12 the rope supporting the body on the plane is so fastened that it is horizontal. Determine the pull on the rope and the pressure on the plane if the inclination of the plane to the horizontal is 30 degrees and the body weighs 120 pounds.

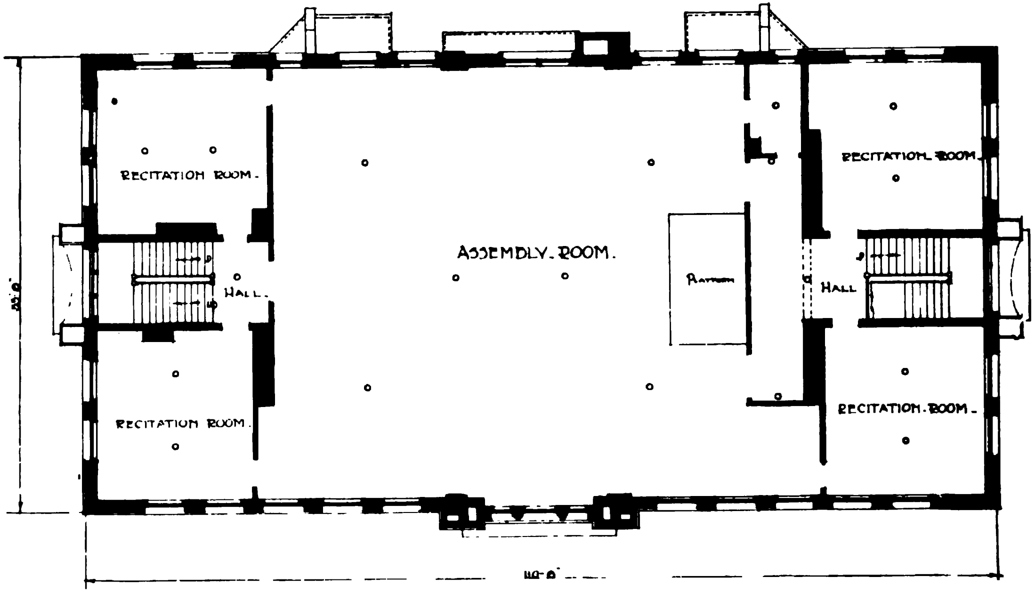
Ans. { Pull = 68.7 pounds.
Pressure = 138 pounds.

3. A sphere weighing 400 pounds rests in a V-shaped trough, the sides of which are inclined at 60 degrees with the horizontal. Compute the pressures on the sphere.

Ans. 400 pounds.

17. **Algebraic Conditions of Equilibrium.** Imagine each one of the forces of a concurrent system in equilibrium replaced by its components along two lines at right angles to each other, horizontal and vertical for example, through the point of concurrence. Evidently the system of components would also be in equilibrium. Now since the components act along one of two lines (horizontal or vertical), all the components along each line must balance among themselves for if either set of components were not balanced, the body would be moved along that line. Hence we state that the conditions of equilibrium of a system of concurrent forces are that the resultants of the two sets of components of the forces along any two lines at right angles to each other must equal zero.

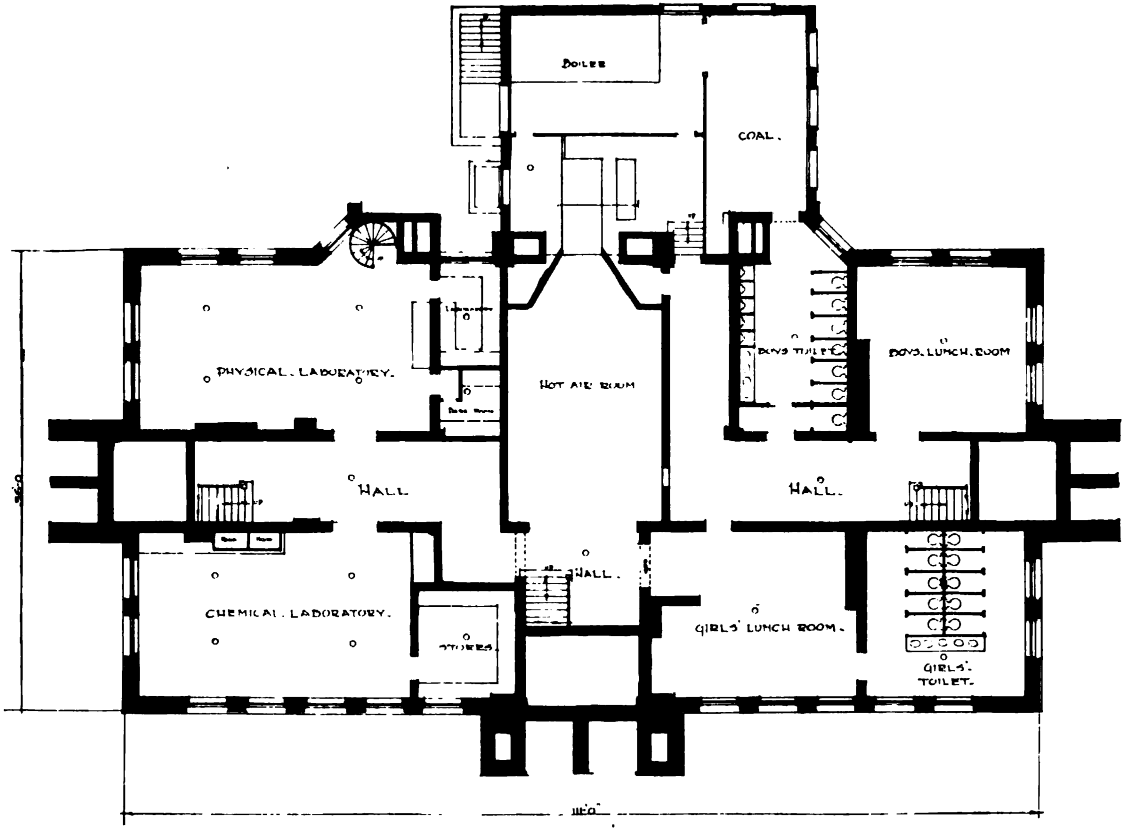
If the components acting in the same direction along either of the two lines be given the plus sign and those acting in the other direction, the negative sign, then it follows from the foregoing that the condition of equilibrium for a concurrent system is that



PLAN. OF. SECOND. FLOOR.

HIGH SCHOOL AT THREE RIVERS, MICH.

J. C. Llewellyn, Architect, Chicago, Ill.
Basement Plan Shown on Opposite Page.



PLAN. OF. BASEMENT.

HIGH SCHOOL AT THREE RIVERS, MICH.

J. C. Llewellyn, Architect, Chicago, Ill.

Second-Floor Plan Shown on Opposite Page. For Exterior and Plan of First Floor, See Page 138

the algebraic sums of the components of the forces along each of two lines at right angles to each other must equal zero.

Examples. 1. It is required to determine the pull on the rope and the pressure on the plane in Example 1, Art. 16 (Fig. 12), it being given that the inclination of the plane to the horizontal is 30 degrees.

Let us denote the pull of the rope by F_1 and the pressure of the plane by F_2. The angles which these forces make the horizontal are 30° and 60° respectively; hence

the horizontal component of $F_1 = F_1 \times \cos 30° = 0.8660\ F_1$,
and " " " " $F_2 = F_2 \times \cos 60° = 0.5000\ F_2$;
also " " " " the weight $= 0$.

The angles which F_1 and F_2 make with the vertical are 60° and 30° respectively, hence

the vertical component of $F_1 = F_1 \times \cos 60° = 0.5000\ F_1$,
and the vertical component of $F_2 = F_2 \times \cos 30° = 0.8660\ F_2$;
also the vertical component of the weight $= 120$.

Since the three forces are in equilibrium, the horizontal and the vertical components are balanced, and hence

$$0.866\ F_1 = 0.5\ F_2$$

and

$$0.5\ F_1 + 0.866\ F_2 = 120.$$

From these two equations F_1 and F_2 may be determined; thus from the first,

$$F_2 = \frac{0.866}{0.5}\ F_1 = 1.732\ F_1.$$

Substituting this value of F_2 in the second equation we have

$$0.5\ F_1 + 0.866 \times 1.732\ F_1 = 120,$$

or

$$2\ F_1 = 120;$$

hence,

$$F_1 = \frac{120}{2} = 60 \text{ pounds},$$

and

$$F_2 = 1.732 \times 60 = 103.92 \text{ pounds}.$$

2. It is required to determine the pulls in the ropes of Fig. 13 by the algebraic method, it being given that the angles which the left- and right-hand ropes make with the ceiling are 30 and 70 degrees respectively.

Let us denote the pulls in the right- and left-hand ropes by F_1 and F_2 respectively. Then

the horizontal component of $F_1 = F_1 \times \cos 70° = 0.342\ F_1$,
the horizontal component of $F_2 = F_2 \times \cos 30° = 0.866\ F_2$,
the horizontal component of the weight $= 0$,
the vertical component of $F_1 = F_1 \times \cos 20° = 0.9397\ F_1$,
the vertical component of $F_2 = F_2 \times \cos 60° = 0.500\ F_2$,
and the vertical component of the weight $= 100$.

Now since these three forces are in equilibrium, the horizontal and the vertical components balance; hence

$$0.342\ F_1 = 0.866\ F_2$$

and

$$0.9397\ F_1 + 0.5\ F_2 = 100.$$

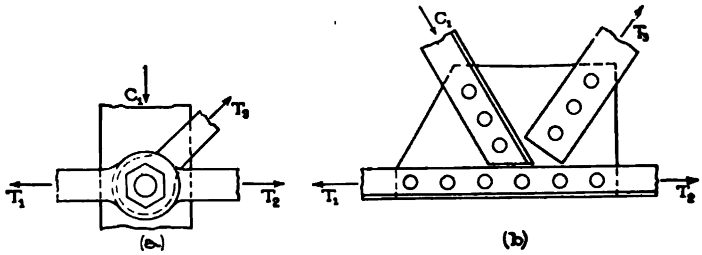


Fig. 15.

These equations may be solved for the unknown forces; thus from the first,

$$F_1 = \frac{0.866}{0.342} F_2 = 2.532\ F_2.$$

Substituting this value of F_1 in the second equation, we get

$$0.9397 \times 2.532\ F_2 + 0.5\ F_2 = 100,$$

or,

$$2.88\ F_2 = 100;$$

hence

$$F_2 = \frac{100}{2.88} = 34.72 \text{ pounds},$$

and

$$F_1 = 2.532 \times 34.72 = 87.91 \text{ pounds}.$$

EXAMPLES FOR PRACTICE.

1. Solve Ex. 1, Art. 16 algebraically. (First determine the angle which the inclined rope makes with the horizontal; you should find it to be 63° 26'.)

2. Solve Ex. 2, Art. 16 algebraically.
3. Solve Ex. 3, Art. 16 algebraically.

IV. ANALYSIS OF TRUSSES; "METHOD OF JOINTS."

18. **Trusses.** A truss is a frame work used principally to support loads as in roofs and bridges. Fig. 16, 25, 26 and 27 represent several forms of trusses. The separate bars or rods, $\overline{12}$, $\overline{23}$, etc. (Fig. 16) are called *members* of the truss and all the parts immediately concerned with the connection of a number of members at one place constitute a *joint*. A "pin joint" is shown in Fig. 15 (*a*) and a "riveted joint" in 15 (*b*).

19. **Truss Loads.** The loads which trusses sustain may be classified into fixed, or dead, and moving or live loads. A fixed, or dead load, is one whose place of application is fixed with reference to the truss, while a moving or live load is one whose place of application moves about on the truss.

Roof truss loads are usually fixed, and consist of the weight of the truss, roof covering, the snow, and the wind pressure, if any. Bridge truss loads are fixed and moving, the first consisting of the weights of the truss, the floor or track, the snow, and the wind pressure, and the second of the weight of the passing trains or wagons.

In this paper we shall deal only with trusses sustaining fixed loads, trusses sustaining moving loads being discussed later.

Weight of Roof Trusses. Before we can design a truss, it is necessary to make an estimate of its own weight; the actual weight can be determined only after the truss is designed. There are a number of formulas for computing the probable weight of a truss, all derived from the actual weights of existing trusses. If W denotes the weight of the truss, l the span or distance between supports and a the distance between adjacent trusses, then for steel trusses

$$W = al\left(\frac{l}{25} + 1\right);$$

and the weight of a wooden truss is somewhat less.

Roof Covering. The beams extending between adjacent trusses to support the roof are called *purlins*. On these there are sometimes placed lighter beams called *rafters* which in turn support *roof boards* or "*sheathing*" and the other covering. Sometimes the purlins are spaced closely, no rafters being used.

The following are weights of roof materials in *pounds per square foot* of roof surface:

Sheathing: Boards, 3 to 5.
Shingling: Tin, 1; wood shingles, 2 to 3; iron, 1 to 3; slate, 10; tiles, 12 to 25.
Rafters: 1.5 to 3.
Purlins: Wood, 1 to 3; iron, 2 to 4.

Snow Loads. The weight of the snow load that may have to be borne depends, of course, on location. It is usually taken from 10 to 30 pounds per square foot of area covered by the roof.

Wind Pressure. Wind pressure per square foot depends on the velocity of the wind and the inclination of the surface on which it blows to the direction of the wind. A horizontal wind blowing at 90 miles per hour produces a pressure of about 40 pounds per square foot on a surface perpendicular to the wind, while on surfaces inclined, the pressures are as follows:

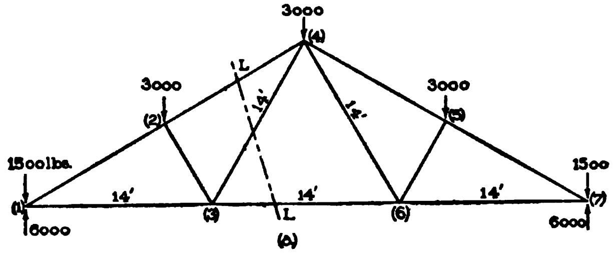


Fig. 16.

10° to the horizontal, 15 pounds per square foot,
20° " " " , 24 " " " "
30° " " " , 32 " " " "
40° " " " , 36 " " " "
50°-90° " " " , 40 " " " "

The wind pressure on an inclined surface is practically perpendicular to the surface.

20. **Computation of "Apex Loads."** The weight of the roof covering including rafters and purlins comes upon the trusses at the points where they support the purlins; likewise the

pressure due to wind and snow. Sometimes all the purlins are supported at joints; in such cases the loads mentioned act upon the truss at its joints. However, the roof, snow, and wind loads are always assumed to be applied to the truss at the upper joints of the trusses. This assumption is equivalent to neglecting the bending effect due to the pressure of those purlins which are not supported at joints. This bending effect can be computed separately.

The weight of the truss itself is assumed to come upon the truss at its upper joints; this, of course, is not exactly correct. Most of the weight does come upon the upper joints for the upper members are much heavier than the lower and the assumption is in most cases sufficiently correct.

Examples. 1. It is required to compute the apex loads for the truss represented in Fig. 16, it being of steel, the roof such that it weighs 15 pounds per square foot, and the distance between adjacent trusses 14 feet.

The span being 42 feet, the formula for weight of truss (Art. 19) becomes

$$14 \times 42 \left(\frac{42}{25} + 1\right) = 1{,}575.84 \text{ pounds.}$$

The length $\overline{14}$ scales about $24\frac{1}{4}$ feet, hence the area of roofing sustained by one truss equals

$$48\tfrac{1}{2} \times 14 = 679 \text{ square feet,}$$

and the weight of the roofing equals

$$679 \times 15 = 10{,}185 \text{ pounds.}$$

The total load equals

$$1{,}575.84 + 10{,}185 = 11{,}760.84 \text{ pounds.}$$

Now this load is to be proportioned among the five upper joints, but joints numbered (1) and (7) sustain only one-half as much load as the others. Hence for joints (1) and (7) the loads equal

$$\frac{1}{8} \text{ of } 11{,}760 = 1{,}470,$$

and for (2), (4) and (5) they equal

$$\frac{1}{4} \text{ of } 11{,}760 = 2{,}940 \text{ pounds.}$$

As the weight of the truss is only estimated, the apex loads would be taken as 1,500 and 3,000 pounds for convenience.

2. It is required to compute the apex loads due to a snow load on the roof represented in Fig. 16, the distance between trusses being 14 feet.

The horizontal area covered by the roof which is sustained by one truss equals

$$42 \times 14 = 588 \text{ square feet.}$$

If we assume the snow load equal to 10 pounds per horizontal square foot, than the total snow load borne by one truss equals

$$588 \times 10 = 5{,}880 \text{ pounds.}$$

This load divided between the upper joints makes

$$\frac{1}{8} \times 5{,}880 = 735 \text{ pounds}$$

at joints (1) and (7); and

$$\frac{1}{4} \times 5{,}860 = 1{,}465 \text{ pounds}$$

at the joints (2), (4), and (5).

3. It is required to compute the apex loads due to wind pressure on the truss represented in Fig. 16, the distance between trusses being 14 ft.

The inclinatien of the roof to the horizontal can be found by measuring the angle from a scale drawing with a protractor or by computing as follows: The triangle $\overline{346}$ is equilateral, and hence its angles equal 60 degrees and the altitude of the triangle equals

$$14 \times \sin 60 = 12.12 \text{ feet.}$$

The tangent of the angle $\overline{413}$ equals

$$\frac{12.12}{21} = 0.577,$$

and hence the angle equals 30 degrees.

According to Art. 19, 32 pounds per square foot is the proper value of the wind pressure. Since the wind blows only on one side of the roof at a given time, the pressure sustained by one truss is the wind pressure on one half of the area of the roof sustained by one truss, that is

$$14 \times 24\tfrac{1}{4} \times 32 = 10,864 \text{ pounds.}$$

One half of this pressure comes upon the truss at joint (2) and one fourth at joints (1) and (4).

EXAMPLES FOR PRACTICE.

1. Compute the apex loads due to weight for the truss represented in Fig. 27 if the roofing weighs 10 pounds per square foot and the trusses (steel) are 12 feet apart.

Ans. As shown in Fig. 27.

2. Compute the apex loads due to a snow load of 20 pounds per square foot on the truss of Fig. 25, the distance between trusses being 15 feet.

Ans. For joints (4) and (7), 1,200 pounds.
For joints (1) and (3), 3,600 pounds.
For joint (2) , 4,800 pounds.

3. Compute the apex loads due to wind for the truss of Fig. 26, the distance between trusses being 15 feet.

Ans. Pressure equals practically 29 pounds per square foot. Load at joint (2) is 4,860 and at joints (1) and (3) 2,430 pounds.

21. **Stress in a Member.** If a truss is loaded only at its joints, its members are under either tension or compression, but the weight of a member tends to bend it also, unless it is vertical. If purlins rest upon members between the joints, then they also bend these members. We have therefore tension members, compression members, and members subjected to bending stress combined with tension or compression. Calling simple tension or compression *direct stress* as in "Strength of Materials," then the process of determining the direct stress in the members is called "analyzing the truss."

22. **Forces at a Joint.** By "forces at a joint" is meant all the loads, weights, and reactions which are applied there and the forces which the members exert upon it. These latter are pushes for compression members and pulls for tension members, in each case acting along the axis of the member. Thus, if the horizontal

and inclined members in Fig. 15 are in tension, they exert pulls on the joint, and if the vertical is a compression member, it exerts a push on the joint as indicated. *The forces acting at a joint are therefore concurrent and their lines of action are always known.*

23. **General Method of Procedure.** The forces acting at a joint constitute a system in equilibrium, and since the forces are concurrent and their lines of action are all known, we can determine the magnitude of two of the forces if the others are all known; for this is the important problem mentioned in Art. 16 which was illustrated there and in Art. 17.

Accordingly, after the loads and reactions on a truss, which is to be analyzed, have been ascertained*, we look for a joint at which only two members are connected (the end joints are usually such). Then we consider the forces at that joint and determine the two unknown forces which the two members exert upon it by methods explained in Arts. 16 or 17. The forces so ascertained are the direct stresses, or stresses, as we shall call them for short, and *they are the values of the pushes or pulls which those same members exert upon the joints at their other ends.*

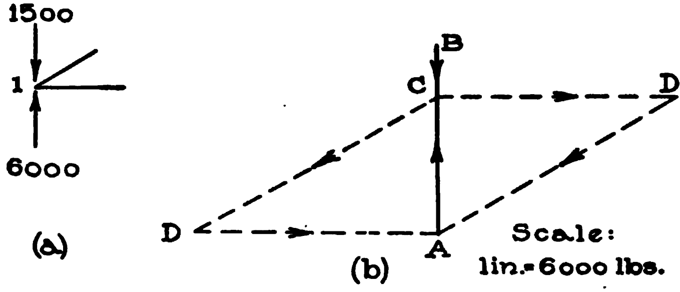


Fig. 17.

Next we look for another joint at which but two unknown forces act, then determine these forces, and continue this process until the stress in each member has been ascertained. We explain further by means of

Examples. 1. It is desired to determine the stresses in the members of the steel truss, represented in Fig. 16, due to its own weight and that of the roofing assumed to weigh 12 pounds per square foot. The distance between trusses is 14 feet.

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* How to ascertain the values of the reactions is explained in Art. 37. For the present their values in any given case are merely stated.

The apex loads for this case were computed in Example 1, Art. 20, and are marked in Fig. 16. Without computation it is plain that each reaction equals one-half the total load, that is, ½ of 12,000, or 6,000 pounds.

The forces at joint (1) are four in number, namely, the left reaction (6,000 pounds), the load applied there (1,500 pounds), and the forces exerted by members $\overline{12}$ and $\overline{13}$. For clearness, we represent these forces so far as known in Fig. 17 (*a*); we can determine the two unknown forces by merely constructing a closed force polygon for all of them. To construct the polygon, we first represent the known forces; thus AB (1 inch long with arrowhead pointing up) represents the reaction and BC (¼ inch long with arrowhead pointing down) represents the load. Then from A and C we draw lines parallel to the two unknown forces and mark their intersection D (or D′). Then the polygon is ABCDA, and CD (1.5 inches = 9,000 pounds) represents the force exerted by the member $\overline{12}$ on the joint and DA (1.3 inches = 7,800 pounds) represents the force exerted by the member $\overline{13}$ on the joint. The arrowheads on BC and CD must point as shown, in order that all may point the same way around, and hence the force exerted by member $\overline{12}$ acts toward the joint and is a push, and that exerted by $\overline{13}$ acts away from the joint and is a pull. It follows that $\overline{12}$ is in compression and $\overline{13}$ in tension.

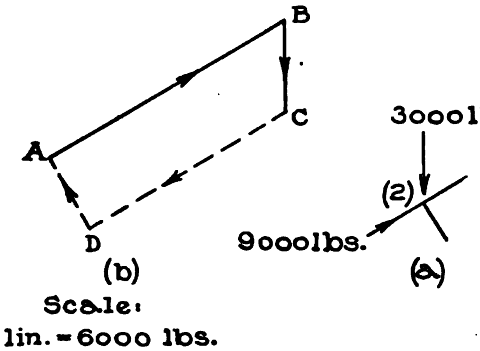

Fig. 18.

If D′ be used, the same results are reached, for the polygon is ABCD′A with arrowheads as shown, and it is plain that CD′ and DA also D′A and CD are equal and have the same sense. But one of these force polygons is preferable for reasons explained later.

Since $\overline{12}$ is in compression, it exerts a push (9,000 pounds) on joint (2) as represented in Fig. 18 (*a*), and since $\overline{13}$ is in tension it exerts a pull (7,800 pounds) on joint (3) as represented in Fig. 19 (*a*).

The forces at joint (2) are four in number, the load (3,000 pounds), the force 9,000 pounds, and the force exerted upon it by the members $\overline{24}$ and $\overline{23}$; they are represented as far as known in Fig. 18 (*a*). We determine the unknown forces by constructing a closed polygon for all of them. Representing the known forces first, draw AB (1.5 inches long with arrowhead pointing up) to represent the 9,000 pound force and BC ($\frac{1}{2}$ inch long with arrowhead pointing down) to represent the load of 3,000 pounds. Next from A and C draw lines parallel to the two unknown forces and mark their intersection D; then the force polygon is ABCDA and the arrowheads on CD and DA must point as shown. CD (1.25 inches = 7,500 pounds) represents the force exerted on joint (2) by $\overline{24}$; since it acts toward the joint the force is a push and member $\overline{24}$ is in compression. DA (0.43 inches = 2,580 pounds) represents the force exerted on the joint by member $\overline{23}$; since the force acts toward the joint it is a push and the member is in compression. Member 23 therefore exerts a push on joint (3) as shown in Fig. 19 (*a*).

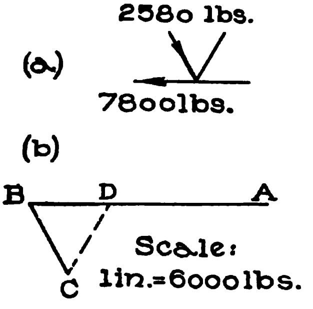

Fig. 19.

At joint (3) there are four forces, 7,800 pounds, 2,580 pounds, and the forces exerted on the joint by members $\overline{34}$ and $\overline{36}$. To determine these, construct the polygon for the four forces. Thus, AB (1.3 inches long with arrowhead pointing to the left) represents the 7,800-pound force and BC (0.43 inches long with arrowheads pointing down) represents the 2,580-pound force. Next draw from A and C two lines parallel to the unknown forces and mark their intersection D; then the force polygon is ABCDA and the arrowhead on CD and DA must point upward and to the right respectively. CD (0.43 inches = 2,580 pounds) represents the force exerted on the joint by member $\overline{34}$; since the force acts away from the joint it is a pull and the member is in tension. DA (0.87 inches = 4,920 pounds) represents the force exerted upon the joint by the member $\overline{36}$; since the force acts away from the joint, it is a pull and the member is in tension.

We have now determined the amount and kind of stress in members $\overline{12}$, $\overline{13}$, $\overline{23}$, $\overline{24}$, $\overline{34}$ and $\overline{36}$. It is evident that the stress in each of the members on the right-hand side is the same as the stress in the corresponding one on the left-hand side; hence further analysis is unnecessary.

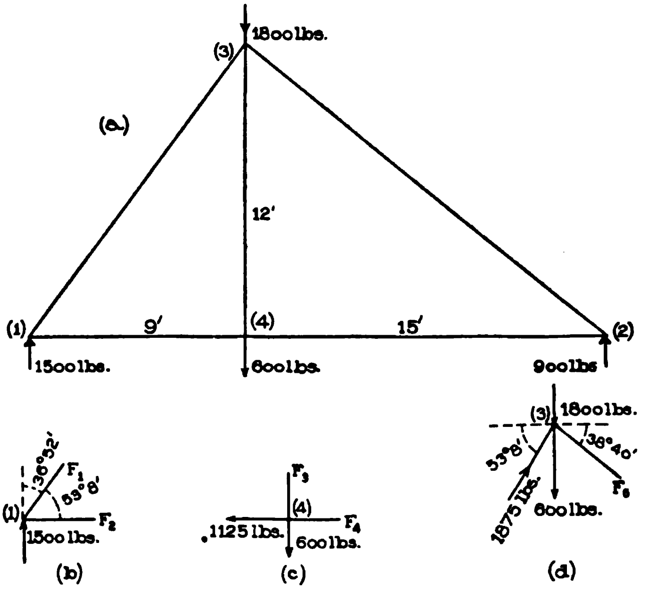

Fig. 20.

2. It is required to analyze the truss represented in Fig. 20 (*a*), the truss being supported at the ends and sustaining two loads, 1,800 and 600 pounds, as shown. (For simplicity we assumed values of the load; the lower one might be a load due to a suspended body. We shall solve algebraically.)

The right and left reactions equal 900 and 1,500 pounds as is shown in Example 1, Page 56. At joint (1) there are three forces, namely, the reaction 1,500 pounds and the forces exerted by members 13 and $\overline{14}$, which we will denote by F_1 and F_2 respect ively. The three forces are represented in Fig. 20 (*b*) as far as they are known. These three forces being in equilibrium, their horizontal and their vertical components balance. Since there are but two horizontal components and two vertical components it follows that (for balance of the components) F_1 must act downward and F_2 toward the right. Hence member $\overline{13}$ pushes on the joint and is under compression while member $\overline{14}$ pulls on the joint and is under tension. From the figure it is plain that

the horizontal component of $F_1 = F_1 \cos 53° 8' = 0.6\ F_1$*,
the horizontal component of $F_2 = F_2$,
the vertical component of $F_1 = F_1 \cos 36° 52' = 0.8\ F_1$,
and the vertical component of the reaction $= 1{,}500$.

Hence $0.6\ F_1 = F_2$, and $0.8\ F_1 = 1{,}500$;

or, $$F_1 = \frac{1{,}500}{0.8} = 1{,}875 \text{ pounds,}$$

and $$F_2 = 0.6 \times 1{,}875 = 1{,}125 \text{ pounds.}$$

Since members $\overline{14}$ and $\overline{13}$ are in tension and compression respectively, $\overline{14}$ pulls on joint (4) as shown in Fig. 20 (*c*) and $\overline{13}$ pushes on joint (3) as shown in Fig. 20 (*d*).

The forces acting at joint (4) are the load 600 pounds, the pull 1,125 pounds, and the forces exerted by members $\overline{34}$ and $\overline{24}$; the last two we will call F_3 and F_4 respectively. The four forces being horizontal or vertical, it is plain without computation that for balance F_4 must be a pull of 1,125 pounds and F_3 one of 600 pounds. Since members $\overline{42}$ and $\overline{43}$ pull on the joint they are both in tension.

Member $\overline{43}$, being in tension, pulls down on joint (3) as shown in Fig. 20 (*d*). The other forces acting on that joint are the load 1,800 pounds, the push 1,875 pounds, the pull 600 pounds, and the force exerted by member $\overline{32}$ which we will call F_5. The only one of these forces having horizontal components are 1,875 and F_5; hence in order that these two components may balance, F_5 must act toward the left. F_5 is therefore a push and the member $\overline{32}$ is under compression.

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* The angles can be computed from the dimensions of the truss; often they can be ascertained easiest by scaling them with a protractor from a large size drawing of the truss.

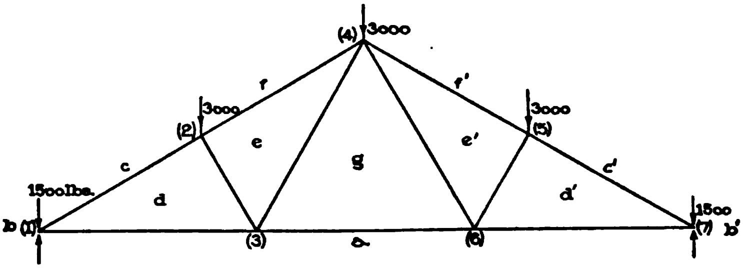
The horizontal component of 1,875 = 1,875 × cos 53° 8′ = 1,125; and the horizontal component of $F_5 = F_5 \times \cos 38° 40' = 0.7808\ F_5$.

Hence $0.7808\ F_5 = 1{,}125$,

or, $$F_5 = \frac{1{,}125}{.7808} = 1{,}440 \text{ pounds.}$$

(This same truss is analyzed graphically later.)

24. **Notation for Graphical Analysis of Trusses.** The notation described in Art. 3 can be advantageously systematized in this connection as follows: Each triangular space in the diagram of the truss and the spaces between consecutive lines of action of the loads and reactions should be marked by a small letter (see Fig. 21 *a*). Then the two letters on opposite sides of any line serve to denote that line and the same large letters are used to denote the force acting in that line. Thus *cd* (Fig. 21 *a*) refers to the member $\overline{12}$ and CD should be used to stand for the force or stress in that member.



(a)

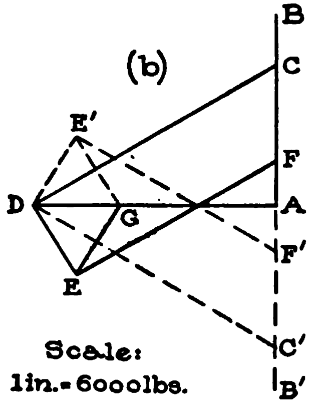
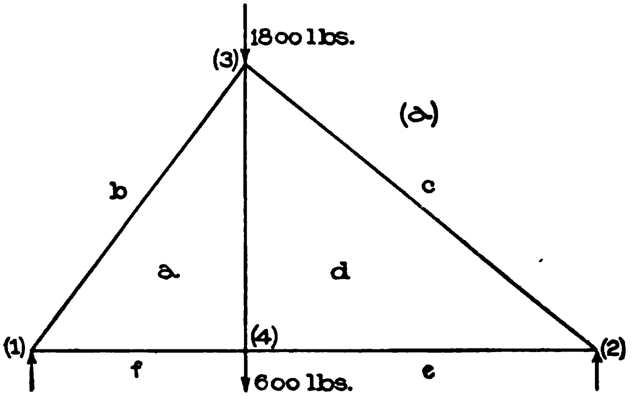


Fig. 21.

25. **Polygon for a Joint.** In drawing the polygon for all the forces at a joint, it is advantageous to represent the forces in the order in which they occur about the joint. Evidently there are always two possible orders thus (see Fig. 20 *d*) F_5, 600, 1,875, and 1,800 is one order around, and F_5, 1,800, 1,875, and 600 is another. The first is called a clockwise order and the second counter-clockwise.

A force polygon for the forces at a joint in which the forces are represented in either order in which they occur about the joint is called *a polygon for the joint*, and it will be called a clockwise or counter-clockwise polygon according as the order followed is clockwise or counter-clockwise. Thus in Fig. 17 (*a*), ABCDA is a clockwise polygon for joint (1). ABCD'A is a polygon for the *forces* at the joint; it is not a polygon for the *joint* because the order in which the forces are represented in that polygon is not the same as either order in which they occur about the joint.



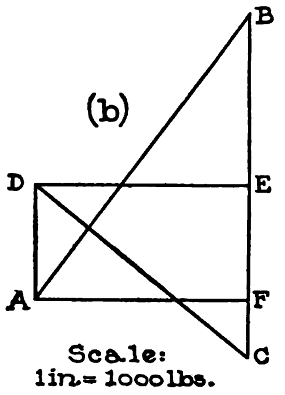


Fig. 22.

(Draw the counter-clockwise polygon for the joint and compare it with ABCDA and ABCD'A.)

26. **Stress Diagrams.** If the polygons for all the joints of a truss are drawn separately as in Example 1, Art. 23, the stress in each member will have been represented twice. It is possible to combine the polygons so that it will be unnecessary to represent the stress in any one member more than once, thus reducing the number of lines to be drawn. Such a combination of force polygons is called a stress diagram.

Fig 21 (b) is a stress diagram for the truss of Fig. 21 (*a*) same as the truss of Fig. 16. It will be seen that the part of the stress diagram consisting of solid lines is a combination of separate polygons previously drawn for the joints on the left half of the truss (Figs. 17, 18 and 19.) It will also be seen that the polygons are all clockwise, but counter-clockwise polygons could be combined into a stress diagram.

To Construct a Stress Diagram for a Truss Under Given Loads.

1. Determine the reactions*.
2. Letter the truss diagram as explained in Art. 24.
3. Construct a force polygon for all the forces applied to the truss (loads and reactions) representing them in the order in which they occur around the truss, clockwise or counter-clockwise. (The part of this polygon representing the loads is called a load line.)
4. On the sides of that polygon, construct the polygons for all the joints. They must be clockwise or counter-clockwise according as the polygon for the loads and reactions is clockwise or counter-clockwise. (The first polygon for a joint must be drawn for one at which but two members are connected—the joints at the supports are usually such. Then one can draw in succession the polygons for joints at which there are not more than two unknown forces until the stress diagram is completed.)

Example. It is desired to construct a stress diagram for the truss represented in Fig. 22 (*a*), it being supported at its ends and sustaining two loads of 1,800 and 600 pounds as shown.

The right and left reactions are 900 and 1,500 pounds as is shown in Example 1, Art. 37. Following the foregoing directions we first letter the truss, as shown. Then, where convenient, draw the polygon for all the loads and reactions, beginning with any force, but representing them in order as previously directed. Thus, beginning with the 1,800-pound load and following the clockwise order for example, lay off a line 1.8 inch in length representing 1,800 pounds (scale 1,000 pounds to an inch); since the line of action of the force is *bc*, the line is to be marked BC and B should be placed at the upper end of the line for a reason which will presently appear. The next force to be represented is the right reaction, 900 pounds ; hence from C draw a line upward and 0.90 inch long. The line of action of this force being *ce*, the line just drawn should be marked CE and since C is already at the lower end, we mark the upper end E. (The reason for placing B at the upper end of the first line is now apparent.) The next force to be represented is the 600-pound load ; therefore we draw from E a line downward and 0.6 inch long, and since the line of action of that force is *ef*, mark the lower end of the line F. The next force to be represented is the left reaction, 1,500 pounds, hence we draw a line 1.5 inches long and upward from F. If the lines have been carefully laid off, the end of the last line should fall at B, that is, the polygon should close.

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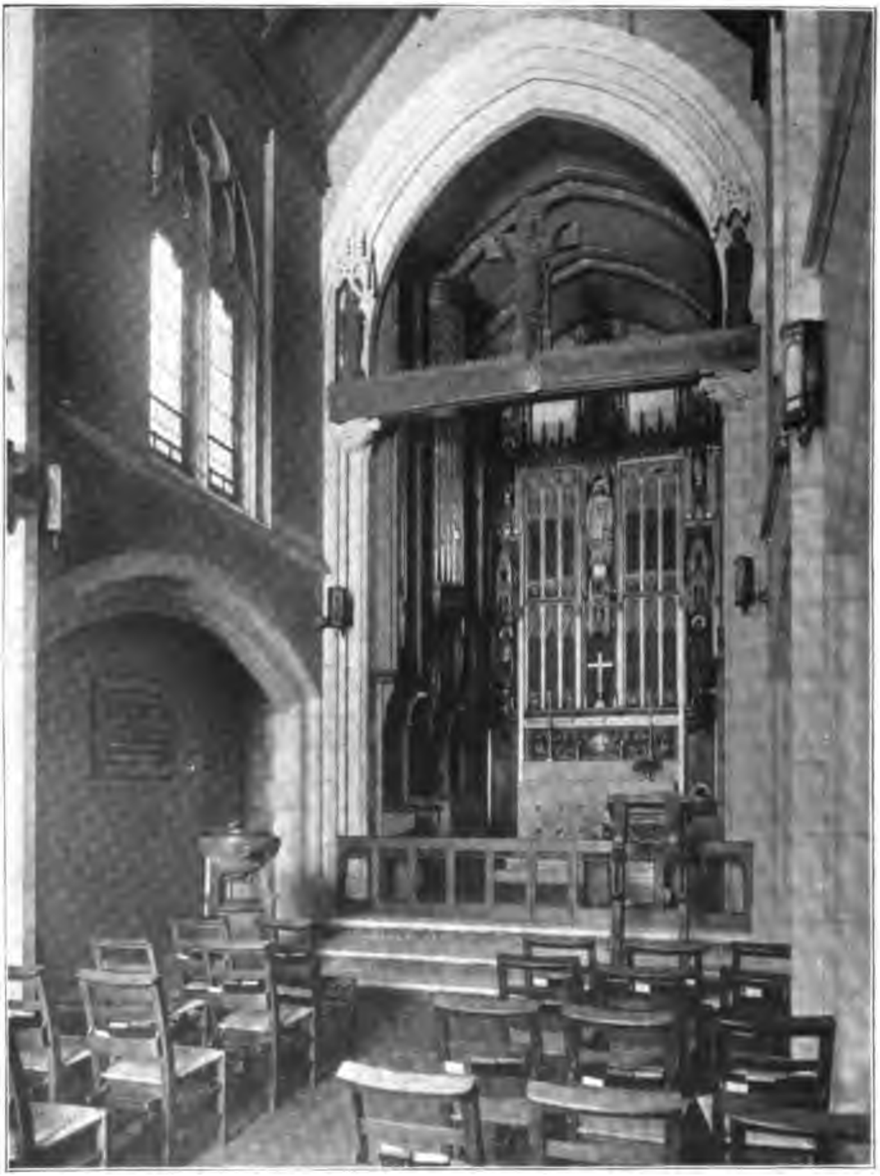
* As already stated, methods for determining reactions are explained in Art. 37; for the present the values of the reactions in any example will be given.

We are now ready to draw polygons for the joints; we may begin at the right or left end as we please but we should bear in mind that the polygons must be clockwise because the polygon for the loads and reactions (BCEFB) is such an one. Beginning at the right end for example, notice that there are three forces there, the right reaction, *de* and *dc*. The right reaction is represented by CE, hence from E draw a line parallel to *de* and from C one parallel to *dc* and mark their intersection D. Then CEDC is the clockwise polygon for the right-hand joint, and since CE acts up, the arrows on ED and DC would point to the left and down respectively. It is better to place the arrows near the joint to which they refer than in the stress diagram; this is left to the student. The force exerted by member *ed* on joint (2) being a pull, *ed* is under tension, and since ED measures 1.12 inches, the value of that tension is 1,120 pounds. The force exerted by member *dc* on joint (2) being a push, *dc* is under compression, and since DC measures 1.44 inches, the value of that compression is 1,440 pounds.

The member *dc* being in compression, exerts a push on the joint (3) and the member *de* being in tension, exerts a pull on the joint (4). Next indicate this push and pull by arrows.

We might now draw the polygon for any one of the remaining joints, for there are at each but two unknown forces. We choose to draw the polygon for the joint (3). There are four forces acting there, namely, the 1,800-pound load, the push (1,440 pounds) exerted by *cd*, and the forces exerted by members *ad* and



GRACE MEMORIAL CHAPEL, CHICAGO, ILL.

Cram, Goodhue & Ferguson, Architects, Boston and New York.
Interior of Gray Pressed Brick; Trimmings of Terra-Cotta of Color and Finish Similar to Bedford Stone. Floor of 9-in. Welsh Quarries; Chancel Floor of Mercer Tile. For Exterior. See Vol. I, Page 10; for Detail of Wood-Carving, See Vol. II. Page 26.

ab, unknown in amount and sense. Now the first two of these forces are already represented in the stress diagram by BC and CD, therefore we draw from D a line parallel to *da* and from B a line parallel to *ba* and mark their intersection A. Then BCDAB is the polygon for the joint, and since the arrowhead on BC and CD would point down and up respectively, DA acts down and AB up; hence place arrowheads in those directions on *da* and *ab* near the joint being considered. These arrows signify that member *da* pulls on the joint and *ba* pushes; hence *da* is in tension and *ba* in compression. Since DA and AB measure 0.6 and 1.88 inches respectively, the values of the tension and compression are 600 and 1,880 pounds.

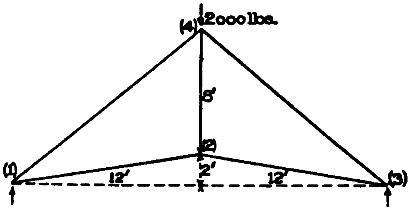


Fig. 23.

Next place arrowheads on *ab* and *ad* at joints (1) and (4) to represent a push and a pull respectively. There remains now but one stress undetermined, that in *af*. It can be ascertained by drawing the polygon for joint 1 or 4; let us draw the latter. There are four forces acting at that joint, namely, the 600-pound load, and the forces exerted by members *ed*, *da*, and *af*. The first three forces are already represented in the drawing by EF, DE and DA, and the polygon for those three forces (not closed) is ADEF. The fourth force must close the polygon, that is, a line from F parallel to *af* must pass through A, and if the drawing has been accurately done, it will pass through A. The polygon for the four forces then is ADEFA, and an arrowhead placed on FA ought to point to the left, but as before, place it in the truss diagram on *af* near joint (4). The force exerted by member *af* on joint (4) being a pull, *af* is under tension, and since AF measures 1.12 inches, the value of the tension is 1,120 pounds.

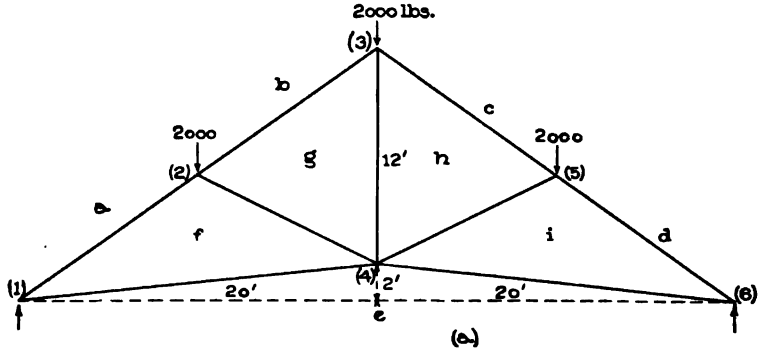
Since *af* is in tension it pulls on joint (1), hence we place an arrowhead on *af* near joint (1) to indicate that pull.

EXAMPLES FOR PRACTICE.

1. Construct a stress diagram for the truss of the preceding Example (Fig. 22*a*) making all the polygons counter-clockwise, and compare with the stress diagram in Fig. 22.

2. Determine the stresses in the members of the truss represented in Fig. 23 due to a single load of 2,000 pounds at the peak.

Ans. { Stresses in $\overline{12}$ and $\overline{23}$ = 1,510 pounds,
Stresses in $\overline{14}$ and $\overline{43}$ = 1,930 pounds,
Stress in $\overline{24}$ = 490 pounds.



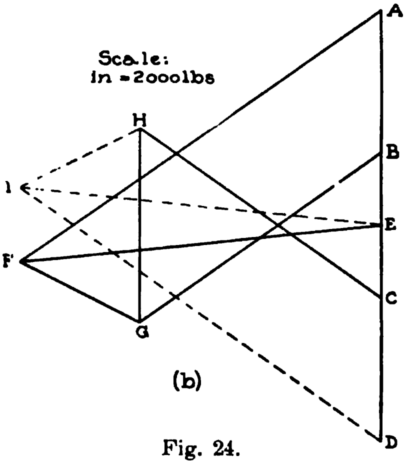


Fig. 24.

27. **Stress Records.** When making a record of the values of the stresses as determined in an analysis of a truss, it is convenient to distinguish between tension and compression by means of the signs plus and minus. Custom differs as to use of the signs for this purpose, but we shall use *plus for tension and minus for compression*. Thus + 4,560 means a tensile stress of 4,560 pounds, and - 7,500 means a compressive stress of 7,500 pounds.

The record of the stresses as obtained in an analysis can be conveniently made in the form of a table, as in Example 1 following, or in the truss diagram itself, as in Example 2 (Fig. 25).

As previously explained, the stress in a member is tensile or compressive according as the member pulls or pushes on the joints between which it extends. If the arrowheads are placed on the lines representing the members as was explained in Example 1 of Art. 26 (Fig. 22), the two arrowheads on any member

> **point toward each other on tension members,**
> **and from each other on compression members.**

If the system of lettering explained in Art. 24 is followed in the analysis of a truss, and if the first polygon (for the loads and reactions) is drawn according to directions (Art. 26), then the system of lettering will guide one in drawing the polygons for the joints as shown in the following illustrations. It must be remembered always that any two parallel lines, one in the truss and one in the stress diagram, must be designated by the same two letters, the first by small letters on opposite sides of it, and the second by the same capitals at its ends.

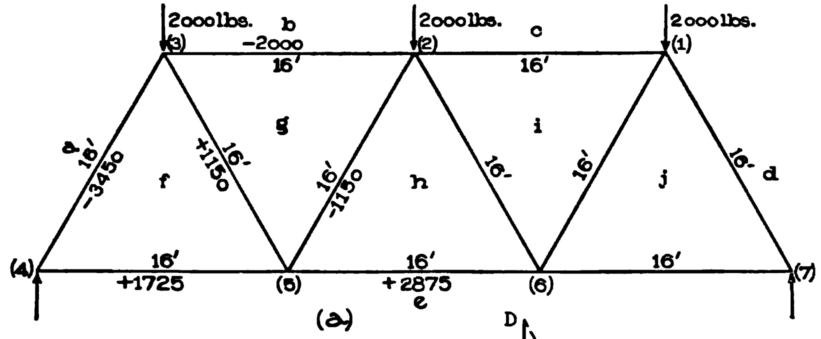
Examples. 1. It is required to construct a stress diagram for the truss represented in Fig. 24 supported at its ends and sustaining three loads of 2,000 pounds as shown. Evidently the reactions equal 3,000 pounds.

Following the directions of Art. 26, we letter the truss diagram, then draw the polygon for the loads and reactions. Thus, to the scale indicated in Fig. 24 (*b*), AB, BC, and CD represent the loads at joints (2), (3) and (5) respectively and DE and EA represent the right and the left reactions respectively. Notice that the polygon (ABCDEA) is a clockwise one.

At joint (1) there are three forces, the left reaction and the forces exerted by the members *af* and *fe*. Since the forces exerted by these two members must be marked AF and EF we draw from A a line parallel to *af* and from E one parallel to *ef* and mark their intersection F. Then EAFE is the polygon for joint (1), and since EA acts up (see the polygon), AF acts down and FE to the right. We, therefore, place the proper arrowheads on *af* and *fe* near (1), and record (see adjoining table) that the stresses in those members are compressive and tensile respectively. Measuring, we find that AF and FE equal 6,150 and 5,100 pounds respectively.

Member....	af	fe	bg	fg	gh
Stress.. ...	- 6,150	+5,100	- 4,100	- 1,875	+2,720

We may next draw the polygon for joint (6) or (2) since there are but two unknown forces at each. At joint (2) for instance, the unknown forces are those exerted by *fg* and *bg*, and the known are the load *ab* (2,000 pounds) and the force exerted by *af*. Since the unknown forces must be marked FG and GB, draw from F a line parallel to *fg*, from B a line parallel to *bg*, and mark their intersection G. Then the polygon for the joint is FABGF, and since AB acts down (see the polygon) BG and GF act down and up respectively. Therefore, place the proper arrowheads on *bg* and *gf* near (2), and record that the stresses in those members are both compressive. Measuring, we find that BG and GF scale 4,100 and 1,875 pounds respectively.



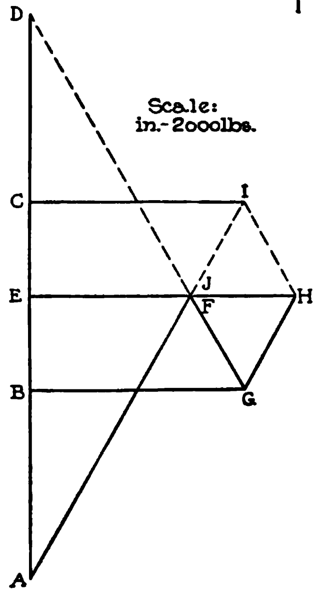


Fig. 25.

Now draw a polygon for joint (3) or (6) since there are but two unknown forces at each joint. At (3) for instance, the unknown forces are those exerted by *ch* and *gh*, the known forces being the load (2,000 pounds) and the force 4,100 pounds, exerted by *bg*. Since the unknown forces must be marked CH and GH, draw from C a line parallel to *ch*, from G one parallel to *gh*, and mark their intersection H. Then

GBCHG is the polygon for the joint, and since BC acts down (see the polygon) CH acts up and HG down. Therefore, place the proper arrowheads on *ch* and *hg* near (3), and record that the stresses in those members are compressive and tensile respectively. Measuring, we find that CH and HG scale 4,100 and 2,720 pounds respectively.

It is plain that the stress in any member on the right-hand side is the same as that in the corresponding member on the left, hence it is not necessary to construct the complete stress diagram.

2. It is required to analyze the truss of Fig. 25 which rests on end supports and sustains three loads each of 2,000 pounds as shown. Each member is 16 feet long.

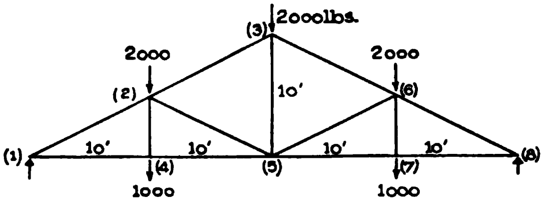


Fig. 26.

Evidently, reactions are each 3,000 pounds. Following directions of Art. 26, first letter the truss diagram and then draw a polygon for the loads and reactions representing them in either order in which they occur about the truss. DCBAED is a counter-clockwise polygon, DC, CB, and BA representing the loads at joints (1), (2) and (3), AE the left reaction, ED the right reaction.

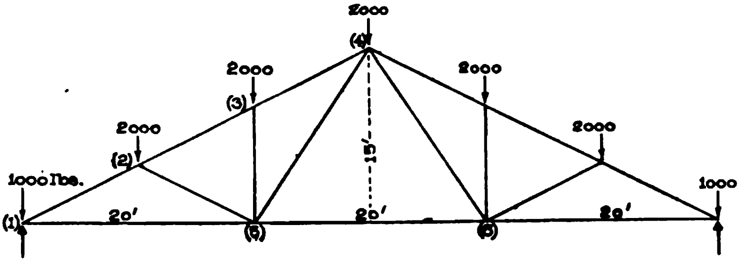


Fig. 27.

The construction of the polygons is carried out as in the preceding illustration, and little explanation is necessary. The polygon for joint (4) is AEFA, EF (1,725 pounds tension) representing the stress in *ef* and FA (3,450 pounds compression) that in *af*. The polygon for joint (3) is BAFGB, FG (1,150 pounds tension) representing the stress in *fg* and GB (2,000 pounds compression) that in *gb*. The polygon for joint (5) is GFEHG, EH (2,875 pounds tension) representing the stress in *eh* and HG (1,150 pounds compression) that in *hg*.

Evidently the stress in any member on the right side of the truss is like that in the corresponding member on the left, therefore it is not necessary to construct the remainder of the stress diagram.

EXAMPLES FOR PRACTICE.

1. Analyze the truss represented in Fig. 26, it being supported at its ends and sustaining three loads of 2,000 and two of 1,000 pounds as represented.

STRESS RECORD.

Member......	$\overline{12}$	$\overline{23}$	$\overline{14}$	$\overline{45}$	$\overline{24}$	$\overline{25}$	$\overline{35}$
Stress.........	-8,950	-5,600	+8,000	+8,000	+1,000	-3,350	+3,000

2. Analyze the truss represented in Fig. 27, it being supported at its ends and sustaining five 2,000-pound loads and two of 1,000 as shown.

STRESS RECORD.

Member.	$\overline{12}$	$\overline{23}$	$\overline{34}$	$\overline{51}$	$\overline{52}$	$\overline{53}$	$\overline{54}$	$\overline{56}$
Stress ...	-8,900	-8,900	-9,400	+10,000	-2,000	-2,000	+4,000	+6,000

28. **Analysis for Snow Loads.** In some cases the apex snow loads are a definite fractional part of the apex loads due to the weights of roof and truss. For instance, in Examples 1 and 2, Pages 25 and 26, it is shown that the apex loads are 1,500 and 3,000 pounds due to weight of roof and truss, and 735 and 1,470 due to snow; hence the snow loads are practically equal to one-half of the permanent dead loads. It follows that the stress in any member due to snow load equals practically one-half of the stress in that member due to the

permanent dead load. The snow load stresses in this case can therefore be obtained from the permanent load stresses and no stress diagram for snow load need be drawn.

In some cases, however, the apex loads due to snow at the various joints are not the *same* fractional part of the permanent load. This is the case if the roof is not all of the same slope, as for instance in Fig. 25 where a part of the roof is flat. In such a case the stresses due to the snow load cannot be determined from a stress diagram for the permanent dead load but a separate stress diagram for the snow load must be drawn. Such diagrams are drawn like those for permanent dead load.

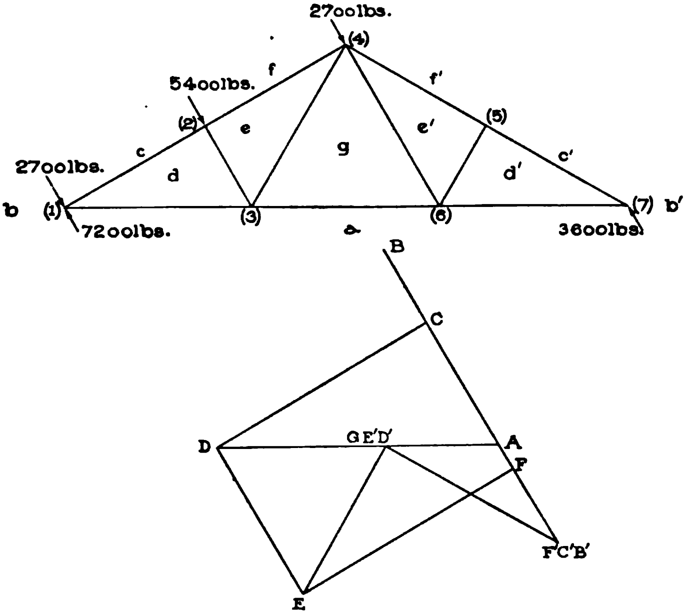


Fig. 28.

29. **Analysis for Wind Loads.** Stresses due to wind pressure cannot be computed from permanent load stresses; they can be most easily determined by means of a stress diagram. Since wind pressure exists only on one side of a truss at a time, the stresses in corresponding members on the right and left sides of a truss are unequal and the whole stress diagram must be drawn in analysis for "wind stresses." Moreover, where one end of the

truss rests on rollers, two stress diagrams must be drawn for a complete analysis, one for wind blowing on the right and one for wind blowing on the left (see Example 2 following).

Examples. 1. It is required to analyze the truss of Fig. 16 for wind pressure, the distance between trusses being 14 feet.

The apex loads for this case are computed in Example 3, Page 26, to be as represented in Fig. 28. Supposing both ends of the truss to be fastened to the supports, then the reactions (due to the wind alone) are parallel to the wind pressure and the right and left reactions equal 3,600 and 7,200 pounds as explained in Example 2, Page 57.

To draw a clockwise polygon for the loads and reactions, we lay off BC, CF, and FF′ to represent the loads at joints (1), (2), and (4) respectively; then since there are no loads at joints (5) and (7) we mark the point F′ by C′ and B′ also; then lay off B′A to represent the reaction at the right end. If the lengths are laid off carefully, AB will represent the reaction at the left end and the polygon is BCFF′C′B′AB.

At joint (1) there are four forces, the reaction, the load, and the two stresses. AB and BC represent the first two forces, hence from C draw a line parallel to *cd* and from A a line parallel to *ad* and mark their intersection D. Then ABCDA is the polygon for the joint and CD and DA represent the two stresses. The former is 7,750 pounds compression and the latter 9,000 pounds tension.

At joint (2) there are four forces, the stress in *cd* (7,750 pounds compression), the load, and the stresses in *fe* and *ed*. As DC and CF represent the stress 7,750 and the load, from F draw a line parallel to *fe* and from D a line parallel to *de*, and mark their intersection E. Then DCFED is the polygon for the joint and FE and ED represent the stresses in *fe* and *ed* respectively. The former is 7,750 pounds and the latter 5,400, both compressive.

At joint (3) there are four forces, the stresses in *ad* (9,000 pounds), *de* (5,400 pounds), *eg* and *ga*. AD and DE represent the first two stresses; hence from E draw a line parallel to *eg* and from A a line parallel to *ag* and mark their intersection G. Then ADEGA is the polygon for the joint and EG and GA represent the stresses in *eg* and *ga* respectively The former is 5,400 and the latter 3,600 pounds, both tensile.

At joint (4) there are five forces, the stresses in *eg* (5,400 pounds) and *ef* (7,750 pounds), the load, and the stresses in *f'e'* and *e'g*. GE, EF and FF' represent the first three forces; hence draw from F' a line parallel to *f'e'* and from G a line parallel to *e'g* and mark their intersection E'. (The first line passes through G, hence E' falls at G). Then the polygon for the joint is GEFF'E'G, and

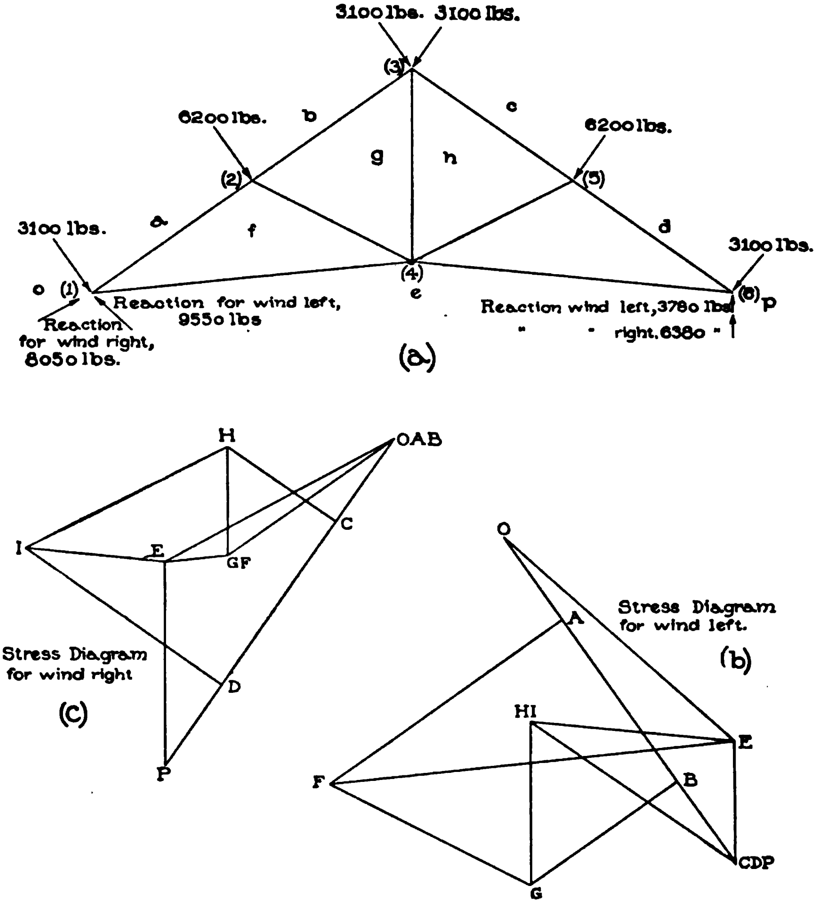

3100 lbs. 3100 lbs.
(3)
6200 lbs.
b
c
6200 lbs.
g
h
(2)
(5)
3100 lbs.
a
f
d
3100 lbs.
(4)
o
(1)
e
(6)
p
Reaction for wind left, 9550 lbs
Reaction for wind right, 8050 lbs.
Reaction wind left, 3780 lbs
" " right, 6380 "
(a)
H
OAB
C
I
E
GF
Stress Diagram for wind right
(c)
D
P
O
A
Stress Diagram for wind left.
(b)
HI
E
F
B
CDP
G

Fig. 29.

F'E' (6,250 pounds compression) represents the stress in *f'e'*. Since E'G = 0, the wind produces no stress in member *ge'*.

At joint (5) three members are connected together and there is no load. The sides of the polygon for the joint must be parallel to the members joined there. Since two of those members are in the same straight line, two sides of the polygon will be parallel and it follows as a consequence that the third side must be zero. Hence the stress in the member *e'd'* equals zero and the stresses in *f'e'* and *d'c'* are equal. This result may be explained slightly differently: Of the stresses in *e'f'*, *e'd'*, and *d'c'* we know the first (6,250) and it is represented by E'F'. Hence we draw from F' a line parallel to *e'd'* and one from E' parallel to *d'e* and mark their intersection D'. Then the polygon for the joint is E'F'C'D'E'; C'D' (6,250 pounds compression) representing the stress in *c'd'*. Since E' and D' refer to the same point, E'D' scales zero and there is no stress in *e'd'*.

The stress in *ad'* can be determined in various ways. Since at joint (6) there are but two forces (the stresses in *ge'* and *e'd'* being zero), the two forces must be equal and opposite to balance. Hence the stress in *d'a* is a tension and its value is 3,600 pounds.

2. It is required to analyze the truss represented in Fig. 24 for wind pressure, the distance between trusses being 15 feet.

The length $\overline{13}$ equals $\sqrt{20^2 + 14^2}$ or

$$\sqrt{400 + 196} = 24.4 \text{ feet.}$$

Hence the area sustaining the wind pressure to be borne by one truss equals $24.4 \times 15 = 366$ square feet.

The tangent of the angle which the roof makes with the horizontal equals $14 \div 20 = 0.7$; hence the angle is practically 35 degrees. According to Art. 19, the wind pressures for slopes of 30 and 40 degrees are 32 and 36 pounds per square foot; hence for 35 degrees it is 34 pounds per square foot. The total wind pressure equals, therefore, $366 \times 34 = 12{,}444$, or practically 12,400 pounds.

The apex load for

joint (2) is ½ of 12,400, or 6,200 pounds,
and for joints (1) and (3), ¼ of 12,400, or 3,100 pounds (see Fig. 29).

When the wind blows from the right the

load for joint (5) is 6,200 pounds, and
for joints (3) and (6) 3,100 pounds.

If the left end of the truss is fastened to its support and the right rests on rollers*, when the wind blows on the left side the right and left reactions equal 3,780 and 9,550 pounds respectively and act as shown. When the wind blows on the right side, the right and left reactions equal 6,380 and 8,050 pounds and act as shown. The computation of these reactions is shown in Example 1, Page 58.

For the wind on the left side, OA, AB, and BC (Fig. 29*b*) represent the apex loads at joints (1), (2) and (3) respectively and CE and EO represent the right and left reactions; then the polygon (clockwise) for the loads and reactions is OABCDPEO. The point C is also marked D and P because there are no loads at joints (5) and (6).

The polygon for joint (1) is EOAFE, AF and FE representing the stresses in *af* and *fe* respectively. The values are recorded in the adjoining table. The polygon for joint (2) is FABGF, BG and GF representing the stresses in *bg* and *fg*. The polygon for joint (3) is GBCHG, CH and HG representing the stresses in *ch* and *hg* respectively. At joint (5) there is no load and two of the members connected there are in the same line; hence there is no wind stress in the third member and the stresses in the other two members are equal. The point H is therefore also marked I to make HI equal to zero. The polygon for joint (5) is HCDIH.

STRESS RECORD.

Member.	Stress, Wind Left.	Stress, Wind Right.
af	- 8,850	- 6,300
fe	+12,700	- 2,000
bg	- 5,600	- 6,300
fg	- 7,000	0
hg	+ 5,100	+3,400
hi	0	- 7,000
ch	- 7,700	- 3,100
ie	+ 6,400	+4,400
di	- 7,700	- 7,500

At joint (4) there are four forces, all known except the one in *ie*. EF, FG, and GH represent the first three; hence the line joining I and E must represent the stress in *ie*. This line, if the drawing has been correctly and accurately made, is parallel to *ie*.

---

* Rollers to allow for free expansion and contraction of the truss would not be required for one as short as this. They are not used generally unless the truss is 55 feet or more in length.

For wind on right side, BC, CD, and DP Fig. 29(*c*) represent the loads at joints (3), (5) and (6) respectively and PE and EB the right and left reactions; then BCDPEB is the polygon for the loads and reactions. The point B is also marked A and O because there are no loads at joints (2) and (1).

The polygon for joint (6) is DPEID, EI and ID representing the stresses in *ei* and *id* respectively. The polygon for joint (5) is CDIHC, IH, and HC representing the stresses in *ih* and *hc* respectively. The polygon for joint (3) is BCHGB, HG, and GB representing the stresses in *hg* and *gb* respectively. The polygon for joint (2) is BGFAB, FA representing the stress in *fa*, and since GF equals zero there is no stress in *gf*.

At joint (1) there are three forces, the left reaction, AF and the stress in *fe*. This third force must close the polygon, so we join F and E and this line represents the stress in *fe*. If the work has been accurately done, FE will be parallel to *fe*.

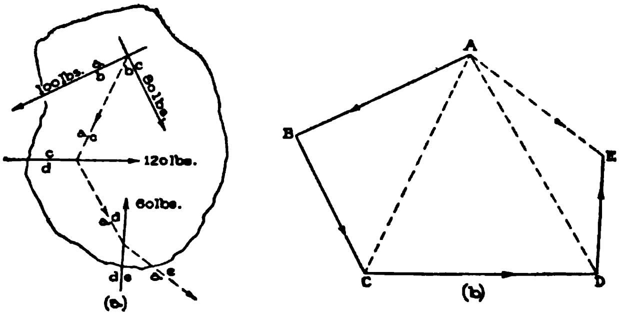


Fig. 30.

EXAMPLE FOR PRACTICE.

Analyze the truss represented in Fig. 26 for wind pressure, the distance between trusses being 15 feet. (See Ex. 3, Page 27, for apex loads.) Assuming both ends of the truss fastened to the supports, the reactions are both parallel to the wind pressure and the reaction on the windward side equals 6,692.5 pounds and the other equals 3,037.5 pounds.

Ans. Stress Record for Wind Left.

Member.	Stress.
12	-9,200
14	+9,460
45	+9,460
24	0
23	-4,806
25	-6,080
35	+2,700
36	-6,080
56	0
57	+4,000
68	-6,080
67	0
78	+4,000

V. COMPOSITION OF NON-CONCURRENT FORCES.

30. **Graphical Composition.** As in composition of concurrent systems, we first compound any two of the forces by means of the Triangle Law (Art. 9), then compound the resultant of these two forces with the third, then compound the resultant of the first three with the fourth and so on until the resultant of all has been found. It will be seen in the illustration that the actual constructions are not quite so simple as for concurrent forces.

Example. It is required to determine the resultant of the four forces (100, 80, 120, and 60 pounds) represented in Fig. 30 (a).

If we take the 100- and 80-pound forces first, and from any convenient point A lay off AB and BC to represent the magnitudes and directions of those forces, then according to the triangle law AC represents the magnitude and direction of their resultant and its line of action is parallel to AC and passes through the point of concurrence of the two forces. This line of action should be marked *ac* and those of the 100- and 80-pound forces, *ab* and *bc* respectively.

If we take the 120-pound force as third, lay off CD to represent the magnitude and direction of that force; then AD represents the magnitude and direction of the resultant of AC and the third force, while the line of action of that resultant is parallel to AD and passes through the point of concurrence of the forces AC and CD. That line of action should be marked *ad* and that of the third force *cd*.

It remains to compound AD and the remaining one of the given forces, hence we lay off DE to represent the magnitude and direction of the fourth force; then AE represents the magnitude and direction of the resultant of AD and the fourth force (also of the four given forces). The line of action of the resultant is parallel to AE and passes through the point of concurrence of the forces AD and DE. That line should be marked *ae* and the line of action of the fourth face *de*.

It is now plain that the magnitude and direction of the resultant is found exactly as in the case of concurrent forces, but finding the line of action requires an extra construction.

31. **When the Forces Are Parallel or Nearly So,** the method of composition explained must be modified slightly because there is no intersection from which to draw the line of action of the resultant of the first two forces.

To make such an intersection available, resolve any one of the given forces into two components and imagine that force replaced by them; then find the resultant of those components and the other given forces by the methods explained in the preceding article. Evidently this resultant is the resultant of the given forces.

Example. It is required to find the resultant of the four parallel forces (50, 30, 40, and 60 pounds) represented in Fig. 31 (*a*).

Choosing the 30-pound force as the one to resolve, lay off AB to represent the magnitude and direction of that force and mark its line of action *ab*. Next draw lines from A and B intersecting at any convenient point O; then as explained in Art. 13, AO and OB (direction from A to O and O to B) represent the magnitudes and directions of two components of the 30-pound force, and the lines of action of those components are parallel to AO and OB and must intersect on the line of action of that force, as at 1. Draw next two such lines and mark them *ao* and *ob* respectively. Now imagine the 30-pound force replaced by its two components and then compound them with the 50-, 40- and 60-pound forces.

In the composition, the second component should be taken as the first force and the first component as the last. Choosing the 50-pound force as the second, lay off BC to represent the magnitude and direction of that force and mark the line of action *bc*. Then OC (direction O to C) represents the magnitude and direction of the resultant of OB and BC, and *oc* (parallel to OC and passing through the point of concurrence of the forces OB and BC) is the line of action.

Choosing the 40-pound force next, lay off CD to represent the magnitude and direction of that force and mark its line of action *cd*. Then OD (direction O to D) represents the magnitude and direction of the resultant of OC and CD, and *od* (parallel to OD and passing through the point of concurrence of the forces OC and CD) is the line of action of it.

Next lay off a line DE representing the magnitude and direction of the 60-pound force and mark the line of action *de*. Then OE (direction O to E) represents the magnitude and direction of the resultant of OD and DE, and *oe* (parallel to OE and passing through the point of concurrence of the forces OD and DE) is the line of action of it.

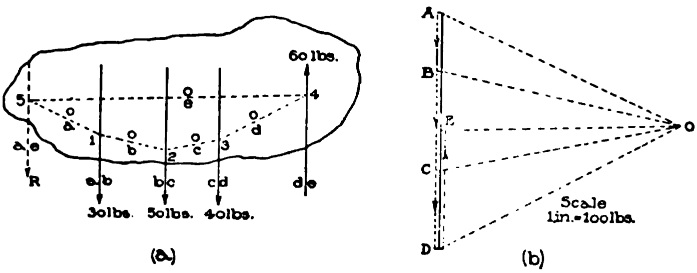


Fig. 31.

It remains now to compound the last resultant (OE) and the first component (AO). AE represents the magnitude and direction of their resultant, and *ae* (parallel to AE and passing through the point of concurrence of the forces OE and AO) is the line of action.

32. **Definitions and Rule for Composition.** The point O (Fig. 31) is called a *pole*, and the lines drawn to it are called *rays*. The lines *oa*, *ob*, *oc*, etc., are called *strings* and *collectively* they are called a *string polygon*. The string parallel to the ray drawn to the beginning of the force polygon (A) is called the first string, and the one parallel to the ray drawn to the end of the force polygon is called the last string.

The method of construction may now be described as follows:

1. Draw a force polygon for the given forces. The line drawn from the beginning to the end of the polygon represents the magnitude and direction of the resultant.

2. Select a pole, draw the rays and then the string polygon. The line through the intersection of the first and last strings parallel to the direction of the resultant is the line of action of the resultant. (In constructing the string polygon, observe carefully that the two strings intersecting on the line of action of any one of the given forces are parallel to the two rays which are drawn to the ends of the line representing that force in the force polygon.)

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the 50-, 70-, 80- and 120-pound forces of Fig. 5.

Ans. { 260 pounds acting upwards 1.8 and 0.1 feet to the right of A and D respectively.

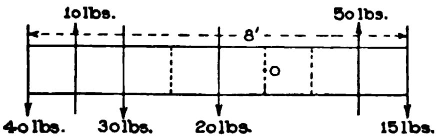


Fig. 32.

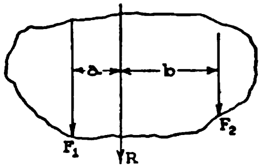


Fig. 33.

2. Determine the resultant of the 40-, 10-, 30- and 20-pound forces of Fig. 32.

Ans. { 80 pounds acting down $1\frac{5}{8}$ feet from left end.

33. **Algebraic Composition.** The algebraic method of composition is best adapted to parallel forces and is herein explained only for that case.

If the plus sign is given to the forces acting in one direction, and the minus sign to those acting in the opposite direction, the magnitude and sense of the resultant is given by the algebraic sum of the forces; the magnitude of the resultant equals the value of the algebraic sum; the direction of the resultant is given by the sign of the sum, thus the resultant acts in the direction which has been called plus or minus according as the sign of the sum is plus or minus.

If, for example, we call up plus and down minus, the algebraic sum of the forces represented in Fig. 32 is

$$-40 + 10 - 30 - 20 + 50 - 15 = -45;$$

hence the resultant equals 45 pounds and acts downward.

The line of action of the resultant is found by means of the principle of moments which is (as explained in "Strength of Materials") that *the moment of the resultant of any number of forces about any origin equals the algebraic sum of the moments of the forces.* It follows from the principle that the arm of the resultant with respect to any origin equals the quotient of the algebraic sum of the moments of the forces divided by the resultant; also the line of action of the resultant is on such a side of the origin that the sign of the moment of the resultant is the same as that of the algebraic sum of the moments of the given forces.

For example, choosing O as origin of moments in Fig. 32, the moments of the forces taking them in their order from left to right are

$$-40\times5 = -200,\ +10\times4 = +40,\ -30\times3 = -90,$$
$$-20\times1 = -20,\ -50\times2 = -100,\ +15\times3 = +45.^*$$

Hence the algebraic sum equals

$$-200 + 40 - 90 - 20 - 100 + 45 = -325 \text{ foot-pounds.}$$

The sign of the sum being negative, the moment of the resultant about O must also be negative, and since the resultant acts down, its line of action must be on the left side of O. Its actual distance from O equals

$$\frac{325}{45} = 7.22 \text{ feet.}$$

EXAMPLES FOR PRACTICE.

1. Make a sketch representing five parallel forces, 200, 150 100, 225, and 75 pounds, all acting in the same direction and 2 feet apart. Determine their resultant.

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* The student is reminded that when a force tends to turn the body on which it acts in the clockwise direction, about the selected origin, its moment is a given a plus sign, and when counter-clockwise, a minus sign.

Ans. { Resultant = 750 pounds, and acts in the same direction as the given forces and 4.47 feet to the left of the 75-pound force.

2. Solve the preceding example, supposing that the first three forces act in one direction and the last two in the opposite direction.

Ans. { Resultant = 150 pounds, and acts in the same direction with the first three forces and 16.3 feet to the left of the 75-pound force.

Two parallel forces acting in the same direction can be compounded by the methods explained in the foregoing, but it is sometimes convenient to remember that the resultant equals the sum of the forces, acts in the same direction as that of the two forces and between them so that the line of action of the resultant divides the distance between the forces inversely as their magnitudes. For example, let F_1 and F_2 (Fig. 33) be two parallel forces. Then if R denotes the resultant and a and b its distances to F_1 and F_2 as shown in the figure,

$$R = F_1 + F_2,$$

and

$$a : b :: F_2 : F_1.$$

34. Couples. Two parallel forces which are equal and act in opposite directions are called a couple. The forces of a couple cannot be compounded, that is, no single force can produce the same effect as a couple. The perpendicular distance between the lines of action of the two forces is called the *arm*, and the product of one of the forces and the arm is called the *moment of the couple.*

A plus or minus sign is given to the moment of a couple according as the couple turns or tends to turn the body on which it acts in the clockwise or counter-clockwise direction.

VI. EQUILIBRIUM OF NON-CONCURRENT FORCES.

35. Conditions of Equilibrium of Non-Concurrent Forces Not Parallel may be stated in various ways; let us consider four. First:

1. The algebraic sums of the components of the forces along each of two lines at right angles to each other equal zero.

2. The algebraic sum of the moments of the forces about any origin equals zero.

Second:

1. The sum of the components of the forces along any line equals zero.
2. The sums of the moments of the forces with respect to each of two origins equal zero.

Third:

The sums of the moments of the forces with respect to each of three origins equals zero.

Fourth:

1. The algebraic sum of the moments of the forces with respect to some origin equals zero.
2. The force polygon for the forces closes.

It can be shown that if any one of the foregoing sets of conditions are fulfilled by a system, its resultant equals zero. Hence each is called a set of conditions of equilibrium for a non-concurrent system of forces which are not parallel.

The first three sets are "algebraic" and the last is "mixed," (1) of the fourth, being algebraic and (2) graphical. There is a set of graphical conditions also, but some one of those here given is usually preferable to a set of wholly graphical conditions.

Like the conditions of equilibrium for concurrent forces, they are used to answer questions arising in connection with concurrent systems known to be in equilibrium. Examples may be found in Art. 37.

36. Conditions of Equilibrium for Parallel Non-Concurrent Forces. Usually the most convenient set of conditions to use is one of the following:

First:

1. The algebraic sum of the forces equals zero, and
2. The algebraic sum of the moments of the forces about some origin equals zero.

Second:

The algebraic sums of the moments of the forces with respect to each of two origins equal zero.

37. Determination of Reactions. The weight of a truss, its loads and the supporting forces or reactions are balanced and constitute a system in equilibrium. After the loads and weight are ascertained, the reactions can be determined by means of conditions of equilibrium stated in Arts. 35 and 36.

The only cases which can be taken up here are the following common ones: (1) The truss is fastened to two supports and (2) The truss is fastened to one support and simply rests on rollers at the other.

Case (1) Truss Fastened to Both Its Supports. If the loads are all vertical, the reactions also are vertical. If the loads are not vertical, we assume that the reactions are parallel to the resultant of the loads.

The algebraic is usually the simplest method for determining the reactions in this case, and two moment equations should be used. Just as when finding reactions on beams we first take moments about the right support to find the left reaction and then about the left support to find the right reaction. As a check we add the reactions to see if their sum equals the resultant load as it should.

Examples. 1. It is required to determine the reactions on the truss represented in Fig. 20, it being supported at its ends and sustaining two vertical loads of 1,800 and 600 pounds as shown.

The two reactions are vertical; hence the system in equilibrium consists of parallel forces. Since the algebraic sum of the moments of all the forces about any point equals zero, to find the left reaction we take moments about the right end, and to find the right reaction we take moments about the left end. Thus, if R_1 and R_2 denote the left- and right-reactions respectively, then taking moments about the right end,

$$(R_1 \times 24) - (1800 \times 15) - (600 \times 15) = 0,$$

or $$24R_1 = 27{,}000 + 9{,}000 = 36{,}000;$$

hence $$R_1 = \frac{36{,}000}{24} = 1{,}500 \text{ pounds.}$$

Taking moment about the left end,

$$-R_2 \times 24 + 1{,}800 \times 9 + 600 \times 9 = 0,$$

or $$24R_2 = 16{,}200 + 5{,}400 = 21{,}600;$$

hence $$R_2 = 900 \text{ pounds.}$$

As a check, add the reactions to see if the sum equals that of the loads as should be the case. (It will be noticed that reactions on trusses and beams under vertical loads are determined in the same manner.)

2. It is required to determine the reactions on the truss represented in Fig. 28 due to the wind pressures shown (2,700, 5,400 and 2,700 pounds), the truss being fastened to both its supports.

The resultant of the three loads is evidently a single force of 10,800 pounds, acting as shown in Fig. 34. The reactions are parallel to this resultant; let R_1 and R_2 denote the left and right reactions respectively.

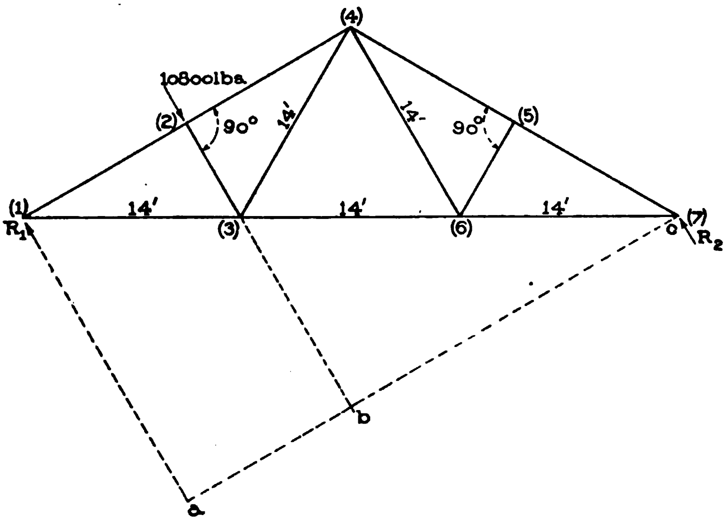

Fig. 34.

The line *abc* is drawn through the point 7 and perpendicular to the direction of the wind pressure; hence with respect to the right support the arms of R_1 and resultant wind pressure are *ac* and *bc*, and with respect to the left support, the arms of R_2 and the resultant wind pressure are *ac* and *ab*. These different arms can be measured from a scale drawing of the truss or be computed as follows: The angle $\overline{17a}$ equals the angle $\overline{417}$, and $\overline{417}$ was shown to be 30 degrees in Example 3, Page 26. Hence

$$ab = 14 \cos 30^\circ,\ bc = 28 \cos 30^\circ,\ ac = 42 \cos 30^\circ.$$

Since the algebraic sums of the moments of all the forces acting on the truss about the right and left supports equal zero,

$$R_1 \times 42 \cos 30^\circ = 10{,}800 \times 28 \cos 30^\circ,$$

and $$R_2 \times 42 \cos 30^\circ = 10{,}800 \times 14 \cos 30^\circ.$$

From the first equation,

$$R_1 = \frac{10{,}800 \times 28}{42} = 7{,}200 \text{ pounds,}$$

and from the second,

$$R_2 = \frac{10{,}800 \times 14}{42} = 3{,}600 \text{ pounds.}$$

Adding the two reactions we find that their sum equals the load as it should.

Case (2) One end of the truss rests on rollers and the other is fixed to its support. The reaction at the roller end is always vertical, but the direction of the other is not known at the outset unless the loads are all vertical, in which case both reactions are vertical.

When the loads are not all vertical, the loads and the reactions constitute a non-concurrent non-parallel system and any one of the sets of conditions of equilibrium stated in Art. 35 may be used for determining the reactions. In general the fourth set is probably the simplest. In the first illustration we apply the four different sets for comparison.

Examples. 1. It is required to compute the reactions on the truss represented in Fig. 29 due to the wind pressures shown on the left side (3,100, 6,200 and 3,100 pounds), the truss resting on rollers at the right end and being fastened to its support at the left.

(*a*) Let R_1 and R_2 denote the left and right reactions. The direction of R_2 (at the roller end) is vertical, but the direction of R_1 is unknown. Imagine R_1 resolved into and replaced by its horizontal and vertical components and call them R_1' and R_1'' respectively (see Fig. 35.) The six forces, R_1', R_1'', R_2 and the three wind pressures are in equilibrium, and we may apply any one of the sets of statements of equilibrium for this kind of a system (see Art. 35) to determine the reactions. If we choose to use the first set we find, resolving forces along a horizontal line,

$$-R_1' + 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 0;$$

resolving forces along a vertical line,

$$+R_1'' + R_2 - 3{,}100 \cos 35^\circ - 6{,}200 \cos 35^\circ - 3{,}100 \cos 35^\circ = 0;$$

taking moments about the left end,

$$+ 6{,}200 \times 12.2 + 3{,}100 \times 24.4 - R_2 \times 40 = 0.$$

From the first equation,

$$R_1' = 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 7{,}113 \text{ pounds},$$

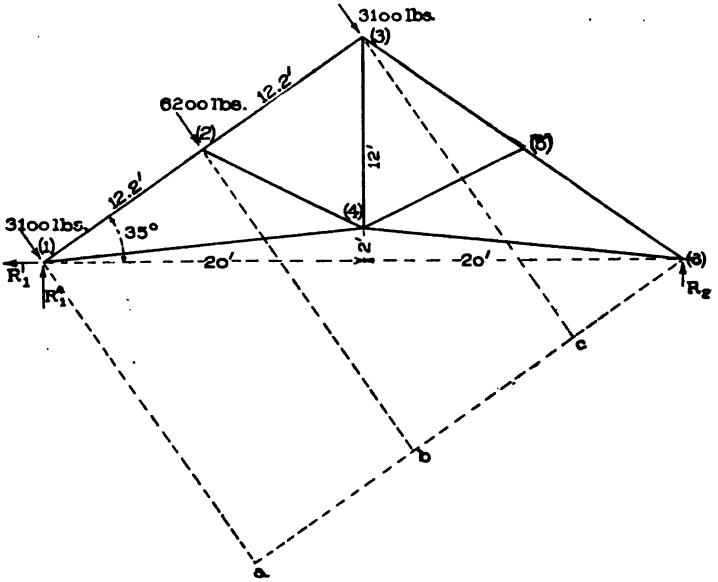

Fig. 35.

and from the third,

$$R_2 = \frac{6{,}200 \times 12.2 + 3{,}100 \times 24.4}{40} = 3{,}782 \text{ pounds}.$$

Substituting this value of R_2 in the second equation we find that

$$R_1'' = 3{,}100 \cos 35^\circ + 6{,}200 \cos 35^\circ + 3{,}100 \cos 35^\circ - 3{,}782$$
$$= 10{,}156 - 3{,}782 = 6{,}374 \text{ pounds}.$$

If desired, the reaction R_1 can now be found by compounding its two components R_1' and R_1''.

(*b*) Using the second set of conditions of equilibrium stated in Art. 35 we obtain the following three "equilibrium equations":
As in (1), resolving forces along the horizontal gives

$$-R_1' + 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 0,$$

and taking moments about the left end,

$$6{,}200 \times 12.2 + 3{,}100 \times 24.4 - R_2 \times 40 = 0.$$

Taking moments about the right end gives

$$R_1'' \times 40 - 3{,}100 \times \overline{a6} - 6{,}200 \times \overline{b6} - 3{,}100 \times \overline{c6} = 0$$

Just as in (*a*), we find from the first and second equations the values of R_1' and R_2. To find R_1'' we need values of the arms $\overline{a6}$, $\overline{b6}$, and $\overline{c6}$. By measurement from a drawing we find that

$$\overline{a6} = 32.7,\ \overline{b6} = 20.5, \text{ and } \overline{c6} = 8.3 \text{ feet.}$$

Substituting these values in the third equation and solving for R_1'' we find that

$$R_1'' = \frac{3{,}100 \times 32.7 + 6{,}200 \times 20.5 + 3{,}100 \times 8.3}{40} = 6{,}355 \text{ pounds.}$$

(*c*) Using the third set of conditions of equilibrium stated in Art. 35 we obtain the following three equilibrium equations:
As in (*b*), taking moments about the right and left ends we get

$$R_1'' \times 40 - 3{,}100 \times 32.7 - 6{,}200 \times 20.5 - 3{,}100 \times 8.3 = 0,$$

and $$-R_2 \times 40 + 6{,}200 \times 12.2 + 3{,}100 \times 24.4 = 0.$$

Choosing the peak of the truss as the origin of moments for the third equation we find that

$$R_1' \times 14 + R_1'' \times 20 - 3{,}100 \times 24.4 - 6{,}200 \times 12.2 - R_2 \times 20 = 0.$$

As in (*b*) we find from the first two equations the values of R_1'' and R_2. These values substituted in the third equation change it to

$$R_1' \times 14 + 6{,}373 \times 20 - 3{,}100 \times 24.4 - 6{,}200 \times 12.2 - 3{,}782 \times 20 = 0$$

or

$$R_1' = \frac{-6{,}373 \times 20 + 3{,}100 \times 24.4 + 6{,}200 \times 12.2 + 3{,}782 \times 20}{14}$$

$= 7{,}104.$*

(*d*) When using the fourth set of conditions we always determine the reaction at the roller end from the moment equation. Then, knowing the value of this reaction, draw the force polygon for all the loads and reactions and thus determine the magnitude and direction of the other reaction.

Taking moments about the left end, we find as in (*a*), (*b*), and (*c*) that $R_2 = 3{,}782$. Then draw AB, BC and CD (Fig. 36) to represent the wind loads, and DE to represent R_2. Since the force polygon for all the forces must close, EA represents the magnitude and direction of the left reaction; it scales 9,550 pounds.

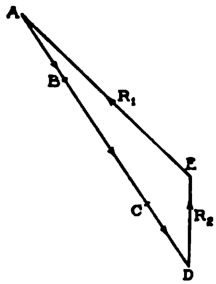

Fig. 36.

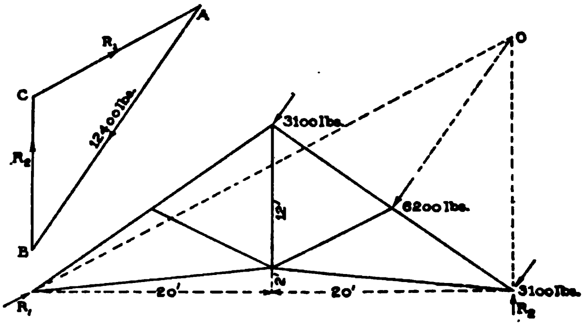

Fig. 37.

2. It is required to determine the reactions on the truss of the preceding illustration when the wind blows from the right.

The methods employed in the preceding illustration might be used here, but we explain another which is very simple. The truss and its loads are represented in Fig. 37. Evidently the resultant of the three wind loads equals 12,400 pounds and acts in the same line with the 6,200-pound load. If we imagine this resultant to replace the three loads we may regard the truss acted upon by three forces, the 12,400-pound force and the reactions, and these three forces as in equilibrium. Now when three forces are in equilibrium they must be concurrent or parallel, and since the resultant load (12,400 pounds) and R_2 intersect at O, the line of action of R_1 must also pass through O. Hence the left reaction acts through the left support and O as shown. We are now ready to determine the values of R_1 and R_2. Lay off AB to represent the resultant load, then from A and B draw lines parallel to R_1 and R_2, and mark their intersection C. Then BC and CA represent the magnitude and directions of R_2 and R_1 respectively; they scale 6,380 and 8,050 pounds.

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* The slight differences in the answers obtained from the different sets of equilibrium equations are due to inaccuracies in the measured arms of some of the forces.

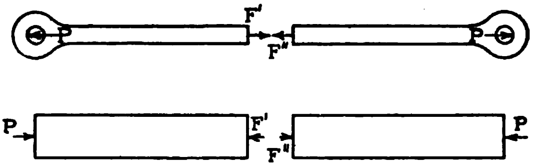

Fig. 38.

EXAMPLE FOR PRACTICE.

1. Determine the reactions on the truss represented in Fig. 26 due to wind pressure, the distance between trusses being 15 feet, supposing that both ends of the truss are fastened to the supports.

Ans. { Reaction at windward end is 6,682½ pounds.
Reaction at leeward end is 3,037½ pounds.

VII. ANALYSIS OF TRUSSES (CONTINUED); METHOD OF SECTIONS.

38. Forces in Tension and Compression Members. As explained in "Strength of Materials" if a member is subjected to forces, any two adjacent parts of it exert forces upon each other which hold the parts together. Figs. 38 (*a*) and 38 (*b*) show how these forces act in a tension and in a compression member. F' is the force exerted on the left part by the right, and F'' that exerted on the right part by the left. The two forces F' and F'' are equal, and in a tension member are pulls while in a compression member they are pushes.

39. Method of Sections. To determine the stress in a member of a truss by the method explained in the foregoing (the "method of joints"), we begin at one end of the truss and draw polygons for joints from that end until we reach one of the joints to which that member is connected. If the member is near the middle of a long truss, such a method of determining the stress in it requires the construction of several polygons. It is sometimes desirable to determine the stress in a member as directly as possible without having first determined stresses in other members. A method for doing this will now be explained; it is called the method of sections.

Fig. 39 (*a*) is a partial copy of Fig. 16. The line LL is intended to indicate a "section" of the truss "cutting" members $\overline{24}$, $\overline{34}$ and $\overline{36}$. Fig. 39 (*b*) and (*c*) represents the parts of the truss to the left and right of the section. By "part of a truss" we mean either of the two portions into which a section separates it when it cuts it completely.

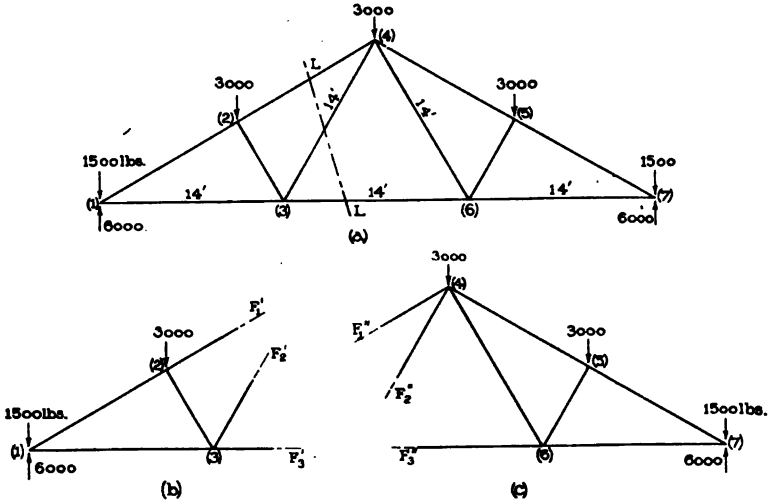

Fig. 39.

Since each part of the truss is at rest, all the forces acting on each part are balanced, or in equilibrium. The forces acting on each part consist of the loads and reactions applied to that part together with the forces exerted upon it by the other part. Thus the forces which hold the part in Fig. 39 (*b*) at rest are the 1,500- and 3,000-pound loads, the reaction 6,000 and the forces which the right part of the cut members exert upon the left parts. These latter forces are marked F_1', F_2' and F_3'; their senses are unknown.

but each acts along the axis of the corresponding member. The forces which hold the part in Fig. 39 (*c*) at rest are the two 3,000-pound loads, the 1,500-pound load, the right reaction 6,000 pounds and the forces which the left parts of the cut members exert upon the right parts. These are marked F_1'', F_2'' and F_3''; their senses are also unknown but each acts along the axis of the corresponding member. The forces F_1' and F_1'', F_2' and F_2'', and F_3' and F_3'' are equal and opposite; they are designated differently only for convenience.

If, in the system of forces acting on either part of the truss, there are not more than three unknown forces, then those three can be computed by "applying" one of the sets of the conditions of equilibrium stated in Art. 35.* In writing the equations of equilibrium for the system it is practically necessary to assume a sense for one or more of the unknown forces. *We shall always assume that such forces are pulls that is, act away from the part of the truss upon which they are exerted. Then if the computed value of a force is plus, the force is really a pull and the member is in tension and if the value is minus, then the force is really a push and the member is in compression.*

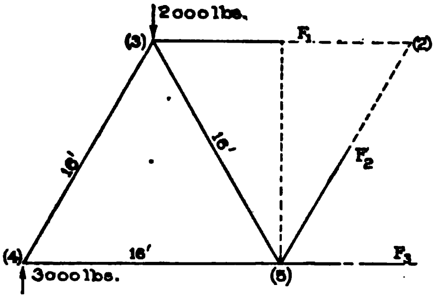

Fig. 40.

To determine the stress in any particular member of a truss in accordance with the foregoing, "pass a section" through the truss cutting the member under consideration, and then apply as many conditions of equilibrium to all the forces acting on either part of the truss as may be necessary to determine the desired force. In passing the section, care should be taken to cut as few members as possible, and never should a section be passed so as to cut more than three, the stresses in which are unknown; neither should the three be such that they intersect in a point.

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* If, however, the lines of action of the three forces intersect in a point then the statement is not true.

Examples. 1. It is required to determine the amount and kind of stress in the member $\overline{24}$ of the truss represented in Fig. 39 (*a*) when loaded as shown.

Having determined the reactions (6,000 pounds each) we pass a section through the entire truss and cutting $\overline{24}$; LL is such a section. Considering the part of the truss to the left of the section, the forces acting upon that part are the two loads, the left reaction and the forces on the cut ends of members $\overline{24}$, $\overline{34}$ and $\overline{36}$ (F_1', F_2', and F_3'). F_1' can be determined most simply by writing a moment equation using (3) as the origin, for with respect to that origin the moments of F_2' and F_3' are zero, and hence those forces will not appear in the equation. Measuring from a large scale drawing, we find that the arm of F_1' is 7 feet and that of the 3,000-pound load is 3.5 feet. Hence

$$(F_1' \times 7)+(6{,}000 \times 14)-(1{,}500 \times 14)-(3{,}000 \times 3.5)=0$$

$$\text{or } F_1'=\frac{-(6{,}000\times 14)+(1{,}500\times 14)+(3{,}000\times 3.5)}{7}=-7{,}500$$

The minus sign means that F_1' is a push and not a pull, hence the member $\overline{24}$ is under 7,500 pounds compression.

The stress in the member $\overline{24}$ may be computed from the part of the truss to the right of the section. Fig. 39 (*c*) represents that part and all the forces applied to it. To determine F_1'' we take moments about the intersection of F_2'' and F_3''. Measuring from a drawing we find that the arm of F_1'' is 7 feet,

that of the load at joint (4) is 7 feet,
that of the load at joint (5) is 17.5 feet,
that of the load at joint (7) is 28 feet,
and that of the reaction is 28 feet.

Hence, assuming F_1'' to be a pull,

$$-(F_1'' \times 7)+(3{,}000 \times 7)+(3{,}000 \times 17.5)+(1{,}500\times 28)-(6{,}000 \times 28)=0,$$

$$\text{or } F_1''=\frac{(3{,}000 \times 7)+(3{,}000\times 17.5)+(1{,}500\times 28)-(6{,}000\times 28)}{7}=-7{,}500$$

The minus sign means that F_1'' is a push, hence the member $\overline{24}$ is under compression of 7,500 pounds, a result agreeing with that previously found.

2. It is required to find the stress in the member *gh* of the truss represented in Fig. 25, due to the loads shown.

If we pass a section cutting *bg*, *gh* and *he*, and consider the left part, we get Fig. 40, the forces on that part being the 2,000-pound load, the left reaction, and the forces F_1, F_2 and F_3 exerted by the right part on the left. To compute F_2 it is simplest to use the condition that the algebraic sum of all the vertical components equals zero. Thus, assuming that F_2 is a pull, and since its angle with the vertical is 30°,

$$F_2 \cos 30^\circ - 2{,}000 + 3{,}000 = 0; \text{ or,}$$

$$F_2 = \frac{2{,}000 - 3{,}000}{\cos 30^\circ} = \frac{-1{,}000}{0.866} = -1{,}154.$$

The minus sign means that F_2 is a push, hence the member is under a compression of 1,154 pounds.

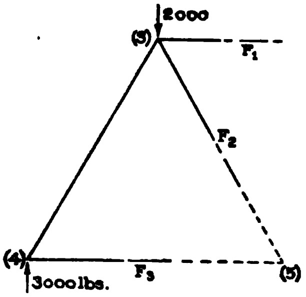


Fig. 41.

3. It is required to determine the stress in the member *bg* of the truss represented in Fig. 25, due to the loads shown.

If we pass a section cutting *bg* as in the preceding illustration, and consider the left part, we get Fig. 40. To compute F_1 it is simplest to write the moment equation for all the forces using joint 5 as origin. From a large scale drawing, we measure the arm of F_1 to be 13.86 feet hence, assuming F_1 to be a pull,

$$F_1 \times 13.86 - 2{,}000 \times 8 + 3{,}000 \times 16 = 0; \text{ or,}$$

$$F_1 = \frac{2{,}000 \times 8 - 3{,}000 \times 16}{13.86} = \frac{-32{,}000}{13.86} = -2{,}427.$$

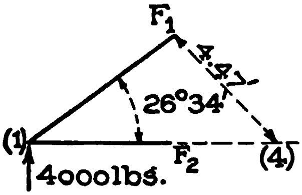


Fig. 42.

The minus sign means that F_1 is a push; hence the member is under a compression of 2,308 pounds.

The section might have been passed so as to cut members *bg*, *fg*, and *fe*, giving Fig. 41 as the left part, and the desired force might be obtained from the system of forces acting on that part (3,000, 2,000, F_1, F_2, and F_3.) To compute F_1 we take moments about the intersection of F_2 and F_3, thus

$$F_1 \times 13.86 - 2{,}000 \times 8 + 3{,}000 \times 16 = 0.$$

This is the same equation as was obtained in the first solution, and hence leads to the same result.

4. It is required to determine the stress in the member $\overline{12}$ of the truss represented in Fig. 26, due to the loads shown.

Passing a section cutting members $\overline{12}$ and $\overline{14}$, and considering the left part, we get Fig. 42. To determine F_1 we may write a moment equation preferably with origin at joint 4, thus:

$$F_1 \times 4.47 + 4{,}000 \times 10 = 0^*;$$

or, $$F_1 = \frac{-4{,}000 \times 10}{4.47} = -8{,}948 \text{ pounds,}$$

the minus sign meaning that the stress is compressive.

F_1 might be determined also by writing the algebraic sum of the vertical components of all the forces on the left part equal to zero, thus:

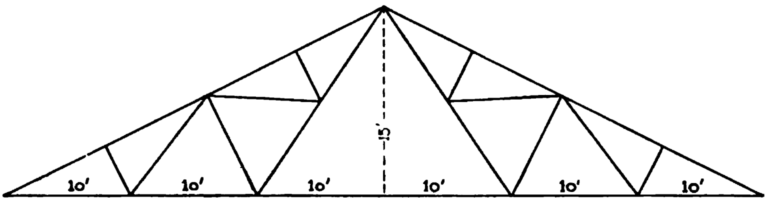


Fig. 43.

$$F_1 \sin 26^\circ\, 34' + 4{,}000 = 0;$$

or, $$F_1 = \frac{-4{,}000}{\sin 26^\circ\, 34'} = \frac{-4{,}000}{0.447} = -8{,}948,$$

agreeing with the first result.

EXAMPLES FOR PRACTICE.

1. Determine by the method of sections the stresses in members $\overline{23}$, $\overline{25}$, and $\overline{45}$ of the truss represented in Fig. 26, due to the loads shown.

Ans. { Stress in $\overline{23}$ = − 5,600 pounds,
Stress in $\overline{25}$ = − 3,350 pounds,
Stress in $\overline{45}$ = + 8,000 pounds.

---

* The arm of F_1 with respect to (4) is 4.47 feet.

2. Determine the stresses in the members $\overline{12}$, $\overline{15}$, $\overline{34}$, and $\overline{56}$ of the truss represented in Fig. 27, due to the loads shown.

Ans. { Stress in $\overline{12}$ = − 11,170 pounds,
Stress in $\overline{15}$ = + 10,000 pounds,
Stress in $\overline{34}$ = − 8,940 pounds,
Stress in $\overline{56}$ = + 6,000 pounds.

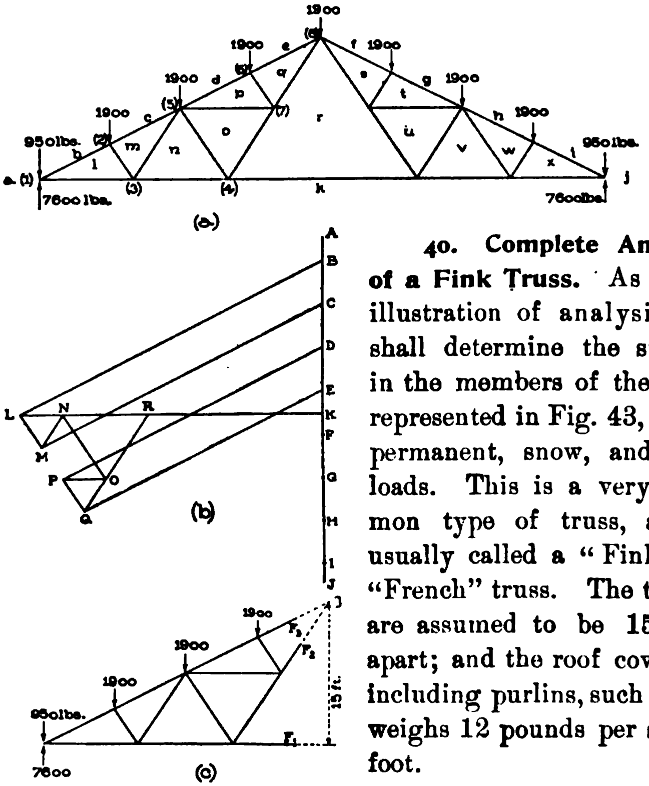


Fig. 44.

40. Complete Analysis of a Fink Truss.

As a final illustration of analysis, we shall determine the stresses in the members of the truss represented in Fig. 43, due to permanent, snow, and wind loads. This is a very common type of truss, and is usually called a "Fink" or "French" truss. The trusses are assumed to be 15 feet apart; and the roof covering, including purlins, such that it weighs 12 pounds per square foot.

The length from one end to the peak of the truss equals

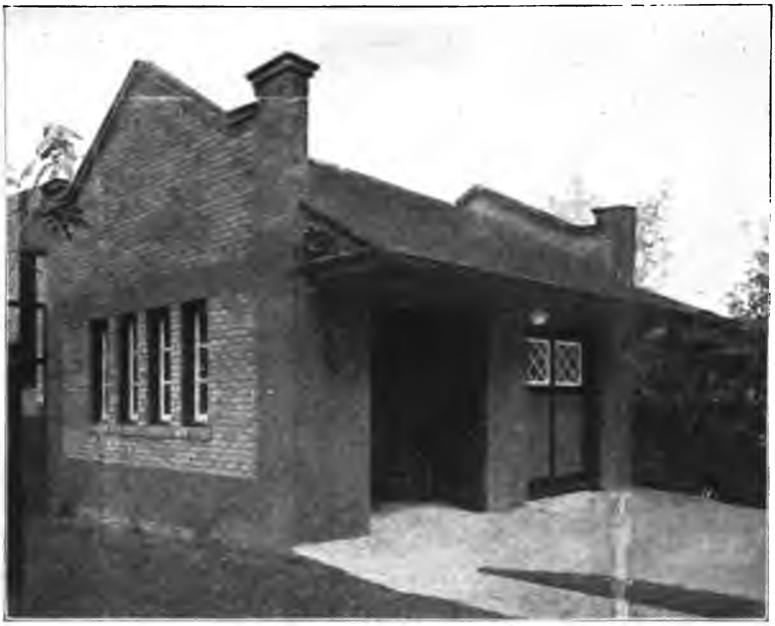
$$\sqrt{15^2 + 30^2} = \sqrt{1,125} = 33.54 \text{ feet},$$

hence the area of the roofing sustained by one truss equals

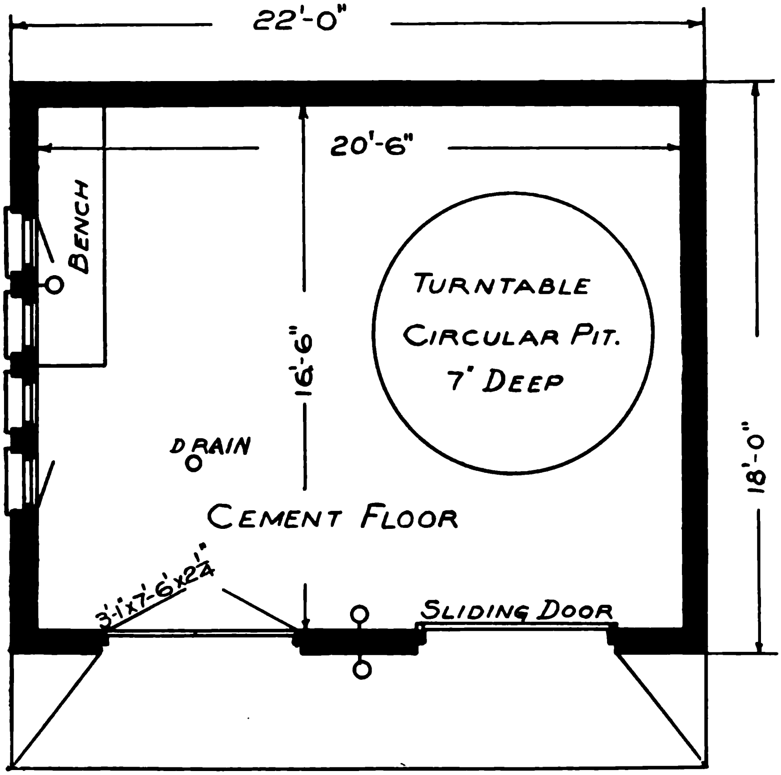
$$(33.54 \times 15)\ 2 = 1,006.2 \text{ square feet},$$

and the weight of that portion of the roof equals

$$1,006.2 \times 12 = 12,074 \text{ pounds}.$$



Walls of Common Brick with Red Mortar Joints and Trimmed with Red Brick.
Slate Roof.



EXAMPLE OF DOUBLE GARAGE FOR E. D. WEARY, CHICAGO, ILL.
W. Carbys Zimmerman, Architect, Chicago, Ill.
Built in 1904.

The probable weight of the truss (steel), according to the formula of Art. 19, is

$$15 \times 60 \left(\frac{60}{25} + 1\right) = 3{,}060 \text{ pounds.}$$

The total permanent load, therefore, equals

$$12{,}074 + 3{,}060 = 15{,}134 \text{ pounds;}$$

the end loads equal $\frac{1}{16}$ of the total, or 950 pounds, and the other apex loads equal $\frac{1}{8}$ of the total, or 1,900 pounds.

Dead Load Stress. To determine the dead load stresses, construct a stress diagram. Evidently each reaction equals one-half the total load, that is 7,600 pounds; therefore ABCDEFGHIJKA (Fig. 44*b*) is a polygon for all the loads and reactions. First, we draw the polygon for joint 1; it is KABLK, BL and LK representing the stress in *bl* and *lk* (see record Page 72 for values). Next draw the polygon for joint 2; it is LBCML, CM and ML representing the stresses in *cm* and *ml*. Next draw the polygon for joint 3; it is KLMNK, MN and NK representing the stresses in *mn* and *nk*.

At each of the next joints (4 and 5), there are three unknown forces, and the polygon for neither joint can be drawn. We might draw the polygons for the joints on the right side corresponding to 1, 2, and 3, but no more until the stress in one of certain members is first determined otherwise. If, for instance, we determine by other methods the stress in *rk*, then we may construct the polygon for joint 4; then for 5, etc., without further difficulty.

To determine the stress in *rk*, we pass a section cutting *rk*, *qr*, and *eq*, and consider the left part (see Fig. 44*c*). The arms of the loads with respect to joint 8 are 7.5, 15, 22.5, and 30 feet; and hence, assuming F_1 to be a pull,

$$-F_1 \times 15 - 1{,}900 \times 7.5 - 1{,}900 \times 15 - 1{,}900 \times 22.5 - 950 \times 30 + 7{,}600 \times 30 = 0; \text{ or,}$$

$$F_1 = \frac{-1{,}900 \times 7.5 - 1{,}900 \times 15 - 1{,}900 \times 22.5 - 950 \times 30 + 7{,}600 \times 30}{15} = 7{,}600 \text{ pounds.}$$

Since the sign of F_1 is plus, the stress in *rk* is tensile.

Now lay off KR to represent the value of the stress in *kr* just found, and then construct the polygon for joint 4. The polygon is KNORK, NO and OR representing the stresses in *no* and *or*. Next draw the polygon for joint 5; it is ONMCDPO, DP and PO representing the stresses in *dp* and *po*. Next draw the polygon for joint 6 or joint 7; for 6 it is PDEQP, EQ and QP representing the stresses in *eq* and *qp*. At joint 7 there is now but one unknown force, namely, that in *qr*. The polygon for the three others at that joint is ROPQ; and since the unknown force must close the polygon, QR must represent that force, and must be parallel to *qr*.

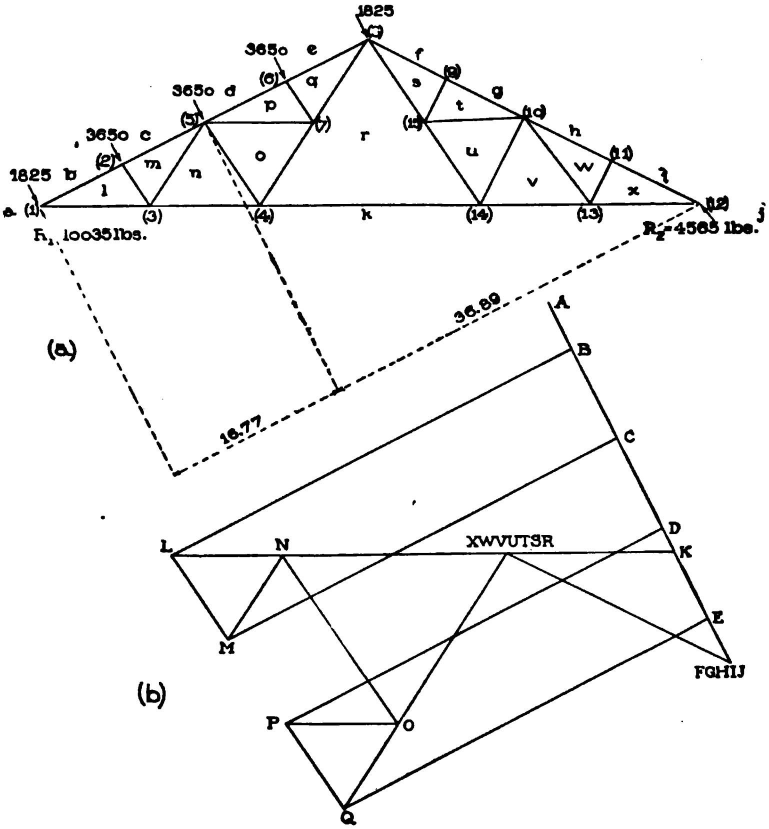


Fig. 45.

On account of the symmetry of loading, the stress in any member on the right side is just like that in the corresponding member on the left; hence, it is not necessary to draw the diagram for the right half of the truss.

Snow-Load Stress. The area of horizontal projection of the roof which is supported by one truss is 60 × 15 = 900 square feet; hence the snow load borne by one truss is 900 × 20 = 18,000 pounds, assuming a snow load of 20 pounds per horizontal square foot. This load is nearly 1.2 times the dead load, and is applied similarly to the latter; hence the snow load stress in any member equals 1.2 times the dead load stress in it. We record, therefore, in the third column of the stress record, numbers equal to 1.2 times those in the second as the snow-load stresses.

Wind Load Stress. The tangent of the angle which the roof makes with the horizontal equals $\frac{15}{30}$ or $\frac{1}{2}$; hence the angle is 26° 34′, and the value of wind pressure for the roof equals practically 29 pounds per square foot, according to Art. 19. As previously explained, the area of the roof sustained by one truss equals 1,006.2 square feet; and since but one-half of this receives wind pressure at one time, the wind pressure borne by one truss equals

$$503.1 \times 29 = 14{,}589.9, \text{ or practically } 14{,}600 \text{ pounds.}$$

When the wind blows from the left, the apex loads are as represented in Fig. 45*a*, and the resultant wind pressure acts through joint 5. To compute the reactions, we may imagine the separate wind pressures replaced by their resultant. We shall suppose that both ends of the truss are fixed; then the reactions will be parallel to the wind pressure. Let R_1 and R_2 denote the left and right reactions respectively; then, with respect to the right end, the arms of R_1 and the resultant wind pressure (as may be scaled from a drawing) are 16.77 + 36.89 and 36.89 feet respectively; and with respect to the left end, the arms of R_2 and the resultant wind pressure are 16.77 + 36.89 and 16.77 feet respectively.

Taking moments about the right end we find that

$$-14{,}600 \times 36.89 + R_1 \times (16.77 + 36.89) = 0;$$

or, $$R_1 = \frac{14{,}600 \times 36.89}{16.77 + 36.89} = 10{,}035 \text{ pounds.}$$

Taking moments about the left end, we find that

$$14{,}600 \times 16.77 - R_2 \times (16.77 + 36.89) = 0;$$

or, $$R_2 = \frac{14{,}600 \times 16.77}{16.77 + 36.89} = 4{,}563 \text{ pounds.}$$

To determine the stresses in the members, we construct a stress diagram. In Fig. 45*b*, AB, BC, CD, DE, and EF represent the wind loads at the successive joints, beginning with joint 1. The point F is also marked G, H, I, and J, to indicate the fact that there are no loads at joints 9, 10, 11, and 12. JK represents the right reaction, and KA the left reaction.

We may draw the polygon for joint 1 or 12; for 1 it is KABLK, BL and LK representing the stresses in *bl* and *lk*. We may next draw the polygon for joint 2; it is LBCML, CM and ML representing the stresses in *cm* and *ml*.

Stress Record.

MEMBER.	STRESSES.					
	Dead Load.	Snow Load.	Wind Left.	Wind Right.	Resultant.	Resultant.
bl	- 14,700	- 17,600	- 16,400	0	- 48,700	- 32,300
cm	- 13,700	- 16,400	- 15,900	0	- 46,000	- 30,100
dp	- 12,600	- 15,100	- 15,400	0	- 43,100	- 28,000
eq	- 11,600	- 13,900	- 14,900	0	- 40,400	- 26,500
lm	- 1,650	- 2,000	- 3,700	0	- 7,350	- 5,350
mn	+ 1,650	+ 2,000	+ 3,700	0	+ 7,350	+ 5,350
no	- 3,300	- 4,000	- 7,400	0	- 14,700	- 10,700
op	+ 1,850	+ 2,200	+ 4,100	0	+ 8,150	+ 5,950
pq	- 1,650	- 2,000	- 3,700	0	- 7,350	- 5,350
rq	+ 5,000	+ 6,000	+ 11,000	0	+ 22,000	+ 16,000
ro	+ 3,400	+ 4,100	+ 7,400	0	+ 14,900	+ 10,800
kl	+ 13,300	+ 16,000	+ 18,300	+ 6,100	+ 47,600	+ 31,600
kn	+ 11,300	+ 13,600	+ 14,200	+ 6,100	+ 39,100	+ 25,500
kr	+ 8,000	+ 9,600	+ 6,100	+ 6,100	+ 23,700	+ 17,600
kv	+ 11,300	+ 13,600	+ 6,100	+ 14,200	+ 39,100	+ 25,500
kx	+ 13,300	+ 16,000	+ 6,100	+ 18,300	+ 47,600	+ 31,600
ru	+ 3,400	+ 4,100	0	+ 7,400	+ 14,900	+ 10,800
rs	+ 5,000	+ 6,000	0	+ 11,000	+ 22,000	+ 16,000
st	- 1,650	- 2,000	0	+ 3,700	- 7,350	- 5,350
tu	+ 1,850	+ 2,200	0	+ 4,100	+ 8,150	+ 5,950
uv	- 3,300	- 4,000	0	- 7,400	- 14,700	- 10,700
vw	+ 1,650	+ 2,000	0	- 3,700	+ 7,350	+ 5,350
wx	- 1,650	- 2,000	0	- 3,700	- 7,350	- 5,350
fs	- 11,600	- 13,900	0	- 14,900	- 40,400	- 26,500
gt	- 12,600	- 15,100	0	- 15,400	- 43,100	- 28,000
hw	- 13,700	- 16,400	0	- 15,900	- 46,000	- 30,100
ix	- 14,700	- 17,600	0	- 16,400	- 48,700	- 32,300

We may draw next the polygon for joint 3; it is KLMNK, MN and NK representing the stresses in *mn* and *nk*. No polygon for a joint on the left side can now be drawn, but we may begin at the right end. For joint 12 the polygon is JKXIJ, KX and XI representing the stresses in *kx* and *xi*.

At joint 11 there are three forces; and since they are balanced, and two act along the same line, those two must be equal and opposite, and the third must equal zero. Hence the point X is also marked W to indicate the fact that XW, or the stress in *xw*, is zero. Then, too, the diagram shows that WH equals XI. Having just shown that there is no stress in *xw*, there are but three forces at joint 13. Since two of these act along the same line, they must be equal and opposite, and the third zero. Therefore the point W is also marked V to indicate the fact that WV, or the stress in *wv*, equals zero. The diagram shows also that VK equals XK. This same argument applied to joints 9, 15, 10, and 14 successively, shows that the stresses in *st*, *tu*, *uv*, *ur*, and *sr* respectively equal zero. For this reason the point X is also marked UTS and R. It is plain, also, that the stresses in *sf* and *tg* equal those in *wh* and *xi*, and that the stress in *kr* equals that in *kv* or *kx*. Remembering that we are discussing stress due to wind pressure only, it is plain, so far as wind pressure goes, that the intermediate members on the right side are superfluous.

We may now resume the construction of the polygons for the joints on the left side. At joint 4, we know the forces in the members *kn* and *kr*; hence there are only two unknown forces there. The polygon for the joint is KNORK, NO and OR representing the stresses in *no* and *or*. The polygon for joint 5 may be drawn next; it is ONMCDPO, DP and PO representing the stresses in *dp* and *po*. The polygon for joint 6 or joint 7 may be drawn next; for 6 it is PDEQP, EQ and QP representing the stresses in *eq* and *qp*. At joint 7 there is but one unknown force, and it must close the polygon for the known forces there. That polygon is ROPQ; hence QR represents the unknown force. (If the work has been correctly and accurately done, QR will be parallel to *qr*).

When the wind blows upon the right side, the values of the reactions, and the stresses in any two corresponding members, are reversed. Thus, when the wind blows upon the left side, the stresses in kl and kx equal 18,300 and 6,100 pounds respectively; and when it blows upon the right they are respectively 6,100 and 18,300 pounds. It is not necessary, therefore, to construct a stress diagram for the wind pressure on the right. The numbers in the fifth column (see table, Page 72) relate to wind right, and were obtained from those in the fourth.

41. Combination of Dead, Snow, and Wind-Load Stresses. After having found the stress in any member due to the separate loads (dead, snow, and wind), we can then find the stress in that member due to any combination of loads, by adding algebraically the stresses due to loads separately. Thus, in a given member, suppose:

Dead-load stress = + 10,000 pounds,
Snow-load " = + 15,000 "
Wind-load " (right) = - 12,000 "
" " (left) = + 4,000 "

Since the dead load is permanent (and hence the dead-load stress also) the resultant stress in the member when there is a snow load and no wind pressure, is

$$+ 10{,}000 + 15{,}000 = + 25{,}000 \text{ pounds (tension)};$$

when there is wind pressure on the right, the resultant stress equals

$$+ 10{,}000 - 12{,}000 = - 2{,}000 \text{ pounds (compression)};$$

when there is wind pressure on the left, the resultant stress is

$$+ 10{,}000 + 4{,}000 = + 14{,}000 \text{ pounds (tension)};$$

and when there is a snow load and wind pressure on the left, the resultant stress is

$$+ 10{,}000 + 15{,}000 + 4{,}000 = + 29{,}000 \text{ pounds (tension).}$$

If all possible combinations of stress for the preceding case be made, it will be seen that the greatest tension which can come upon the member is 29,000 pounds, and the greatest compression is 2,000 pounds.

In roof trusses it is not often that the wind load produces a "reversal of stress" (that is, changes a tension to compression, or *vice versa*); but in bridge trusses the rolling loads often produce reversals in some of the members. In a record of stresses the reversals of stress should always be noted, and also the value of the greatest tension and compression for each one.

The numbers in the sixth column of the record (Page 72) are the values of the greatest resultant stress for each member. It is sometimes assumed that the greatest snow and wind loads will not come upon the truss at the same time. On this assumption the resultant stresses are those given in the seventh column.

EXAMPLE FOR PRACTICE.

1. Compile a complete record for the stresses in the truss of Fig. 24, for dead, snow, and wind loads. See Example 1, Article 27, for values of dead-load stresses, and Example 2, Article 29, for values of the wind-load stresses. Assume the snow load to equal 1.2 times the dead load.

After the record is made, compute the greatest possible stress in each member, assuming that the wind load and snow load will not both come upon the truss at the same time.

The greatest resultant stresses are as follows:

Member.	*af*	*fe*	*bg*	*fg*	*gh*	*hi*	*hc*	*ie*	*id*
Resultant....	-14,950	+17,800	-10,400	-8,900	+7,800	-8.900	-11.800	+11,500	-13,800

42. Truss Sustaining a Roof of Changing Slope. Fig 46 represents such a truss. The weight of the truss itself can be estimated by means of the formula of Art. 19. Thus if the distance between trusses equals 12 feet, the weight of the truss equals

$$W = 12 \times 32\,(\frac{32}{25} + 1) = 875 \text{ pounds.}$$

The **weight of the roofing** equals the product of the area of the roofing and the weight per unit area. The area equals 12 times the sum of the lengths of the members $\overline{12}$, $\overline{23}$, $\overline{34}$, and $\overline{45}$, that is, $12 \times 36\frac{1}{2} = 438$ square feet. If the roofing weighs 10 pounds per square foot, then the weight of roofing sustained by one truss equals $438 \times 10 = 4{,}380$ pounds. The total dead load then equals

$$875 + 4{,}380 = 5{,}255 \text{ pounds;}$$

and the apex dead loads for joints 2, 3, and 4 equal:

$$\frac{1}{4} \times 5{,}255 = 1{,}314 \text{ (or approximately 1,300) pounds;}$$

while the loads for joints 1 and 5 equal

$$\frac{1}{8} \times 5{,}255 = 657 \text{ (or approximately 650) pounds.}$$

The **snow loads** for the joints are found by computing the snow load on each separate slope of the roof. Thus, if the snow weighs 20 pounds per square foot (horizontal), the load on the portion $\overline{12}$ equals 20 times the area of the horizontal projection of the portion of the roof represented by $\overline{12}$. This horizontal projection equals 8×12 (= 96) square feet; snow load equals 96×20 (= 1,920) pounds. This load is to be equally divided between joints 1 and 2.

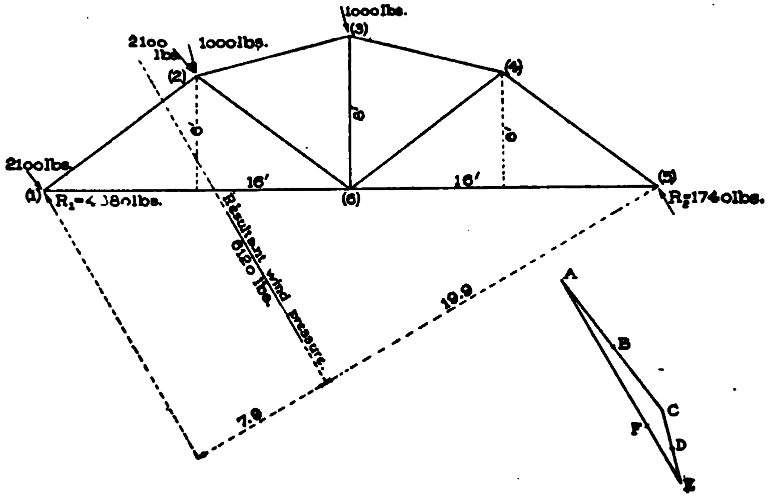


Fig. 46.

In a similar way the snow load borne by $\overline{23}$ equals 20 times the area of the horizontal projection of the roof represented by $\overline{23}$; this horizontal projection equals 8×12 (= 96) square feet as before, and the snow load hence equals 1,920 pounds also. This load is to be equally divided between joints 2 and 3.

Evidently the loads on parts $\overline{34}$ and $\overline{45}$ also equal 1,920 pounds each; hence the apex loads at joints 1 and 5 equal 960 pounds and at joints 2, 3 and 4, 1,920 pounds.

The wind load must be computed for each slope of the roof separately. The angles which $\overline{12}$ and $\overline{23}$ make with the horizontal, scale practically 37 and 15 degrees. According to the table of wind pressures (Art. 19), the pressures for these slopes equal about 35 and 20 pounds per square foot respectively. Since member $\overline{12}$ is 10 feet long, the wind pressure on the 37-degree slope equals $10 \times 12 \times 35 = 4{,}200$ pounds.

This force acts perpendicularly to the member $\overline{12}$, and is to be equally divided between joints 1 and 2 as represented in the figure. Since the member $\overline{23}$ is $8\frac{1}{4}$ feet long, the wind pressure on the 15-degree slope equals

$$8\tfrac{1}{4} \times 12 \times 20 = 1{,}980 \text{ or approximately } 2{,}000 \text{ pounds.}$$

This pressure acts perpendicularly to member $\overline{23}$, and is to be equally divided between joints 2 and 3 as represented.

The stress diagram for dead, snow, or wind load for a truss like that represented in Fig. 46, is constructed like those previously explained; but there are a few points of difference in the analysis for wind stress, and these will be explained in what follows.

Example. Let it be required to determine the stresses in the truss of Fig. 46, due to wind loads on the left as represented.

It is necessary to ascertain the reactions due to the wind loads; therefore, find the resultant of the wind pressures, see Art. 32; it equals 6,120 pounds and acts as shown. Now, if both ends of the truss are fastened to the supports, then the reactions are parallel to the resultant wind pressure, and their values can be readily found from moment equations. Let R_1 and R_2 denote the left and right reactions respectively; then, since the arms of R_1 and the resultant wind pressure with respect to the right end equal 27.8 and 19.9 feet respectively,

$$R_1 \times 27.8 = 6{,}120 \times 19.9 = 121{,}788\ ;$$

hence, $$R_1 = \frac{121{,}788}{27.8} = 4{,}380 \text{ pounds approximately.}$$

Since the arms of R_2 and the resultant wind pressure with respect to the left support are 27.8 and 7.9 feet respectively,

$$R_2 \times 27.8 = 6{,}120 \times 7.9 = 48{,}348\ ;$$

hence, $R_2 = \frac{48,348}{27.8} = 1,740$ pounds approximately.

The next step is to draw the polygon for the loads and reactions; so we draw lines AB, BC, CD, and DE to represent the loads at joint 1, the two at joint 2, and that at joint 3, respectively; and then EF to represent the right reaction. (If the reactions have been correctly determined and the drawing accurately done, then FA will represent the left reaction.)

The truss diagram should now be lettered (agreeing with the letters on the polygon just drawn), and then the construction of the stress diagram may be begun. Since this construction presents no points not already explained, it will not be here carried out.

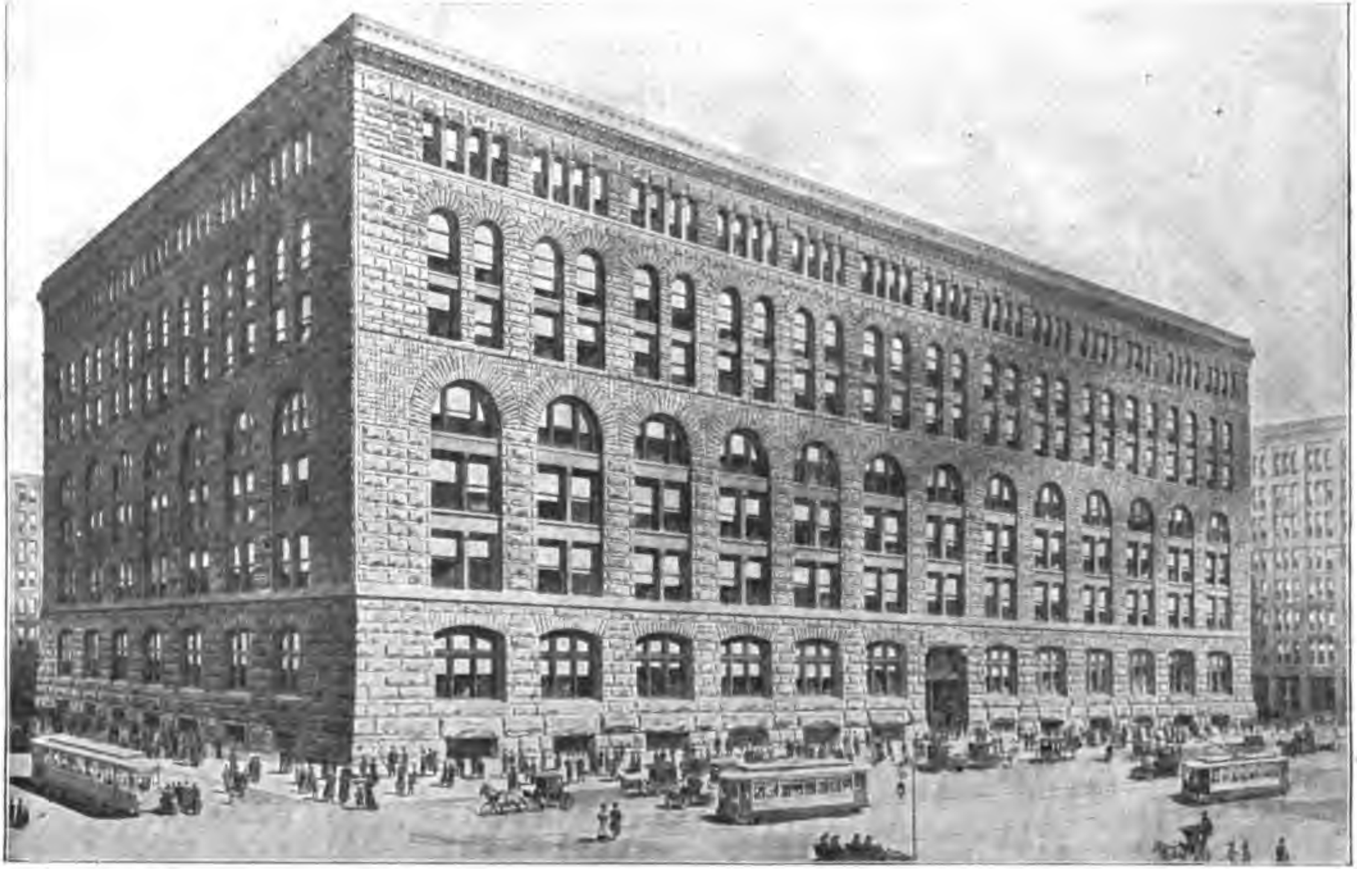
EXAMPLE FOR PRACTICE.

Analyze the truss of Fig. 46 for dead, snow, and wind loads as computed in the foregoing, and compute the greatest resultant stress in each member due to combined loads, assuming that the snow and wind do not act at the same time.

Stress Record.

Ans.

Member.	Dead.	Snow.	Wind Left.	Wind Right.	Resultant.
12	-3,250	-4,800	-3,450	-2,500	-8,050
23	-2,700	-4,000	-2,850	-3,100	-6,700
16	+2,600	+3,850	+3,750	+1,150	+6,450
26	0	0	-2,000	+1,250	-2,000 +1,250
36	0	0	+ 450	+ 450	+ 450
46	0	0	+1,250	-2,000	+1,250 -2 000
56	+2,600	+3,850	+1,150	+3,750	+6,450
43	-2,700	-4,000	-3,100	-2,850	-6,700
54	-3,250	-4,800	-2,500	-3,450	-8,050



MARSHALL FIELD WHOLESALE STORE, CHICAGO, ILL.
H. H. Richardson and Shepley, Rutan & Coolidge, Architects. Granite Exterior.

MASONRY CONSTRUCTION.

PART I.

STRUCTURAL MATERIALS.

Classification of Natural Stones. The rocks from which the stones for building are selected are classified according to (1) their geological position, (2) their physical structure, and (3) their chemical composition.

Geological Classification. The geological position of rocks has but little connection with their properties as building materials. As a general rule, the more ancient rocks are the stronger and more durable; but to this there are many notable exceptions. According to the usual geological classification rocks are divided into three classes, viz.:

Igneous, of which greenstone (trap), basalt, and lava are examples.

Metamorphic, comprising granite, slate, marble, etc.

Sedimentary, represented by sandstones, limestones, and clay.

Physical Classification. With respect to the structural character of their large masses, rocks are divided into two great classes: (1) the unstratified, (2) the stratified, according as they do or do not consist of flat layers.

The *unstratified* rocks are for the most part composed of an aggregation of crystalline grains firmly cemented together. Granite, trap, basalt, and lava are examples of this class. All the unstratified rocks are composed as it were of blocks which separate from each other when the rock decays or when struck violent blows. These natural joints are termed the *line of cleavage* or *rift*, and in all cutting or quarrying of unstratified rocks the work is much facilitated by taking advantage of them.

The *stratified* rocks consist of a series of parallel layers, evidently deposited from water, and originally horizontal, although in most cases they have become more or less inclined and curved by the action of disturbing forces. It is easier to divide them at the planes of division between these layers than elsewhere. Besides its principal layers or strata, a mass of stratified rock is in general capable of division into thinner layers; and, although the surfaces of division of the thinner layers are often parallel to those of the strata, they are also often oblique or even perpendicular to them. This constitutes a *laminated* structure.

Laminated stones resist pressure more strongly in a direction perpendicular to their laminæ than parallel to them; they are more tenacious in a direction parallel to their laminæ than perpendicular to them; and they are more durable with the edges than with the sides of their laminæ exposed to the weather. Therefore in building they should be placed with their laminæ or "beds" perpendicular to the direction of greatest pressure, and with the edges of these laminæ at the face of the wall.

Chemical Classification. The stones used in building are divided into three classes, each distinguished by the predominant mineral which forms the chief constituent, viz.:

Silicious stones, of which granite, gneiss, and trap are examples.

Argillaceous stones, of which clay, slate, and porphyry are examples.

Calcareous stones, represented by limestones and marbles.

REQUISITES FOR GOOD BUILDING STONE.

The requisites for good building stone are durability, strength, cheapness, and beauty.

Durability. The durability of stone is a subject upon which there is very little reliable knowledge. The durability will depend upon the chemical composition, physical structure, and the position in which the stone is placed in the work. The same stone will vary greatly in its durability according to the nature and extent of the atmospheric influences to which it is subjected.

The sulphur acids, carbonic acid, hydrochloric acid, and traces of nitric acid, in the smoky air of cities and towns, and the carbonic acid in the atmosphere ultimately decompose any stone of which either carbonate of lime or carbonate of magnesia forms a considerable part.

Wind has a considerable effect upon the durability of stone. High winds blow sharp particles against the face of the stone and thus grind it away. Moreover, it forces the rain into the pores of the stone, and may thus cause a considerable depth to be subject to the effects of acids and frost.

In winter water penetrates porous stones, freezes, expands, and disintegrates the surface, leaving a fresh surface to be similarly acted upon.

Strength is generally an indispensable attribute, especially under compression and cross-strain.

Cheapness is influenced by the ease with which the stone can be quarried and worked into the various forms required. Cheapness is also affected by abundance, facility of transportation, and proximity to the place of use.

Appearance. The requirement of beauty is that it should have a pleasing appearance. For this purpose all varieties containing much iron should be rejected, as they are liable to disfigurement from rust-stains caused by the oxidation of the iron under the influence of the atmosphere.

TESTS FOR STONE.

The relative enduring qualities of different stones are usually ascertained by noting the weight of water they absorb in a given time. The best stones as a rule absorb the smallest amount of water.

Some stones, however, come from the quarry soaked with water and in that condition are very soft and easily worked. Upon exposure to the atmosphere they gradually dry out and become very hard and durable. The Bedford limestone of Indiana forms an example of this kind of stone, and the stone in many of the public buildings throughout the United States may be seen in the process of "weathering", indicated by the mottled appearance of the walls.

To determine the absorptive power, dry a specimen and weigh it carefully, then immerse it in water for 24 hours and weigh again. The increase in weight will be the amount of absorption.

TABLE 1.

Absorptive Power of Stones.

	Percentage of Water Absorbed.
Granites	0.06 to 0.15
Sandstones	0.41 " 5.48
Limestones	0.20 " 5.00
Marbles	0.08 " 0.16

Effect of Frost (*Brard's Test*). To ascertain the effect of frost, small pieces of the stone are immersed in a concentrated boiling solution of sulphate of soda (Glauber's salts), and then hung up for a few days in the air. The salt crystallizes in the pores of the stone, sometimes forcing off bits from the corners and arrises, and occasionally detaching larger fragments.

The stone is weighed before and after submitting it to the test. The difference of weight gives the amount detached by disintegration. The greater this is, the worse is the quality of the stone.

Effect of the Atmosphere (*Acid Test*). Soaking a stone for several days in water containing 1 per cent of sulphuric and hydrocholric acids will afford an idea as to whether it will stand the atmosphere of a large city. If the stone contains any matter likely to be dissolved by the gases of the atmosphere, the water will be more or less cloudy or muddy.

A drop or two of acid on the surface of a stone will create an intense effervescence if there is a large proportion present of carbonate of lime or magnesia.

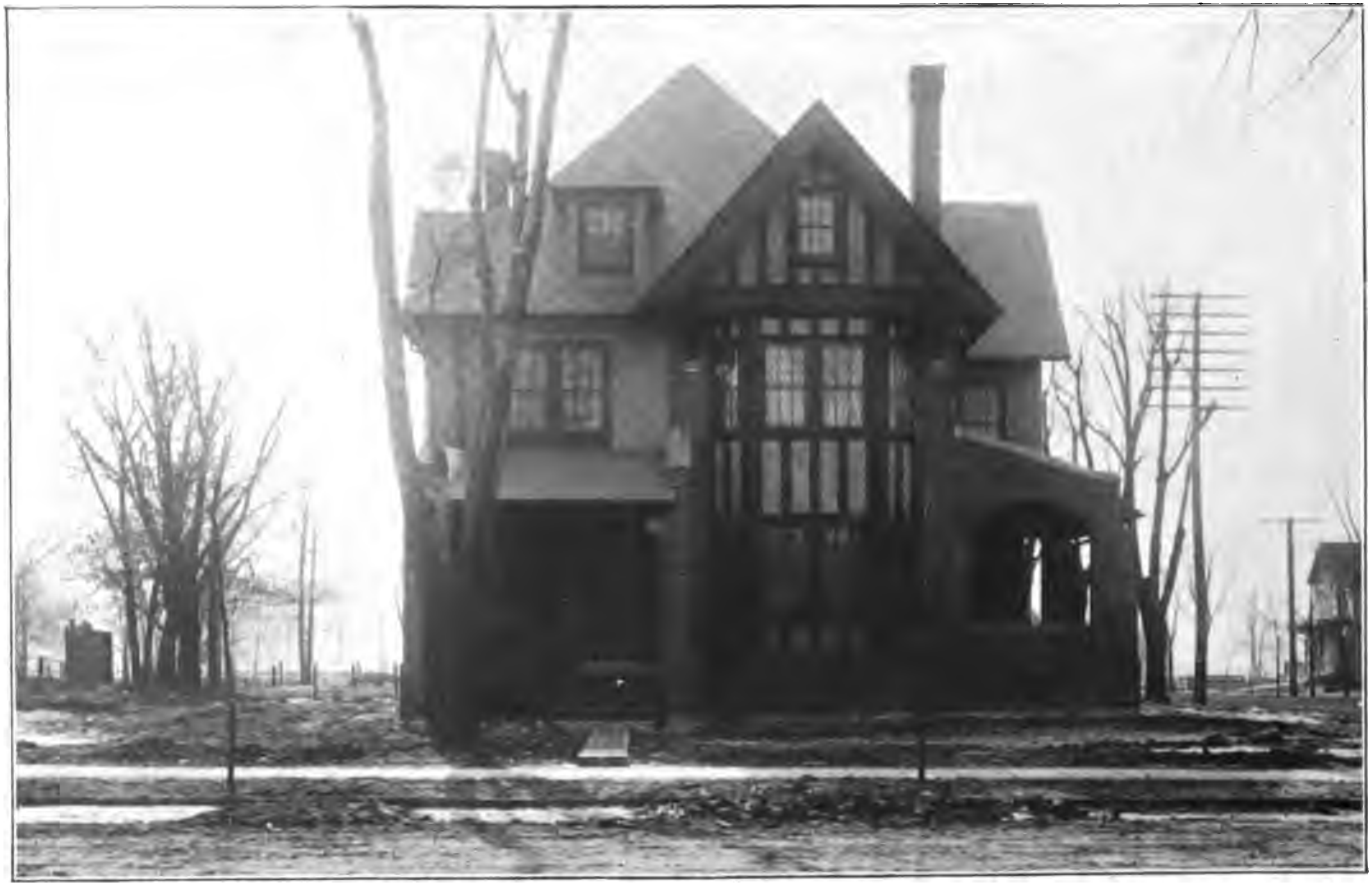
PRESERVATION OF STONE.

A great many preparations have been used for the prevention of the decay of building stones, as paint, coal-tar, oil, beeswax, rosin, paraffine, soft-soap, soda, etc. All of the methods are expensive, and there is no evidence to show that they afford permanent protection to the stone.

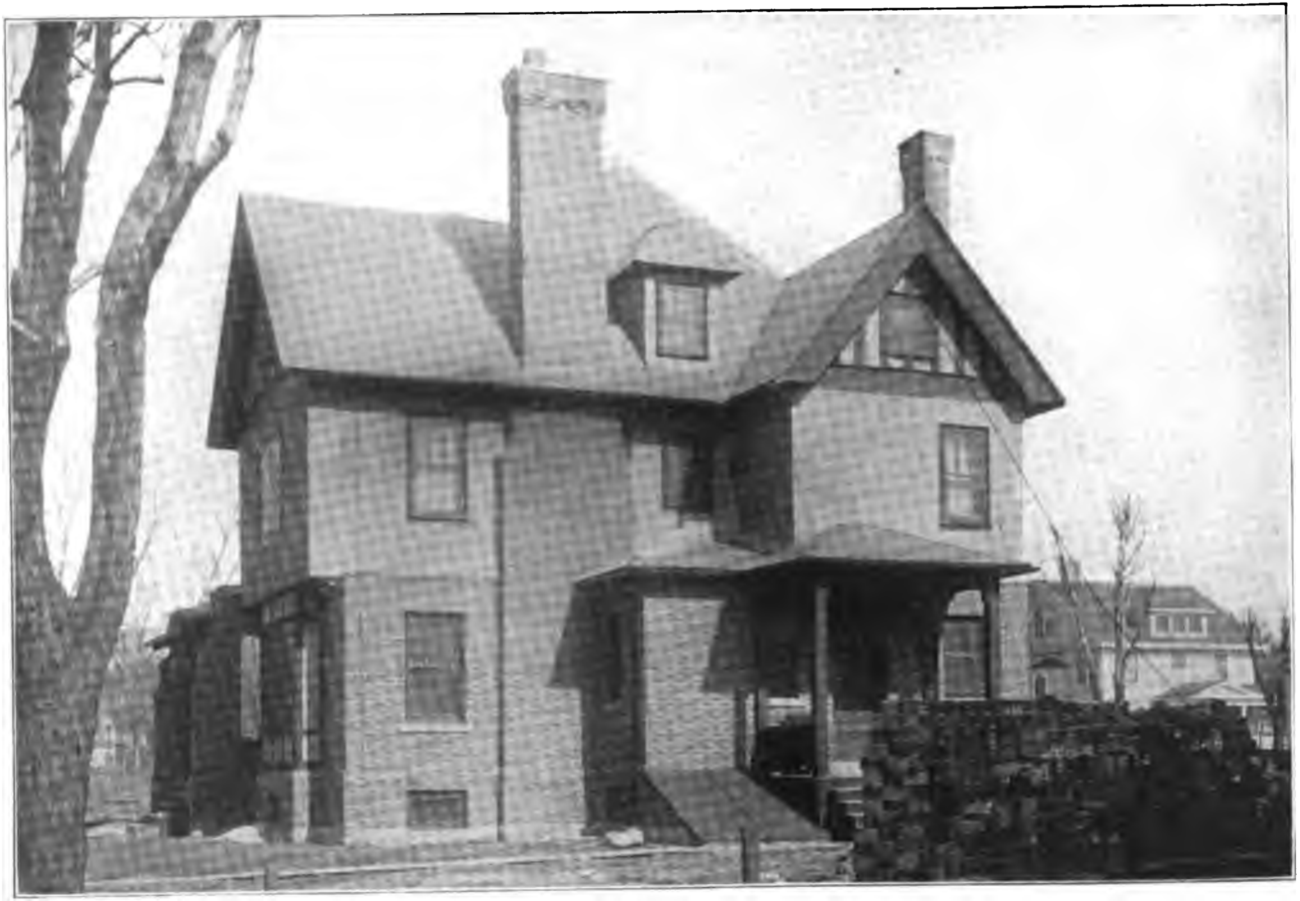
ARTIFICIAL STONES.

Brick is an artificial stone made by submitting clay, which has been suitably prepared and moulded into shape, to a temperature of sufficient intensity to convert it into a semi-vitrified state. The quality of the brick depends upon the kind of clay used and upon the care bestowed on its preparation.

The clays of which brick is made are chemical compounds consisting of silicates of alumina, either alone or combined with other substances, such as iron, lime, soda, potash, magnesia, etc., all of which influence the character and quality of the brick, according as one or the other of those substances predominates.



FRONT VIEW OF HOUSE AT ST. PAUL, MINN.
John T. Comes, Architect, Pittsburg, Pa.
Cost of House, $7,500. For Plans, See Page 234.



REAR VIEW OF HOUSE AT ST. PAUL, MINN.
John T. Comes, Architect, Pittsburg, Pa.
For Plans, See Page 234.

TABLE 2.

Specific Gravity, Weight and Resistance to Crushing of Stones.

Kinds of Stone	Specific Gravity	Weight, pounds per cubic foot	Resistance to crushing, pounds per sq. in.
Granite—			
minimum	2.60	163	12,000
maximum	2.80	176	35,000
Trap—			
minimum	2.86	178	19,000
maximum	3.03	189	24,000
Gneiss, average	2.70	168	19,600
Syenite, average	2.64	167	30,740
Sandstones—			
minimum	2.23	137	5,000
maximum	2.75	170	18,000
Limestones—			
minimum	1.90	118	7,000
maximum	2.75	175	20,000
Marbles—			
minimum	2.62	165	8,000
maximum	2.95	179	20,000

Iron gives hardness and strength; hence the red brick of the Eastern States is often of better quality than the white and yellow brick made in the West. *Silicate of lime* renders the clay too fusible and causes the bricks to soften and to become distorted in the process of burning. *Carbonate of lime* is at high temperatures changed into caustic lime, renders the clay fusible, and when exposed to the action of the weather absorbs moisture, promotes disintegration, and prevents the adherence of the mortar. *Magnesia* exerts but little influence on the quality; in small quantities it renders the clay fusible; at 60° F. its crystals lose their water of crystallization, and cold water decomposes them, forming an insoluble hydrate in the form of a white powder. In air-dried brick this action causes cracking. The *alkalies* are found in small quantities in the best of clays; their presence tends to promote softening, and this goes on the more rapidly if it has been burned at too low a temperature. *Sand* mixed with the clay in moderate quantity (one part of sand to four of clay is about the best proportion) is beneficial, as tending to prevent excessive shrinking in the fire. Excess of sand destroys the cohesion and renders the brick brittle and weak.

MANUFACTURE OF BRICK.

The manufacture of brick may be classified under the following heads:

Excavation of the clay, either by manual or mechanical power.

Preparation of the clay consists in (*a*) removing stones and mechanical impurities; (*b*) tempering and moulding, which is now done almost wholly by machinery. There is a great variety of machines for tempering and moulding the clay, which, however, may be grouped into three classes, according to the condition of the clay when moulded: (1) soft-mud machines, for which the clay is reduced to a soft mud by adding about one quarter of its volume of water; (2) stiff-mud machines, for which the clay is reduced to a stiff mud; (3) dry-clay machines, with which the dry or nearly dry clay is forced into the moulds by a heavy pressure without having been reduced to a plastic mass. These machines may also be divided into two classes, according to the method of filling the moulds: (1) those in which a continuous stream of clay is forced from the pug-mill through a die and is afterwards cut up into bricks; and (2) those in which the clay is forced into moulds moving under the nozzle of the pug-mill.

Drying and Burning. The bricks, having been dried in the open air or in a drying-house, are burned in kilns; the time of burning varies with the character of the clay, the form and size of the kiln, and the kind of fuel, from six to fifteen days.

Color of Bricks depends upon the composition of the clay, the moulding sand, temperature of burning, and volume of air admitted to the kiln. Pure clay free of iron will burn *white,* and mixing of chalk with the clay will produce a like effect. Iron produces a tint ranging from *red* and *orange* to *light yellow,* according to the proportion of the iron.

A large proportion of oxide of iron mixed with pure clay will produce a *bright red,* and where there is from 8 to 10 per cent, and the brick is exposed to an intense heat, the oxide fuses and produces a *dark blue* or *purple,* and with a small volume of manganese and an increased proportion of the oxide the color is darkened even to a *black.*

A small volume of lime and iron produces a *cream color,* an increase of iron produces *red,* and an increase of lime *brown.* Magnesia in presence of iron produces *yellow,* and clay containing alkalies and burned at a high temperature produces a *bluish green.*

The best quality of building brick and probably the majority of paving brick or block, are manufactured from shale. The process of manufacture is similar to that of clay-brick, the shale being first ground very fine. If the shale is nearly free from impurities, the resulting product will be a cream colored brick. To give the brick any desired color, the shale is mixed with clay containing the proper proportions of lime, iron, or magnesia, giving almost any shade from a cream to a dark wine color or even a black.

Classification of Brick: Bricks are classified according to (1) the way in which they are moulded; (2) their position in the kiln while being burned; and (3) their form or use.

The method of moulding gives rise to the following terms:

Soft-mud Brick. One moulded from clay which has been reduced to a soft mud by adding water. It may be either hand-moulded or machine-moulded.

Stiff-mud Brick. One moulded from clay in the condition of stiff mud. It is always machine-moulded.

Pressed Brick. One moulded from dry or semi-dry clay.

Re-pressed Brick. A soft-mud brick which, after being partially dried, has been subjected to an enormous pressure. It is also called, but less appropriately, pressed brick. The object of the re-pressing is to render the form more regular and to increase the strength and density.

Sanded Brick. Ordinarily, in making soft-mud brick, sand is sprinkled into the moulds to prevent the clay from sticking; the brick is then called sanded brick. The sand *on the surface* is of no advantage or disadvantage. In hand-moulding, when sand is used for this purpose, it is certain to become mixed with the clay and occur in streaks in the finished brick, which is very undesirable.

Machine-made Brick. Brick is frequently described as "machine made"; but this is very indefinite, since all grades and kinds are made by machinery.

When brick was generally burned in the old-style up-draught kiln, the classification according to position was important; but with the new styles of kilns and improved methods of burning, the quality is so nearly uniform throughout the kiln that the classification is less important. Three grades of brick are taken from the old-style kiln:

Arch or Clinker Bricks. Those which form the tops and sides of the arches in which the fire is built. Being overburned and partially vitrified, they are hard, brittle, and weak.

Body, Cherry, or Hard Bricks. Those taken from the interior of the pile. The best bricks in the kiln.

Salmon, Pale, or Soft Bricks. Those which form the exterior of the mass. Being underburned, they are too soft for ordinary work, unless it be for filling. The terms *salmon* and *pale* refer to the color of the brick, and hence are not applicable to a brick made of a clay that does not burn red. Although nearly all brick-clays burn red, yet the localities where the contrary is true are sufficiently numerous to make it desirable to use a different term in designating the *quality.* There is not necessarily any relation between color, and strength and density. Brick-makers naturally have a prejudice against the term *soft brick,* which doubtless explains the nearly universal prevalence of the less appropriate term salmon.

The form or use of bricks gives rise to the following classification:

Compass Brick. Those having one edge shorter than the other. Used in lining shafts, etc.

Feather-edge Brick. Those of which one edge is thinner than the other. Used in arches; and more properly, but less frequently, called *voussoir* brick.

Front or Face Brick. Those which, owing to uniformity of size and color, are suitable for the face of the walls of buildings. Sometimes face bricks are simply the best ordinary brick; but generally the term is applied only to re-pressed or pressed brick made especially for this purpose. They are a little larger than ordinary bricks.

Sewer Brick. Ordinary hard brick, smooth, and regular in form.

Kiln-run Brick. All the brick that are set in the kiln when burned.

Hard Kiln-run Brick. Brick burned hard enough for the face of outside walls, but taken from the kiln unselected.

Rank of Bricks. In *regularity of form* re-pressed brick ranks first, dry-kiln brick next, then stiff-mud brick, and soft-mud brick last. Regularity of form depends largely upon the method of burning.

The *compactness and uniformity* of texture, which greatly influence the durability of brick, depend mainly upon the method of moulding. As a general rule, hand-moulded bricks are best in this respect, since the clay in them is more uniformly tempered before being moulded; but this advantage is partially neutralized by the presence of sand-seams. Machine-moulded soft-mud bricks rank next in compactness and uniformity of texture. Then come machine-moulded stiff-mud bricks, which vary greatly in durability with the kind of machine used in their manufacture. By some of the machines the brick is moulded in layers (parallel to any face, according to the kind of machine) which are not thoroughly cemented, and which separate under the action of frost. The dry-clay brick comes last. However, the relative value of the products made by different processes varies with the nature of the clay used.

TABLE 3.

Size and Weight of Bricks.

The variations in the dimensions of brick render a table of exact dimensions impracticable.

As an exponent, however, of the ranges of their dimensions, the following averages are given:

Baltimore front Wilmington front Washington front	8¼ in. × 4⅛ in. × 2⅜ in.
Croton front	8½ in. × 4 in. × 2¼ in.
Maine ordinary	7½ in. × 3⅜ in. × 2⅜ in.
Milwaukee ordinary	8½ in. × 4⅛ in. × 2⅜ in.
North River, N. Y	8 in. × 3½ in. × 2¼ in.
English	9 in. × 4½ in. × 2½ in.

The Standard Size as adopted by the National Brickmakers' Association and the National Traders and Builders' Association is for common brick 8¼ × 4 × 2¼ inches, and for face brick 8⅜ × 4⅛ × 2¼ inches.

Re-pressed Brick weighs about 150 lb. per cubic foot, common brick 125, inferior soft 100. Common bricks will average about 4½ lb. each.

Hollow Brick, used for interior walls and furring, are usually of the following dimensions:

Single, 8 in. long, 3⅝ in. wide, 2¼ in. thick.
Double, 8 " " 7½ " " 4½ " "
Treble, 8 " " 7¼ " " 7¼ " "

Roman Brick, 12 in. long, 4 to 4½ in. wide, 1½ in. thick.

TABLE 4.

Specific Gravity, Weight, and Resistance to Crushing of Brick.

Designation of brick.	Specific gravity.	Weight per cubic foot, pounds.	Resistance to Crushing, pounds per sq. in.
Best pressed..........	2.4	150	5,000 to 14,973
Common hard;.......	1.6 to 2.0	125	5,000 to 8,000
Soft.................	1.4	100	450 to 600

Fire-Brick are used wherever high temperatures are to be resisted. They are made from fire-clay by processes very similar to those adopted in making ordinary brick. Fire-clay is also used in the manufacture of paving-blocks or pavers, especially in Western Indiana; and many of the streets of our Western cities are laid with fire-clay block, forming a smooth and durable roadway.

Fire-clay may be defined as native combinations of hydrated silicates of alumina, mechanically associated with silica and alumira in various states of subdivision, and sufficiently free from silicates of the alkalies and from iron and lime to resist vitrification at high temperatures. The presence of oxide of iron is very injurious; and, as a rule, the presence of 6 per cent justifies the rejection of the brick. The presence of 3 per cent of combined lime, soda, potash, and magnesia should be a cause for rejection. The sulphide of iron—pyrites—is even worse than the substances first named.

A good fire-clay should contain from 52 to 80 per cent of silica and 18 to 35 per cent of alumina and have a uniform texture, a somewhat greasy feel, and be free from any of the alkaline earths.

Good fire brick should be uniform in size, regular in shape, homogeneous in texture and composition, strong, and infusible and break with a uniform and regular fracture.

A properly burnt fire-brick is of a uniform color throughout its mass. A dark central patch and concentric rings of various shades of color are due mainly to the different states of oxidation of the iron and partly to the presence of unconsumed carbonaceous matter, and indicates that the brick was burned too rapidly.

Fire-brick are made in various forms to suit the required work. A straight brick measures 9 × 4½ × 2½ inches and weighs about 7 lb., or 120 lb. per cubic foot; specific gravity 1.93. One cubic foot of wall requires 17 9-inch bricks; one cubic yard requires 460. One ton of fire-clay should be sufficient to lay 3000 ordinary bricks. English fire-bricks measure 9 × 4½ × 2¼ inches.

To secure the best results fire-brick should be laid in the same clay from which they are manufactured. It should be used as a thin paste, and not as mortar: the thinner the joint the better the furnace wall. The brick should be dipped in water as they are used, so that when laid they will not absorb the water from the clay paste. They should then receive a thin coating of the prepared fire-clay, and be carefully placed in position with as little of the fire-clay as possible.

CEMENTING MATERIALS.

Composition. All the cementing materials employed in buildings are produced by the burning of natural or artificial mixtures of limestone with clay or siliceous material. The active substances in this process and the ones which are necessary for the production of a cement, are the burned lime, the silica and the alumina, all of which enter into chemical combination with one another under the influence of a high temperature.

Classification. Owing to the varying composition of the raw materials, which range from pure carbonate of lime to stones containing variable proportions of silica, alumina, magnesia, oxide of iron, manganese, etc., and the different methods employed for burning, the product possesses various properties which regulate its behavior when treated with water, and render necessary certain precautions in its manipulation and use, and furnishes a basis for division into three classes; namely, common lime, hydraulic lime, and hydraulic cements, the individual peculiarities of which will be taken up later.

Common lime is distinguished from hydraulic lime by its failure to set or harden under water, a property which is possessed by hydraulic lime to a greater or less degree.

The limes are distinguished from the cements by the former falling to pieces (slaking) on the application of water, while the latter must be mechanically pulverized before they can be used.

The hydraulic cements are divided into two classes, namely; *natural* and *artificial*. The first class includes all hydraulic substances produced from natural mixtures of lime and clay, by a burning process which has not been carried to the point of vitrification, and which still contain more or less free lime.

The artificial cements are generally designated by the name "Portland" and comprise all the cements produced from natural or artificial mixtures of lime and clay, lime and furnace slag, etc., by a burning process which is carried to the point of vitrification.

The hydraulic cements do not slake after calcination, differing materially in this particular from the limes proper. They can be formed into paste with water, without any sensible increase in volume, and little, if any, production of heat, except in certain instances among those varieties which contain the maximum amount of lime. They do not shrink in hardening, like the mortar of rich lime, and can be used with or without the addition of sand, although for the sake of economy sand is combined with them.

All the limes and cements in practical use have one feature in common, namely, the property of "setting" or "hardening" when combined with a certain amount of water. The setting of a cement is, in general, a complex process, partly chemical in its nature and partly mechanical. Broadly stated, the chemical changes which occur may be said to afford opportunity for the mechanical changes which result in hardening, rather than themselves to cause the hardening. The chemical changes are, therefore, susceptible of wide variation without materially influencing the result. The crumbling which calcined lime undergoes on being slaked is simply a result of the mechanical disintegrating action of the evolved steam. In some cements of which plaster of Paris may be taken as the type, water simply combines with some constituent of the cement already present. In others, of which Portland cement is the most important example, certain chemical reactions must first take place. These reactions give rise to substances which, as soon as formed, combine with water and constitute the true cementitious material. The quantity of water used should be regulated according to the kind of cement, since every cement has a certain capacity for water. However, in practice an excess of about 50 per cent must be used to aid manipulation.

The rapidity of setting (denominated activity) varies with the character of the cement, and is influenced to a great extent by the temperature, and also, but in less degree, by the purity of the water.

Sea water hinders the setting of some cements, and some cements, which are very hard in fresh water, only harden slightly in sea water or even remain soft. Cements which require more than one-half hour to set are called "slow-setting", all others "quick-setting". As a rule the natural cements are quick- and the Portlands slow-setting. None of the cements attain their maximum hardness until some time has elapsed. For good Portland 15 days usually suffices for complete setting, but the hardening process may continue for a year or more.

The form and fineness of the cement particles are of great importance in the setting of the cement, and affect the cementing and economic value. Coarse particles have no setting power and act as an adulterant. In consequence of imperfect pulverization some cements only develop three-fourths of their available activity, one-fourth of the cement consisting of grains so coarse as to act merely like so much sand. The best cement when separated from its fine particles will not harden for months after contact with water, but sets at once if previously finely ground.

In a mortar or concrete composed of a certain quantity of inert material bound together by a cementing material it is evident that to obtain a strong mortar or concrete it is essential that each piece of aggregate shall be entirely surrounded by the cementing material, so that no two pieces are in actual contact. Obviously, then, the finer a cement the greater surface will a given weight cover, and the more economy will there be in its use. The proper degree of fineness is reached when it becomes cheaper to use more cement in proportion to the aggregate than to pay the extra cost of additional grinding.

Use. Common lime is used almost exclusively in making mortar for architectural masonry. Natural cement is used for masonry where great ultimate strength is not as important as initial strength and in masonry protected from the weather. Portland cement is used for foundations and for all important engineering structures requiring great strength or which are subject to shock; also for all sub-aqueous construction.

LIMES.

Rich Limes. The common fat or rich limes are those obtained by calcining pure or very nearly pure carbonate of lime. In slaking they augment to from two to three and a half times that of the original mass. They will not harden under water, or even in damp places excluded from contact with the air. In the air they harden by the gradual formation of carbonate of lime, due to the absorption of carbonic acid gas.

The pastes of fat lime shrink in hardening to such a degree that they cannot be employed for mortar without a large dose of sand.

Poor Limes. The poor or meagre limes generally contain silica, alumina, magnesia, oxide of iron, sometimes oxide of manganese, and in some cases traces of the alkalies, in relative proportions which vary considerably in different localities. In slaking they proceed sluggishly, as compared with the rich limes—the action only commences after an interval of from a few minutes to more than an hour after they are wetted; less water is required for the process, and it is attended with less heat and increase of volume than in the case of fat limes.

Hydraulic Limes. The hydraulic limes, including the three subdivisions, viz., *slightly hydraulic, hydraulic,* and *eminently hydraulic,* are those containing after calcination sufficient of such foreign constituents as combine chemically with lime and water to confer an appreciable power of setting or hardening under water without the access of air. They slake still slower than the meagre limes, and with but a small augmentation of volume, rarely exceeding 30 per cent of the original bulk.

Lime is shipped either in bulk or in barrels. If in bulk, it is impossible to preserve it for any considerable length of time. A barrel of lime usually weighs about 230 lb. net, and will make about three tenths of a cubic yard of stiff paste. A bushel weighs 75 lb.

NATURAL CEMENT.

Rosendale or *natural* cements are produced by burning in draw-kilns at a heat just sufficient in intensity and duration to expel the carbonic acid from argillaceous or silicious limestones containing less than 77 per cent of carbonate of lime, or argillo-magnesian limestone containing less than 77 per cent of both carbonates, and then grinding the calcined product to a fine powder between millstones.

Characteristics. The natural cements have a porous, globular texture. They do not heat up nor swell sensibly when mixed with water. They set quickly in air, but harden slowly under water, without shrinking, and attain great strength with well-developed adhesive force.

Setting. A pat made with the minimum amount of water should set in about 30 minutes.

Fineness. At least 93 per cent must pass through a No. 50 sieve.

Weight. Varies from 49 to 56 pounds per cubic foot.

Specific Gravity about 2.70.

Tensile Strength. Neat cement, one day, from 40 to 80 pounds. Seven days, from 60 to 100 pounds. One year, from 300 to 400 pounds.

PORTLAND CEMENT.

Portland Cement is produced by burning, with a heat of sufficient intensity and duration to induce incipient vitrification, certain argillaceous limestones, or calcareous clays, or an artificial mixture of carbonate of lime and clay, and then reducing the burnt material to powder by grinding. Fully 95 per cent of the Portland cement produced is artificial. The name is derived from the resemblance which hardened mortar made of it bears to a stone found in the isle of Portland, off the south coast of England.

The quality of Portland cement depends upon the quality of the raw materials, their proportion in the mixture, the degree to which the mixture is burnt, the fineness to which it is ground, and the constant and scientific supervision of all the details of manufacture.

Characteristics. The color should be a dull bluish or greenish gray, caused by the dark ferruginous lime and the intensely green manganese salts. Any variation from this color indicates the presence of some impurity; blue indicates an excess of lime; dark green, a large percentage of iron; brown, an excess of clay; a yellowish shade indicates an underburned material.

Fineness. It should have a clear, almost floury feel in the hand; a gritty feel denotes coarse grinding.

Specific Gravity is between 3 and 3.05. As a rule the strength of Portland cement increases with its specific gravity.

Tensile Strength. When moulded neat into a briquette and placed in water for seven days it should be capable of resisting a tensile strain of from 300 to 500 pounds per square inch.

Setting. A pat made with the minimum amount of water should set in not less than three hours, nor take more than six hours.

Expansion and Contraction. Pats left in the air or placed in water should during or after setting show neither expansion nor contraction, either by the appearance of cracks or change of form.

A cement that possesses the foregoing properties may be considered a fair sample of Portland cement and would be suitable for any class of work.

Overlimed Cement is likely to gain strength very rapidly in the beginning and later to lose its strength, or if the percentage of free lime be sufficient it will ultimately disintegrate.

Blowing or Swelling of Portland cement is caused by too much lime or insufficient burning. It also takes place when the cement is very fresh and has not had time to cool.

Adulteration. Portland cement is adulterated with slag cement and slaked lime. This adulteration may be distinguished by the light specific gravity of the cement, and by the color, which is of a mauve tint in powder, while the inside of a water-pat when broken is deep indigo. Gypsum or sulphate of lime is also used as an adulterant.

TESTING CEMENTS.

The quality or constructive value of a cement is generally ascertained by submitting a sample of the particular cement to a series of tests. The properties usually examined are the *color, weight, activity, soundness, fineness* and *tensile strength.* Chemical analysis is sometimes made, and specific gravity test is substituted for that of weight. Tests of compression and adhesion are also sometimes added. As these tests cannot be made upon the site of the work, it is usual to sample each lot of cement as it is delivered and send the samples to a testing laboratory.

Sampling Cement. The cement is sampled by taking a small quantity (1 to 2 lb.) from the center of the package. The number of packages sampled in any given lot of cement will depend upon the character of the work, and varies from every package to 1 in 5 or 1 in 10. When the cement is brought in barrels the sample is obtained by boring with an auger either in the head or center of the barrel, drawing out a sample, then closing the hole with a piece of tin firmly tacked over it. For drawing out the sample a brass tube sufficiently long to reach the bottom of the barrel is used. This is thrust into the barrel, turned around, pulled out, and the core of cement knocked out into the sample-can, which is usually a tin box with a tightly fitting cover.

Each sample should be labelled, stating the number of the sample, the number of bags or barrels it represents, the brand of the cement, the purpose for which it is to be used, the date of delivery, and date of sampling.

FORM OF LABEL.

Sample No..................................
No. of Barrels..................................
Brand ....
To be used..................................
Delivered................ *Sampled*...............
By...................

The sample should be sent at once to the testing office, and none of the cement should be used until the report of the tests is received.

After the report of the tests is received the rejected packages should be conspicuously marked with a "C" and should be removed without delay; otherwise they are liable to be used.

Color. The color of a cement indicates but little, since it is chiefly due to oxides of iron and manganese, which in no way affect the cementitious value; but for any given kind variations in shade may indicate differences in the character of the rock or in the degree of burning. The natural cements may have almost any color from the very light straw colored "Utica" through the brown "Louisville", to chocolate "Rosendale". The artificial Portlands are usually a grayish blue or green, but never chocolate colored.

Weight. For any particular cement the weight varies with the degree of heat in burning, the degree of fineness in grinding, and the density of packing. The finer a cement is ground the more bulky it becomes, and consequently the less it weighs. Hence light weight may be caused by laudable fine grinding or by objectionable underburning. Other things being the same, the harder-burned varieties are the heavier.

The weight per unit of volume is usually determined by sifting the cement into a measure as lightly as possible, and striking the top level with a straight edge. In careful work the height of fall should be recorded. Since the cement absorbs moisture, the sample must be taken from the interior of the package. The weight per cubic foot is neither exactly constant, nor can it be determined precisely. The approximate weight of cement per cubic foot is as follows:

Portland, English and German	77 to 90 lb.
" fine-ground French	69 "
" American	92 " 95 "
Rosendale	49 " 56 "
Roman	54 "

A bushel contains 1.244 cubic feet. The weight of a bushel can be obtained sufficiently close by adding 25 per cent to the weight per cubic foot.

Fineness. The cementing and economic value of a cement is affected by the degree of fineness to which it is ground. Coarse particles in a cement have no setting power and act as an adulterant.

The fineness of a cement is determined by measuring the percentage which will not pass through sieves of a certain number of meshes per square inch. Three sieves are generally used, viz.:

No. 50,	2,500	meshes	per	square	inch.
No. 74,	5,476	"	"	"	"
No. 100,	10,000	"	"	"	"

Activity denotes the speed with which a cement begins to *set*. Cements differ widely in their rate and manner of *setting*. Some occupy but a few minutes in the operation, and others require several. Some begin setting immediately and take considerable time to complete the set, while others stand for a considerable time with no apparent action and then set very quickly. The point at which the set is supposed to begin is *when the stiffening of the mass first becomes perceptible,* and the end of the set is when cohesion extends through the mass sufficiently to offer such resistance to any change of form as to cause rupture before any deformation can take place.

Test of Activity. To test the activity mix the cement with 25 to 30 per cent of its weight of clean water, having a temperature of between 65° F. and 70° F., to a stiff plastic mortar, and make one or two cakes or pats 2 or 3 inches in diameter and about $\frac{1}{2}$ inch in thickness. As soon as the cakes are prepared, immerse in water at 65° F., and note the time required for them to set hard enough to bear respectively a $\frac{1}{12}$-inch wire loaded to weigh $\frac{1}{4}$ pound, and a $\frac{1}{24}$-inch wire loaded to weigh 1 pound. When the cement bears the light weight, it is said to have begun to set; when it bears the heavy weight, it is said to have entirely set. The apparatus employed for this test is shown in Fig. 1, and is called "Vicat's Needle apparatus".

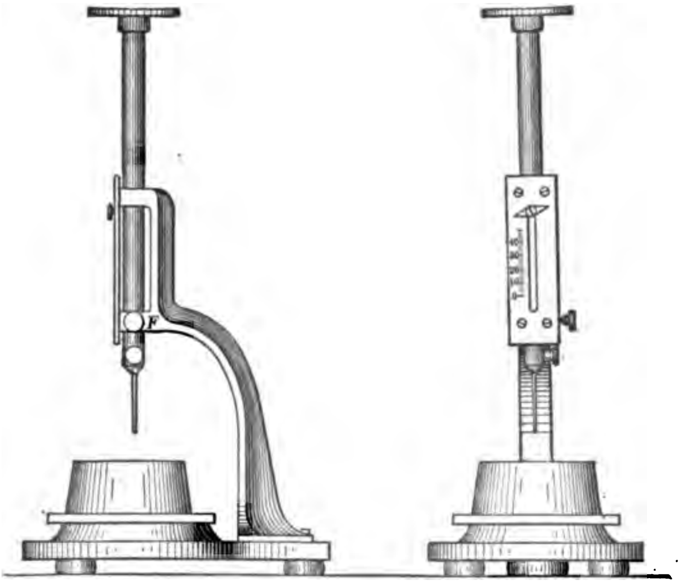


Fig. 1. Vicat's Needle Apparatus.

Quick and Slow Setting. The aluminous natural cements are commonly "quick-setting," though not always so, as those containing a considerable percentage of sulphuric acid may set quite slowly. The magnesian and Portland varieties may be either "quick" or "slow". Specimens of either variety may be had that will set at any rate, from the slowest to the most rapid, and no distinction can be drawn between the various classes in this regard.

Water containing sulphate of lime in solution retards the setting, This fact has been made use of in the adulteration of cement, powdered gypsum being mixed with it to make it slow-setting, greatly to the injury of the material.

The temperature of the water used affects the time required for setting; the higher the temperature, within certain limits, the more rapid the *set*. Many cements which require several hours to set when mixed with water at a temperature of 40° F. will set in a few minutes if the temperature of the water be increased to 80° F. Below a certain inferior limit, ordinarily from 30° to 40° F., the cement will not set, while at a certain upper limit, in many cements between 100° and 140° F., a change is suddenly made from a very rapid to a very slow rate, which then continually decreases as the temperature increases, until practically the cement will not set.

The quick-setting cements usually set so that experimental samples can be handled within 5 to 30 minutes after mixing. The slow-setting cements require from 1 to 8 hours. Freshly ground cements set quicker than older ones.

Soundness denotes the property of not expanding or contracting or cracking or checking in setting. These effects may be due to free lime, free magnesia, or to unknown causes. Testing soundness is, therefore, determining whether the cement contains any *active* impurity. An inert adulteration or impurity affects only its economic value; but an active impurity affects also its strength and durability.

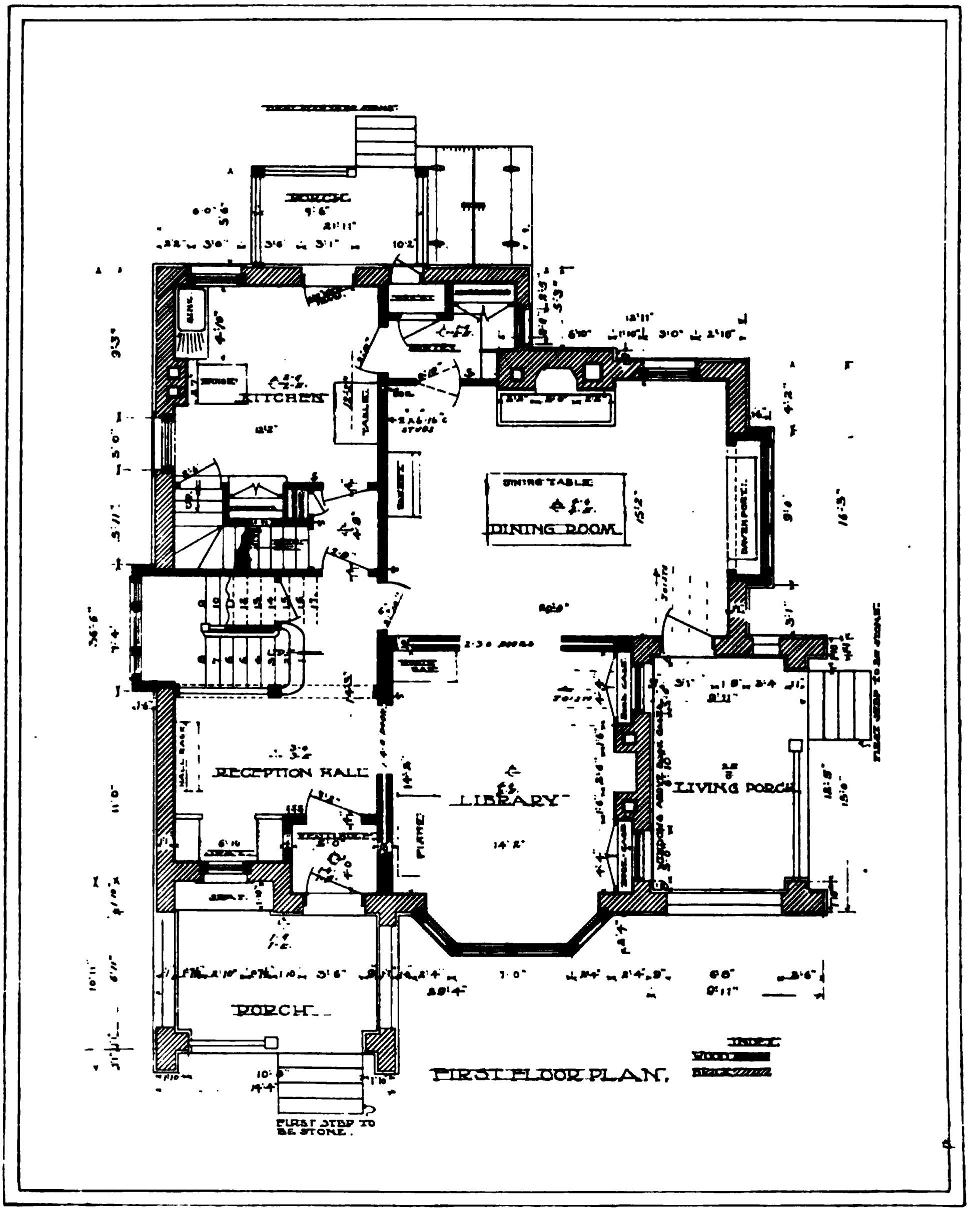
For the purpose of determining the amount of contraction or expansion the "Bauschinger" apparatus, Fig. 2, is used. A mould is used in which the test bars of cement are formed.

Tests of Soundness. The soundness of a cement may be determined by cold tests, so-called, the cement being at ordinary temperature; or by accelerated or hot tests.

To make the cold tests, prepare small cakes or pats of neat cement, 3 or 4 inches in diameter and about one-half inch thick at the center, tapering to a thin edge. Place the samples upon a piece of glass and cover with a damp cloth for a period of 24 hours and then immerse glass and all in water for a period of 28 days if possible, keeping watch from day to day to see if the samples show any cracks or signs of distortion.

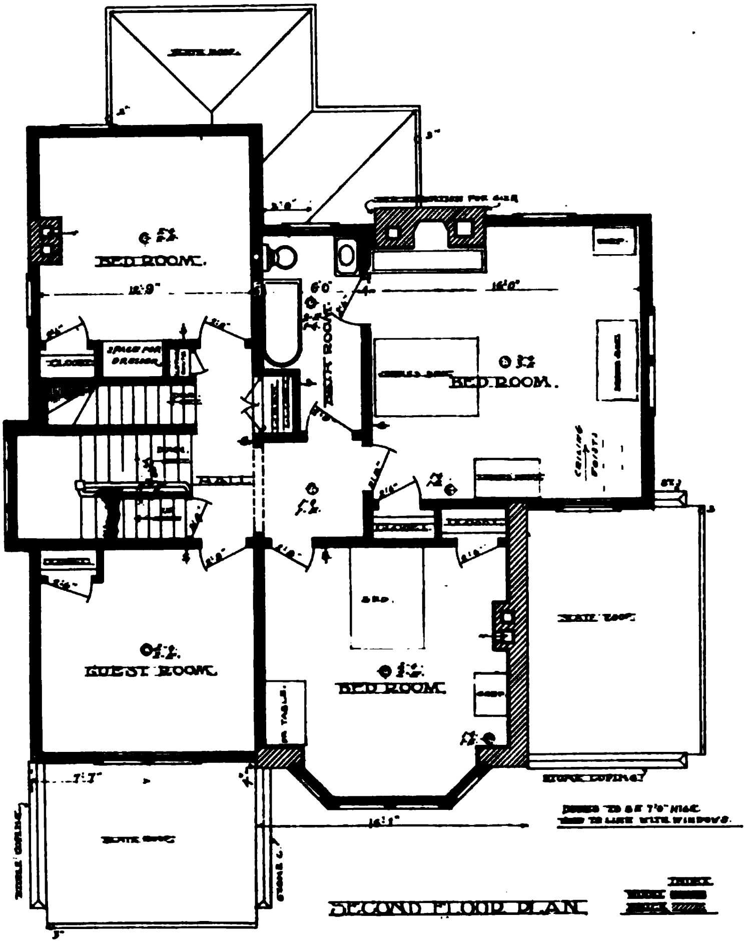
The first indication of inferior quality is the loosening of the pat from the glass, which usually takes place in one or two days. Good cement will remain firmly attached to the glass for two weeks at least.

The ordinary tests, extending over a proper interval, often fail to detect unsoundness, and circumstances may render the ordinary



HOUSE AT ST. PAUL, MINN.

John T. Comes, Architect, Pittsburg, Pa.
Cost of House, $7,500. For Exteriors, See Page 218.



HOUSE AT ST. PAUL, MINN.

John T. Comes, Architect, Pittsburg, Pa.

For Exteriors, See Page 218.

tests impossible from lack of time. Under such circumstances resort must be had to accelerated tests, which may be made in several ways.

Warm-Water Test. Prepare the sample as before, and after allowing it to set, immerse in water maintained at a temperature of from 100° to 115° F. If the specimen remains firmly attached to the glass and shows no cracks, it is probably sound.

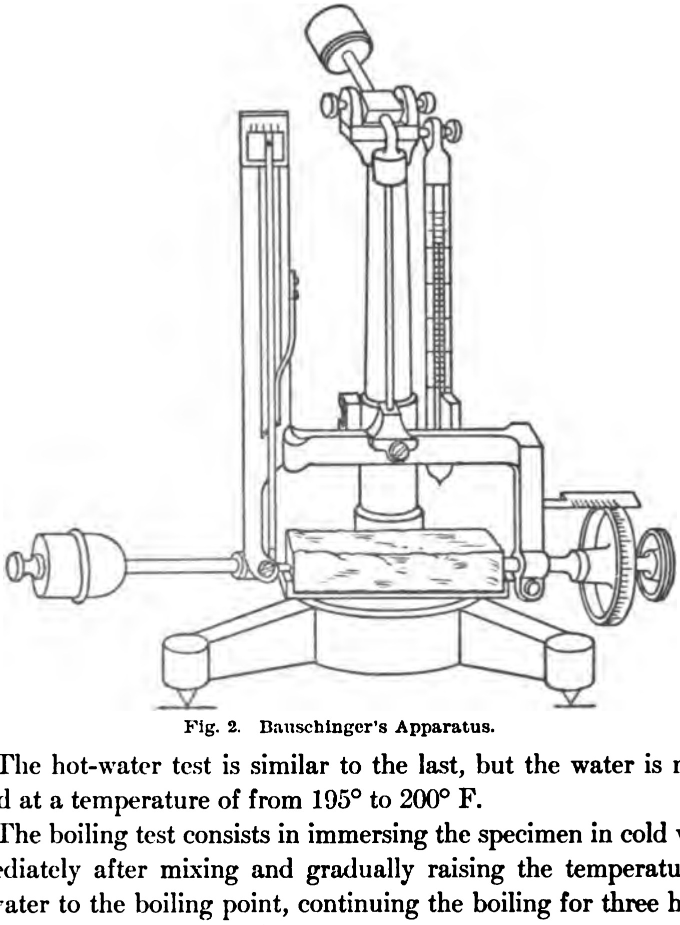


Fig. 2. Bauschinger's Apparatus.

The hot-water test is similar to the last, but the water is maintained at a temperature of from 195° to 200° F.

The boiling test consists in immersing the specimen in cold water immediately after mixing and gradually raising the temperature of the water to the boiling point, continuing the boiling for three hours.

For an emergency test, the specimen may be prepared as before, and after setting may be held under a steam cock of a boiler and live steam discharged upon it.

The results of accelerated tests must not be accepted too literally, but should be interpreted in the light of judgment and experience.

The cracking or contortion of the specimen (sometimes called "blowing"), is due to the hydration and consequent expansion of the lime or magnesia. If the effect is due to lime, the cement can be improved by exposure to the air, thus allowing the free lime to slake. This treatment is called "cooling the cement". The presence of uncombined magnesia is more harmful than that of lime.

Some idea of the quality of a cement may be gained by exposing to the air a small cake of neat cement mortar and observing its color. "A good Portland cement should be uniform bluish gray throughout, yellowish blotches indicate poor cement".

Tests of soundness should not only be carefully conducted, but should extend over considerable time. Occasionally cement is found which seems to meet the usual tests for soundness, strength, etc., and yet after considerable time loses all coherence and falls to pieces.

Strength. The strength is usually determined by submitting a specimen of known cross-section (generally one square inch) to a tensile strain. The reason for adopting a tensile test is that since even the weakest cement cannot be crushed, in ordinary practice, by direct compression, and since cement is not used in places where cross strain is brought to bear upon it, torsion being out of the question, the only valuable results can be derived from tests for tensile strength. In case of a cracking wall the strain is that of tension due to the difference of the direction of the strain caused by the sinking of one part of the wall.

In comparing different brands of cement great care must be exercised to see that the same kind and quality of sand is used in each case, as difference in the sand will cause difference in the results. To obviate this a standard sand is generally used. This consists of crushed quartz of such a degree of fineness that it will all pass a No. 20 sieve (400 meshes to the square inch; wire No. 28 Stubbs' gauge) and be caught on a No. 30 sieve (900 meshes to the square inch; wire No. 31 Stubbs' gauge).

Valuable and probably as reliable comparative tests can be made with the sand which is to be used for making the mortar in the proposed work. Specimens of neat cement are also used for testing, they can be handled sooner and will show less variation than specimens composed of cement and sand.

The cement is prepared for testing by being formed into a stiff paste by the addition of just sufficient water for this purpose. When sand is to be added, the exact proportions should be carefully determined by weight and thoroughly and intimately mixed with the cement in a dry state before the water is added; and, so far as possible, all the water that is necessary to produce the desired consistency should be added at once and thereafter the manipulation with the spatula or trowel should be rapid and thorough. The mortar so obtained is filled into a mould of the form and dimensions shown in Fig. 3. These moulds are usually of iron or brass. Wooden moulds, if well oiled to prevent their absorbing water, answer the purpose well for temporary use, but speedily become unfit for accurate work. In filling the mould care must be exercised to complete the filling before incipient setting begins.

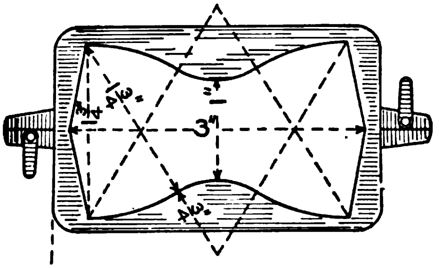


Fig. 3. Briquette Mould.

The moulds while being charged and manipulated, should be laid on glass, slate, or some other non-absorbent material. The specimen, now called the "briquette", should be removed from the mould as soon as it is hard enough to stand it, without breaking. The briquettes are then immersed in water, where they should remain constantly covered until tested. If they are exposed to the air, the water may be carried away by evaporation and leave the mortar a pulverant mass. Also, since the mortar does not ordinarily set as rapidly under water as in the air (owing to the difference in temperature), it is necessary for accurate work to note the time of immersion, and also to break the briquette as soon as it is taken from the water. Cement ordinarily attains a greater strength when allowed to set under water, but attains it more slowly.

Age of Briquette for Testing. It is customary to break part of the briquettes at the end of seven days, and the remainder at the end of twenty-eight days. As it is sometimes impracticable to wait twenty-eight days, tests are often made at the end of one and seven days, respectively. The ultimate strength of the cement is judged by the increase in strength between the two dates. A minimum strength for the two dates is usually specified.

Testing the Briquettes. When taken out of the water the briquettes are subjected to a tensile strain until rupture takes place in a suitably devised machine. There are several machines on the market for this purpose. Fig. 4 represents one which is extensively used.

To make a test, hang the cup *F* on the end of the beam *D*, as shown in the illustration. See that the poise *R* is at the zero mark, and balance the beam by turning the ball *L*. Fill the hopper *B* with fine shot, place the specimen in the clamps *N N*, and adjust the hand wheel *P* so that the graduated beam *D* will rise nearly to the stop *K*.

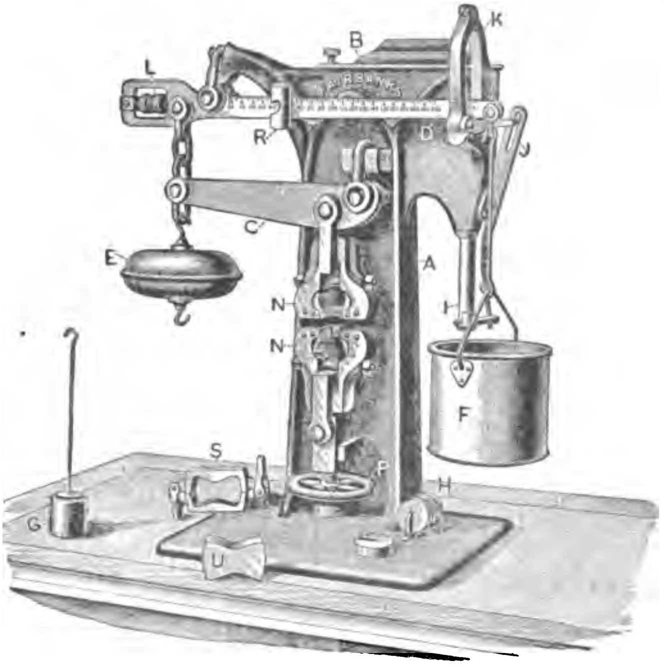


Fig. 4. Cement Testing Machine.

Open the automatic valve *J* so as to allow the shot to run slowly into the cup *F*. Stand back and leave the machine to make the test.

When the specimen breaks, the graduated beam *D* drops and closes the valve *J*, remove the cup with the shot in it, and hang the counterpoise weight *G* in its place.

Hang the cup *F* on the hook under the large ball *E*, and proceed to weigh the shot in the regular way, using the poise *R* on the graduated beam *D*, and the weights *H* on the counterpoise weight *G*. The result will show the number of pounds required to break the specimen.

TABLE 5.

Tensile Strength of Cement Mortar.

Age of mortar when tested.	Average tensile strength in pounds per square inch.			
	Portland.		Rosendale.	
CLEAR CEMENT.	Min.	Max.	Min.	Max.
One hour, or until set, in air, the remainder of the time in water:				
1 *day*	100	140	40	80
One day in air, the remainder of the time in water:				
1 *week*	250	550	60	100
4 *weeks*	350	700	100	150
1 *year*	450	800	300	400
1 PART CEMENT TO 1 PART SAND.				
One day in air, the remainder of the time in water:				
1 *week*	......	......	30	50
4 *weeks*	......	......	50	80
1 *year*	......	......	200	300
1 PART CEMENT TO 3 PARTS SAND.				
One day in air, the remainder of the time in water:				
1 *week*	80	125	......	......
4 *weeks*	100	200	......	......
1 *year*	200	350	......	......

CEMENTS—MEMORANDA.

Cement is shipped in barrels or in cotton or paper bags.

The usual dimensions of a barrel are: length 2 ft. 4 in., middle diameter 1 ft. 4½ in., end diameter 1 ft. 3½ in.

The bags hold 50, 100, or 200 pounds.

A barrel weighs about as follows:

Rosendale, N. Y.	300 lb. net
Rosendale, Western	265 "
Portland	375 "

A barrel of Rosendale cement contains about 3.40 cubic feet and will make from 3.70 to 3.75 cubic feet of stiff paste, or 79 to 83 pounds will make about one cubic foot of paste. A barrel of Rosendale cement (300 lb.) and two barrels of sand (7½ cubic feet) mixed with about half a barrel of water will make about 8 cubic feet of mortar, sufficient for

192 square feet of mortar-joint ½ inch thick.
288 " " " " ⅜ " "
384 " " " " ¼ " "
768 " " " " ⅛ " "

A barrel of Portland cement contains about 3.25 to 3.35 cubic feet—100 pounds will make about one cubic foot of stiff paste.

A barrel of cement measured loosely increases considerably in bulk. The following results were obtained by measuring in quantities of two cubic feet:

1 bbl.	Norton's Rosendale gave	4.37 cu. ft.
"	Anchor Portland gave	3.65 "
"	Sphinx Portland gave	3.71 "
"	Buckeye Portland gave	4.25 "

Preservation of Cements. Cements require to be stored in a dry place protected from the weather; the packages should not be placed directly on the ground, but on boards raised a few inches from it. If necessary to stack it out of doors a platform of planks should first be made and the pile covered with canvas. Portland cement exposed to the atmosphere will absorb moisture until it is practically ruined. The absorption of moisture by the natural cements will cause the development of carbonate of lime, which will interfere with the subsequent hydration.

MISCELLANEOUS CEMENTS.

Slag Cements are those formed by an admixture of slaked lime with ground blast-furnace slag. The slag used has approximately the composition of an hydraulic cement, being composed mainly of silica and alumina, and lacking a proper proportion of lime to render it active as a cement. In preparing the cement the slag upon coming from the furnace is plunged into water and reduced to a spongy form from which it may be readily ground. This is dried and ground to a fine powder. The powdered slag and slaked lime are then mixed in proper proportions and ground together, so as to very thoroughly distribute them through the mixture. It is of the first importance in a slag cement that the slag be very finely ground, and that the ingredients be very uniformly and intimately incorporated.

Both the composition and methods of manufacture of slag cements vary considerably in different places. They usually contain a higher percentage of alumina than Portland cements, and the materials are in a different state of combination, as, being mixed after the burning, the silicates and aluminates of lime formed during the burning of Portland cement cannot exist in slag cement.

The tests for slag cement are that briquettes made of one part of cement and three parts of sand by weight shall stand a tensile strain of 140 pounds per square inch (one day in air and six in water), and must show continually increasing strength after seven days, one month, etc. At least 90 per cent must pass a sieve containing 40,000 meshes to the square inch, and must stand the boiling test.

Pozzuolanas are cements made by a mixture of volcanic ashes with lime, although the name is sometimes applied to mixed cements in general. The use of pozzuolana in Europe dates back to the time of the Romans.

Roman Cement is a natural cement manufactured from the septaria nodules of the London Clay formation; it is quick-setting, but deteriorates with age and exposure to the air.

MORTAR.

Ordinary Mortar is composed of lime and sand mixed into a paste with water. When cement is substituted for the lime, the mixture is called *cement mortar.*

Uses. The use of mortar in masonry is to bind together the bricks or stones, to afford a bed which prevents their inequalities from bearing upon one another and thus to cause an equal distribution of pressure over the bed. It also fills up the spaces between the bricks or stones and renders the wall weather tight. It is also used in concrete as a matrix for broken stones or other bodies to be amalgamated into one solid mass; and for plastering and other purposes.

The quality of mortar depends upon the character of the materials used in its manufacture, their treatment, proportions, and method of mixing.

Proportions. The proportion of cement to sand depends upon the nature of the work and the necessity for the development of strength or imperviousness. The relative quantities of sand and cement should also depend upon the nature of the sand; fine sand requires more cement than coarse. Usual proportions are:

Lime mortar, 1 part of lime to 4 parts of sand.

Natural cement mortar, 1 part cement to 2 or 3 parts of sand.

Portland cement mortar, 1 part cement to 2, 3, or 4 parts of sand, according to the character of the work.

Sand for Mortar. The sand used *must be clean,* that is, free from clay, loam, mud, or organic matter; *sharp,* that is, the grains must be angular and not rounded as those from the beds of rivers and the seashore; *coarse,* that is, it must be large-grained, but not too uniform in size.

The best sand is that in which the grains are of different sizes; the more uneven the sizes the smaller will be the amount of voids, and hence the less cement required.

The *cleanness* of sand may be tested by rubbing a little of the dry sand in the palm of the hand, and after throwing it out noticing the amount of dust left on the hand. The cleanness may also be judged by pressing the sand between the fingers while it is damp; if the sand is clean it will not stick together, but will immediately fall apart when the pressure is removed.

The *sharpness* of sand can be determined approximately by rubbing a few grains in the hand or by crushing it near the ear and noting if a grating sound is produced; but an examination through a small lens is better.

To determine the presence of Salt and Clay. Shake up a small portion of the sand with pure distilled water in a perfectly clean stoppered bottle, and allow the sand to settle; add a few drops of pure nitric acid and then add a few drops of solution of nitrate of silver. A white precipitate indicates a tolerable amount of salt; a faint cloudiness may be disregarded.

The presence of clay may be ascertained by agitating a small quantity of the sand in a glass of clear water and allowing it to stand for a few hours to settle; the sand and clay will separate into two well-defined layers.

Screening. Sand is prepared for use by screening to remove the pebbles and coarser grains. The fineness of the meshes of the screen depends upon the kind of work in which the sand is to be used.

Washing. Sand containing loam or earthy matters is cleansed by washing with water, either in a machine specially designed for the purpose and called a sand-washer, or by agitating with water in tubs or boxes provided with holes to permit the dirty water to flow away.

Water for Mortar. The water employed for mortar should be fresh and clean, free from mud and vegetable matter. Salt water may be used, but with some natural cements it may retard the setting, the chloride and sulphate of magnesia being the principal retarding elements. Less sea-water than fresh will be required to produce a given consistency.

Quantity. The quantity of water to be used in mixing mortar can be determined only by experiment in each case. It depends upon the nature of the cement, upon that of the sand and of the water, and upon the proportions of sand to cement, and upon the purpose for which the mortar is to be used.

Fine sand requires more water than coarse to give the same consistency. Dry sand will take more water than that which is moist, and sand composed of porous material more than that which is hard. As the proportion of sand to cement is increased the proportion of water to cement should also increase, but in a much less ratio.

The amount of water to be used is such that the mortar when thoroughly mixed shall have a plastic consistency suitable for the purpose for which it is to be used.

The addition of water, little by little, or from a hose, should not be allowed.

Cement Mortar. In mixing cement mortar the cement and sand are first thoroughly mixed dry, the water then added, and the whole worked to a uniformly plastic condition.

The quality of the mortar depends largely upon the thoroughness of the mixing, the great object of which is to so thoroughly incorporate the ingredients that no two grains of sand shall lie together without an intervening layer or film of cement. To accomplish this the cement must be uniformly distributed through the sand during the dry mixing.

The mixers usually fail to thoroughly intermix the dry cement and sand, and to lighten the labor of the wet mixing they will give an overdose of water.

Packed cement when measured loose increases in volume to such an extent that a nominal 1 to 3 mortar is easily changed to an actual 1 to 4. The specifications should prescribe the manner in which the materials are to be measured, *i.e.*, packed or loose.

The quantity of sand will also vary according to whether it is measured in a wet or dry condition, packed or loose. On work of sufficient importance to justify some sacrifice of convenience the sand and cement should be proportioned by weight instead of by volume.

In mixing by hand a platform or box should be used; the sand and cement should be spread in layers with a layer of sand at the bottom, then turned and mixed with shovels until a thorough incorporation is effected. The dry mixture should then be spread out, a bowl-like depression formed in the center and all the water required poured into it. The dry material from the outside of the basin should be thrown in until the water is taken up and then worked into a plastic condition, or the dry mixture may be shovelled to one end of the box and the water poured into the other end. The mixture of sand and cement is then drawn down with a hoe, small quantities at a time, and mixed with the water until enough has been added to make a good stiff mortar.

In order to secure proper manipulation of the materials on the part of the workmen it is usual to require that the whole mass shall be turned over a certain number of times with the shovels, both dry and wet.

The mixing wet with the shovels must be performed quickly and energetically. The paste thus made should be vigorously worked with a hoe for several minutes to insure an even mixture. The mortar should then leave the hoe clean when drawn out of it, and very little should stick to the steel.

A large quantity of cement and sand should not be mixed dry and left to stand a considerable time before using, as the moisture in the sand will to some extent act upon the cement, causing a partial setting.

Upon large works mechanical mixers are frequently employed with the advantage of at once lessening the labor of manipulating the material and insuring good work.

Retempering Mortar. Masons very frequently mix mortar in considerable quantities, and if the mass becomes stiffened before being used, by the setting of the cement, add water and work it again to a soft or plastic condition. After this second tempering the cement is much less active than at first, and will remain for a longer time in a workable condition.

This practice is condemned by engineers, and is not usually allowed in good engineering construction. Only sufficient quantity of mortar should be mixed at once as may be used before the cement takes the initial set. Reject all mortar that has set before being placed in the work.

Freezing of Mortar. It does not appear that common *lime mortar* is seriously injured by freezing, provided it remains frozen until it has fully set. The freezing retards, but does not entirely suspend the setting. Alternate freezing and thawing materially damages the strength and adhesion of lime mortar.

Although the strength of the mortar is not decreased by freezing, it is not always permissible to lay masonry during freezing weather; for example, if, in a thin wall, the mortar freezes before setting and afterwards thaws on one side only, the wall may settle injuriously.

Mortar composed of one part *Portland cement* and three parts sand is entirely uninjured by freezing and thawing.

Mortar made of *cements* of the *Rosendale type*, in any proportion, is entirely ruined by freezing and thawing.

Mortar made of overclayed cement (which condition is indicated by its quicker setting), of either the Portland or Rosendale type, will not withstand the action of frost as well as one containing less clay, for since the clay absorbs an excess of water, it gives an increased effect to the action of frost.

In making cement mortar during freezing weather it is customary to add salt or brine to the water with which it is mixed. The ordinary rule is: Dissolve 1 pound of salt in 18 gallons of water when the temperature is at 32° F., and add 1 ounce of salt for each degree of lower temperature.

The use of salt, and more especially of sea-water, in mortar is objectionable in exposed walls, since the accompanying salts usually produce efflorescence.

The practice of adding hot water to lime mortar during freezing weather is undesirable. When the very best results are sought the brick or stone should be warmed—enough to thaw off any ice upon the surface is sufficient—before being laid. They may be warmed either by laying them on a furnace, or by suspending them over a slow fire, or by wetting with hot water.

TABLE 6.

Amount of Cement and Sand Required for One Cubic Yard of Mortar.

Composition of mortar by volumes.		Cement.* Number of barrels.		Sand, cubic yards.
Cement.	Sand.	Portland or Ulster County Rosendale.	Western Rosendale.	
1	0	7.14	6.43	0.00
1	1	4.16	3.74	0.58
1	2	2.85	2.57	0.80
1	3	2.00	1.80	0.90
1	4	1.70	1.53	0.95
1	5	1.25	1.13	0.97
1	6	1.18	1.06	0.98
		Cement. Number of Pounds.†		
1	0	2675	2140	0.00
1	1	1440	1150	0.67
1	2	900	720	0.84
1	3	675	540	0.94
1	4	525	420	0.98
1	5	425	340	0.99
1	6	355	285	1.00

* Packed cement and loose sand.

† Loose cement and loose sand.

CONCRETE.

Concrete is a species of artificial stone composed of (1) the matrix, which may be either lime or cement mortar, usually the latter, and (2) the aggregate, which may be any hard material, as gravel, shingle, broken stone, shells, brick, slag, etc.

The aggregate should be of different sizes, so that the smaller shall fit into the voids between the larger. This requires less mortar and with good aggregate gives a stronger concrete. Broken stone is the most common aggregate.

Gravel and shingle should be screened to remove the larger-sized pebbles, dirt, and vegetable matter, and should be washed if they contain silt or loam. The broken stone if mixed with dust or dirt must be washed before use.

Strength of Concrete. The resistance of concrete to crushing ranges from about 600 to 1400 pounds per sq. in. It depends upon the kind and amount of cement and upon the kind, size, and strength of the aggregate. The transverse strength ranges between 50 and 400 pounds.

Weight of Concrete. A cubic yard weighs from 2,500 to 3,000 pounds according to its composition.

PROPORTIONS OF MATERIALS FOR CONCRETE.

To manufacture one cubic yard of concrete the following quantities of materials are required:

BROKEN-STONE-AND-GRAVEL CONCRETE.

Broken-stone 50% of its bulk voids....	1	cubic yard
Gravel to fill voids in the stone........	½	" "
Sand to fill voids in the gravel.........	¼	" "
Cement to fill voids in the sand........	⅛	" "

BROKEN-STONE CONCRETE.

Broken stone 50% of its bulk voids....	1	cubic yard
Sand to fill voids in the stone.........	½	" "
Cement to fill voids in the sand........	¼	" "

GRAVEL CONCRETE.

Gravel ⅓ of its bulk voids.............	1	cubic yard
Sand to fill voids in the gravel........	⅓	" "
Cement to fill voids in the sand........	⅙	" "

Concrete composed of 1 part Rosendale cement, 2 parts of sand, and 5 parts of broken stone requires:

Broken stone	0.92 cubic yard
Sand	0.37 " "
Cement	1.43 barrels

The usual proportions of the materials in concrete are:

ROSENDALE CEMENT CONCRETE.

Rosendale cement	1 part
Sand	2 parts
Broken stone	3 to 4 "

PORTLAND CEMENT CONCRETE.

Portland cement	1 part
Sand	2 to 3 parts
Broken stone or gravel	3 to 7 "

To make 100 cubic feet of concrete of the proportions 1 to 6 will require 5 bbl. cement (original package) and 4.4 yards of stone and sand.

Mixing Concrete. The concrete may be mixed by hand or machinery. In hand-mixing the cement and sand are mixed dry. About half the sand to be used in a batch of concrete is spread evenly over the mortar-board, then the dry cement is spread evenly over the sand, and then the remainder of the sand is spread on top of the cement. The sand and cement are then mixed with a hoe or by turning and re-turning with a shovel. It is very important that the sand and cement be thoroughly mixed. A basin is then formed by drawing the mixed sand and cement to the outer edges of the board, and the whole amount of water required is poured into it. The sand and cement are then thrown back upon the water and thoroughly mixed with the hoe or shovel into a stiff mortar and then levelled off. The broken stone or gravel should be sprinkled with sufficient water to remove all dust and thoroughly wet the entire surface. The amount of water required for this purpose will vary considerably with the absorbent power of the stone and the temperature of the air. The wet stone is then spread evenly over the top of the mortar and the whole mass thoroughly mixed by turning over with the shovel. Two, three, or more turnings may be necessary. It should be turned until every stone is coated with mortar, and the entire mass presents the uniform color of the cement, and the mortar and stones are uniformly distributed. When the aggregate consists of broken brick or other porous material it should be thoroughly wetted and time allowed for absorption previous to use; otherwise it will take away part of the water necessary to effect the setting of the cement.

When the concrete is ready for use it should be quite coherent and capable of standing at a steep slope without the water running from it.

The rules and the practice governing the mixing of concrete vary as widely as the proportion of the ingredients. It may be stated in general that if too much time is not consumed in mixing the wet materials a good result can be obtained by any of the many ways practised, if only the mixing is thorough. With four men the time required for mixing one cubic yard is about ten minutes.

Whatever the method adopted for mixing the concrete, it is advisable for the inspector to be constantly present during the operation, as the temptation to economize on the cement and to add an excess of water to lighten the labor of mixing is very great.

Laying Concrete. Concrete is usually deposited in layers, the thickness of which is generally stated in the specifications for the particular work (the thickness varies between 6 and 12 in.). The concrete must be carefully deposited in place. A very common practice is to tip it from a height of several feet into the trench. This process is objected to by the best authorities on the ground that the heavy and light portions separate while falling, and that the concrete is, therefore, not uniform throughout its mass.

The best method is to wheel the concrete in barrows, immediately after mixing, to the place where it is to be laid, gently tipping or sliding it into position and at once ramming it.

The ramming should be done before the cement begins to set, and should be continued until the water begins to ooze out upon the upper surface. When this occurs it indicates a sufficient degree of compactness. A gelatinous or quicksand condition of the mass indicates that too much water was used in mixing. Too severe or long-continued pounding injures the strength by forcing the stones to the bottom of the layers and by distributing the incipient "set" of the cement.

The rammers need not be very heavy, 10 to 15 lb. will be sufficient. Square ones should measure from 6 to 8 in. on a side and round ones from 8 to 12 in. in diameter.

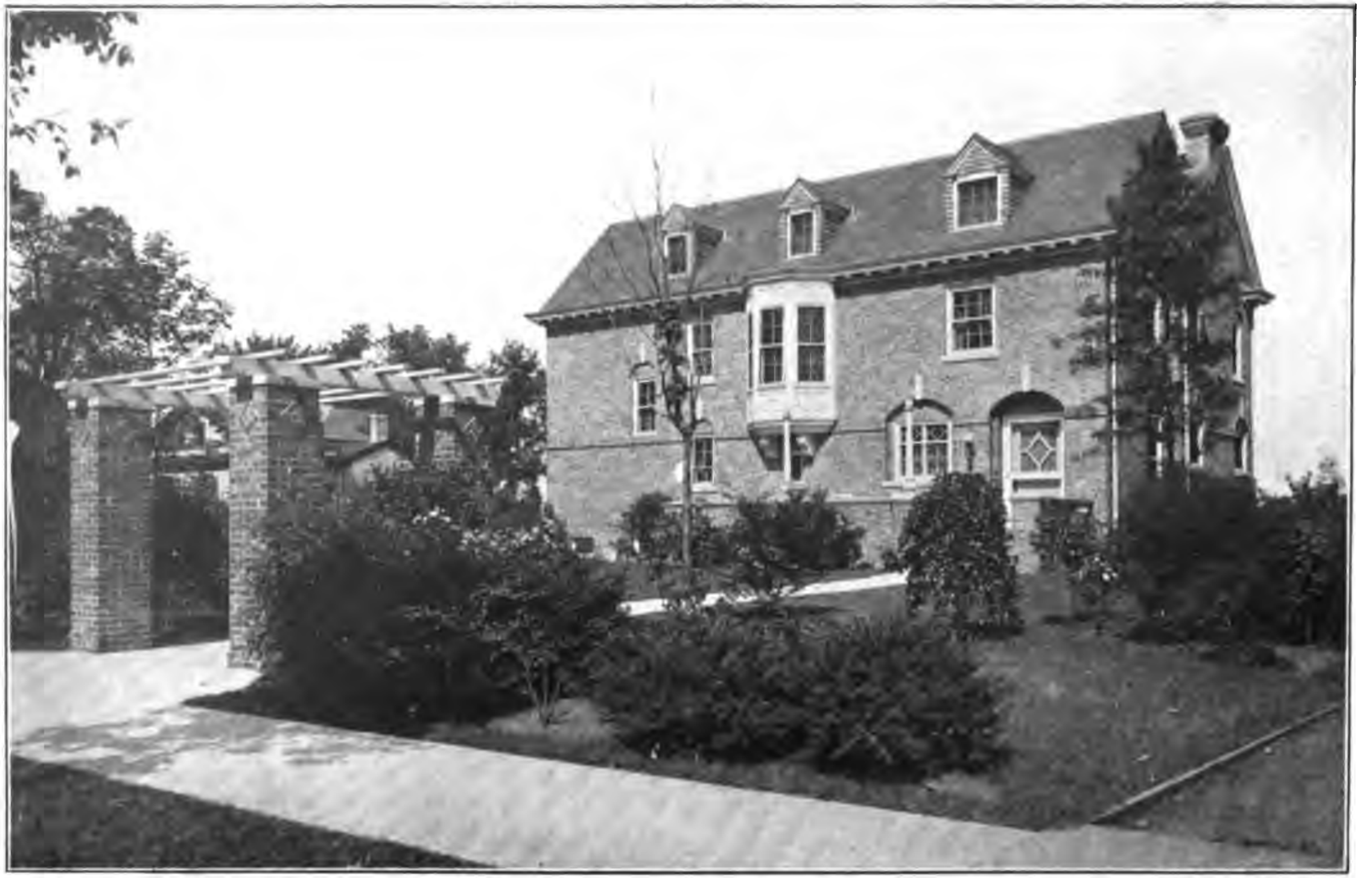
After each layer has been rammed it should be allowed sufficient time to "set", without walking on it or in other ways disturbing it. If successive layers are to be laid the surface of the one already set should be swept clean, wetted, and made rough by means of a pick for the reception of the next layer.

Great care should be observed in joining the work of one day to that of the next. The last layer should be thoroughly compacted and left with a slight excess of mortar. It should be finished with a level surface, and when partially set should be scratched with a pointed stick and covered with planks, canvas, or straw. In the morning, immediately before the application of the next layer, the surface should be swept clean, moistened with water, and painted with a wash of neat cement mixed with water to the consistency of cream. This should be put on in excess and brushed thoroughly back and forth upon the surface so as to insure a close contact therewith.

Depositing Concrete Under Water. In laying concrete under water an essential requisite is that the materials shall not fall from any height through the water, but be deposited in the allotted place in a compact mass; otherwise the cement will be separated from the other ingredients and the strength of the work be seriously impaired. If the concrete is allowed to fall through the water its ingredients will be deposited in a series, the heaviest—the stone—at the bottom, and the lightest—the cement—at the top. A fall of even one foot causes an appreciable separation.

A common method of depositing concrete under water is to place it in a V-shaped box of wood or plate iron, which is lowered to the bottom with a crane. The box is so constructed that on reaching the bottom a latch operated by a rope reaching to the surface can be drawn out, thus permitting one of the sloping sides to swing open and allow the concrete to fall out. The box is then raised and refilled.

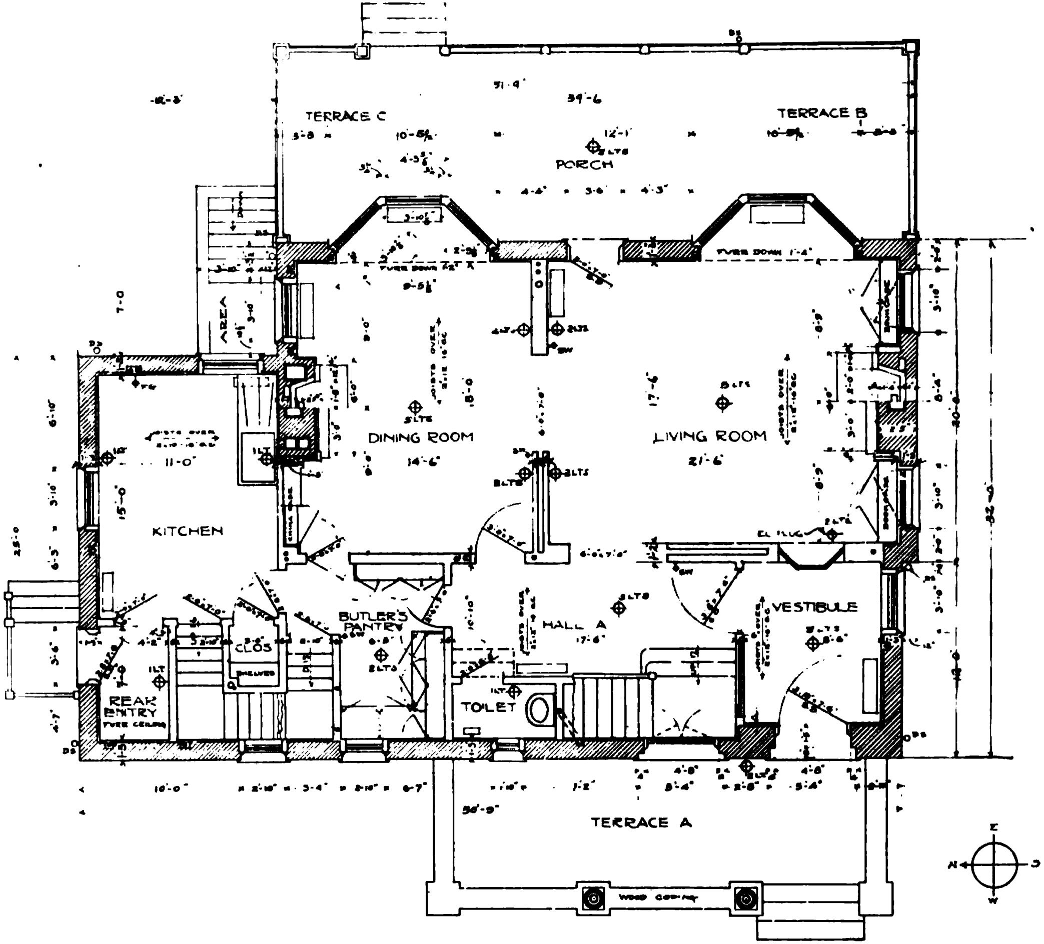
A long box or tube, called a *tremie,* is also used. It consists of a tube open at top and bottom built in detachable sections, so that the length may be adjusted to the depth of water. The tube



RESIDENCE AT KENOSHA, WIS.

Pond & Pond, Architects, Chicago, Ill.

Wall of Purple Continental Brick; Roof of Shingles. First-Floor Plan Shown on Opposite Page.



FIRST FLOOR PLAN.

RESIDENCE AT KENOSHA, WIS.

Pond & Pond, Architects, Chicago, Ill.
Built in 1905. Cost, $17,000. Exterior Shown on Opposite Page. For Interiors, See Vol. IV, Page 218, and Vol. II, Page 170.

is suspended from a crane or movable frame running on a track, by which it is moved about as the work progresses. The upper end is hopper-shaped, and is kept above the water; the lower end rests on the bottom. The tremie is filled in the beginning by placing the lower end in a box with a movable bottom, filling the tube, lowering all to the bottom, and then detaching the bottom of the box. The tube is kept full of concrete by more being thrown in at the top as the mass issues from the bottom.

Concrete is also successfully deposited under water by enclosing it in paper bags and lowering or sliding them down a chute into place. The bags get wet and the pressure of the concrete soon bursts them, thus allowing the concrete to unite into a solid mass. Concrete is also sometimes deposited under water by enclosing it in open-cloth bags, the cement oozing through the meshes sufficiently to unite the whole into a single mass.

Concrete should not be deposited in running water unless protected by one or other of the above-described methods; otherwise the cement will be washed out.

Concrete deposited under water should not be rammed, but if necessary may be levelled with a rake or other suitable tool immediately after being deposited.

When concrete is deposited in water a pulpy, gelatinous fluid is washed from the cement and rises to the surface. This causes the water to assume a milky hue. The French engineers apply the term *laitance* to this substance. It is more abundant in salt water than in fresh. The theory of its formation is that the immersed concrete gives up to the water, free caustic lime, which precipitates magnesia in a light and spongy form. This precipitate sets very slowly, and sometimes scarcely at all, and its interposition between the layers of concrete forms strata of separation. The proportion of laitance is greatly diminished by using large immersion-boxes, or a tremie, or paper or cloth bags.

Asphaltic Concrete is composed of asphaltic mortar and broken stone in the proportion of 5 parts of stone to 3 parts of mortar. The stone is heated to a temperature of about 250° F. and added to the hot mortar. The mixing is usually performed in a mechanical mixer.

The material is laid hot and rammed until the surface is smooth. Care is required that the materials are properly heated, that the place where it is to be laid is absolutely dry and that the ramming is done before it chills or becomes set. The rammers should be heated in a portable fire.

CLAY PUDDLE.

Clay puddle is a mass of clay and sand worked into a plastic condition with water. It is used for filling coffer-dams, for making embankments and reservoirs water-tight, and for protecting masonry against the penetration of water from behind.

Quality of Clay. The clays best suited for puddle are opaque, and not crystallized, should exhibit a dull earthy fracture, exhale when breathed upon a peculiar faint odor termed "argillaceous," should be unctuous to the touch, free from gritty matter, and form a plastic paste with water.

The important properties of clay for making good puddle are its tenacity or cohesion and its power of retaining water. The tenacity of a clay may be tested by working up a small quantity with water into a thoroughly plastic condition, and forming it by hand into a roll about 1 to 1½ inches in diameter by 10 or 12 inches in length. If such a roll is sufficiently cohesive not to break on being suspended by one end while wet the tenacity of the material is ample.

To test its power of retaining water one to two cubic yards should be worked with water to a compact homogeneous plastic condition, and then a hollow should be formed in the center of the mass capable of holding four or five gallons of water. After filling the hollow with water it should be covered over to prevent evaporation and left for about 24 hours, when its capability of holding water will be indicated by the presence or absence of water in the hollow.

The clay should be freed from large stones and vegetable matter, and just sufficient sand and water added to make a homogeneous mass. If there is too little sand the puddle will crack by shrinkage in drying, and if too much it will be permeable.

Puddling. The operation of puddling consists in chopping the clay in layers of about 3 inches thick with spades, aided by the addition of sufficient water to reduce it to a pasty condition. After each chop and before withdrawing the spade it should be given a lunging motion so as to permit the water to pass through.

The spade should pass through the upper layer into the lower layer so as to cause the layers to bond together.

The test for thorough puddling is that the spade will pass through the layer with ease, which it will not do if there are any dry hard lumps.

Sometimes in place of spades, harrows are used, each layer of clay being thoroughly harrowed aided by water and then rolled with a grooved roller to compact it.

The finished puddle should not be exposed to the drying action of the air, but should be covered with a layer of dry clay or sand.

FOUNDATIONS.

The foundation is the most critical part of a masonry structure. The failures of masonry work due to faulty workmanship or to an insufficient thickness of the walls are rare in comparison with those due to defective foundations. When it is necessary, as so frequently it is at the present day, to erect gigantic edifices—as high buildings or long-span bridges—on weak and treacherous soils, the highest constructive skill is required to supplement the weakness of the natural foundation by such artificial preparations as will enable it to sustain the load with safety.

Natural Foundations. The soils comprised under this head may be divided into two classes. (1) Those whose stability is not affected by water, and which are firm enough to support the structure, such as rock, compact gravels, and hard clay, and (2) soils which are firm enough to support the weight of the structure, but whose stability is affected by water, such as loose gravels, sand, clay and loam.

Foundations on Rock. To prepare a rock foundation, all that is generally necessary is to cut away the loose and decayed portions and to dress the surface so exposed to a plane as nearly perpendicular to the direction of the pressure as practicable; or, if the rock forms an inclined plane, to cut a series of plane surfaces, like those of steps, for the walls to rest upon. If there are any fissures in the rock they should be filled with concrete.

Foundations on Gravel, Etc. In dealing with soils of this kind usually nothing more is required than to cover them with a layer of concrete of width and depth sufficient to distribute the weight properly.

Foundations on Sand. Sand is almost incompressible so long as it is not allowed to spread out laterally, but as it has no cohesion, and acts like a fluid when exposed to running water, it must be treated with great caution.

Foundations on Clay. Clay is much affected by the action of water, and hence the ground should be well drained before the work is begun, and the trenches so arranged that water does not remain in them. In general, the less a soil of this kind is exposed to the action of the air, and the sooner it is protected from exposure, the better for the work. The top of the footings must be carried below the frost line to prevent heaving, and for the same reason the outside face of the wall should be built with a slight batter and perfectly smooth. The frost line attains a depth of six feet in some of the northern states.

The bearing power of clay and loamy soils may be greatly increased: (1) By increasing the depth. (2) By drainage. This may be accomplished by a covering of gravel or sand, the thickness depending upon the plasticity of the soil, and by surrounding the foundation walls with a tile drain as in Fig. 5. If springs are encountered the water may be excluded by sheet pilings, puddling or plugging the spring with concrete. (3) By consolidating the soil. This may be done by driving short piles close together, or by driving piles, then withdrawing them and filling the space immediately with damp sand well rammed. If the soil is very loose and wet, sand will not be effective, and concrete will be found more satisfactory.

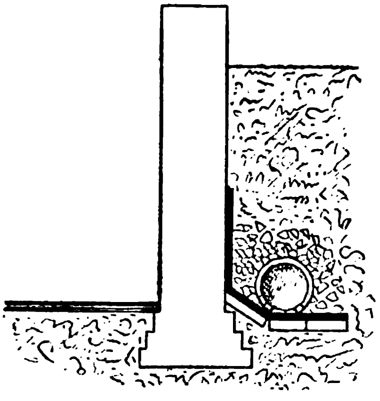


Fig. 5. Drainage of Foundation Walls.

Artificial Foundations. When the ground in its natural state is too soft to bear the weight of the proposed structure, recourse must be had to artificial means of support, and, in doing this, whatever mode of construction is adopted, the principle must always be that of extending the bearing surface as much as possible.

Foundations on Mud, silt, marshy or compressible soils are generally formed in one of three ways: (1) By driving piles in which the footings are supported. (2) By spreading the footings either by layers of timber, steel beams, or concrete, or a combination of either. (3) By sinking caissons of iron or steel, excavating the soil from the interior, and filling with concrete.

Foundations in Water are formed in several ways: (1) Wholly of piles. (2) Solid foundations laid upon the surface of the ground by means of cribs, caissons, or piles, and grillage. (3) Solid foundations laid *below* the surface, the ground being made dry by cofferdams or caissons. (4) Where the site is perfectly firm, and there is no danger of the work being undermined by scour, foundations are started on the surface, the inequalities being first removed by blasting or dredging. The simplest foundation of this class is called "Random" work or *Pierre perdue.* It is formed by throwing large masses of stone upon the site until the mass reaches the surface of the water, above which the work can be carried on in the usual manner. Large rectangular blocks of stone or concrete are also used, the bottom being first simply leveled and the blocks carefully lowered into place.

PILE FOUNDATIONS.

Timber Piles are generally round, the diameter at the butt varying from 9 to 18 inches. They should be straight-grained and as free from knots as possible. The variety of timber is usually selected according to the character of the soil. Where the piles will be always under water and where the soil is soft, spruce and hemlocks are used. For firm soils the hard pines, fir, elm and beech are preferable. Where the piles will be alternately wet and dry, white or black oak and yellow pine are used. Piles exposed to tide water are generally driven with the bark on. It is customary to fix an iron hoop to the heads of piles to prevent their splitting, and also to have them shod with either cast- or wrought-iron shoes.

Timber piles when partly above and partly under water, decay rapidly at the water line owing to the alternations of dryness and moisture. In tidal waters they are destroyed by the marine worm called the "teredo navalis." To preserve timber in such situations several processes are in use. The one most extensively employed is known as "creosoting," which consists of injecting creosote or dead oil of coal tar into the pores of the timber.

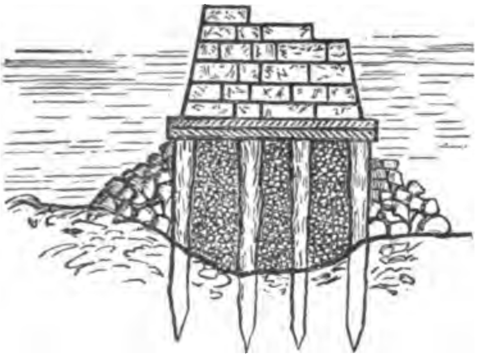


Fig. 6. Timber Pile Foundation.

The frame of timbers placed on the top of the piles is called the grillage. The piles are sawed off square below low water, a timber called a cap is placed on the ends of the piles and fastened with drift bolts, and transverse timbers called strips are placed on the caps and drift-bolted to them. As many courses as necessary may be added, each at right angles to the one below it, the top courses being either laid close together to form a floor or else covered with heavy plank to receive the masonry.

In some cases the grillage is omitted, a layer of concrete being used instead, with the heads of the piles embedded therein, as shown in Fig. 7. The name grillage is also applied to a combination of steel beams and concrete.

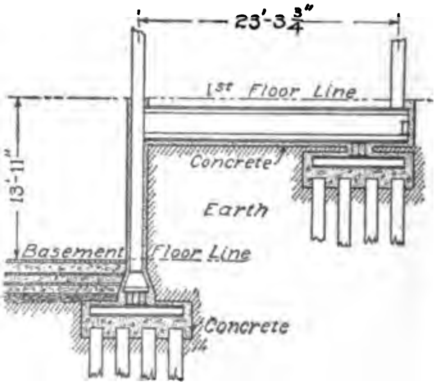


Fig. 7. Timber Piles, Concrete Capping, and I-Beam Grillage.

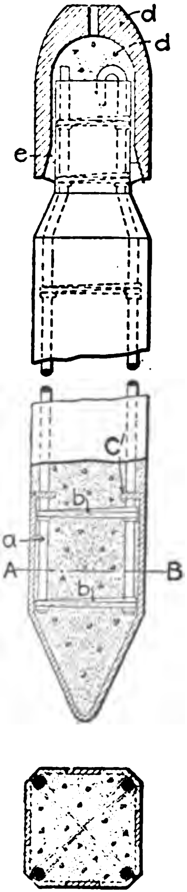
Iron and Steel Piles. Cast iron, wrought iron, and steel are employed for ordinary bearing piles, sheet piles, and for cylinders. Iron cylinders are usually sunk either by dredging the soil from the inside or by the pneumatic process.

Cast-iron piles are used as substitutes for wooden ones. Lugs or flanges are usually cast on the sides of the piles, to which bracing may be attached for securing them in position. A wooden block is laid on top of the pile to receive the blows of the hammer, and after being driven a cap with a socket in its lower side is placed upon the pile to receive the load. Solid rolled-steel piles are driven in the same manner as timber piles, either with a hammer, machine or water-jet.

Screw Piles are piles which are screwed into the stratum in which they are to stand. They are ordinary piles of timber or iron (the latter usually hollow), to the bottom of which a screw disk, consisting of a single turn of the spiral, similar to the bottom turn of an auger, is fastened by bolts or pins. Instead of driving these piles into the ground they are forced in by turning with levers or machinery suitable for the purpose. The screw disks vary in diameter from 1 to 6 feet. The water jet is sometimes employed by applying it to the under, upper, or both sides of the disk for the purpose of reducing the resistance.

Concrete Piles. Two methods of forming these piles are in use. (1) The piles are made in moulds and carried to the place of use and driven in the same manner as timber piles. (2) Holes are made in the ground and filled with concrete.

Moulded Concrete Piles. Fig. 8 shows the moulded pile. This pile is made in moulds and contains four vertical rods *a* at the corners, the rods are stayed by loops or hooks *b* of large wire sprung into place across the sides of the pile and held transversely by horizontal strips of thin metal. The feet of the piles are either wedge shaped or pyramidal and are protected by cast-iron points with side plates which turn in at *c* to lock with the concrete. The upper ends of the piles are shouldered in to give clearance for the driving cap *d*. This is a cast steel hood which fits loosely around the neck of the pile, and is filled with dry sand or a bag of sawdust *d'* retained by a clay ring and hemp jacket *e* at the bottom of the cap.



Section A-B.

Fig. 8. Moulded Concrete Pile.

The sand absorbs the impact of the hammer so as to permit the piles to be driven safely, and it raises the hood sufficiently above the top of the pile to permit the reinforcement rods to extend beyond the concrete for connection with the superstructure.

Concrete Piles Formed in Place. Fig. 9 shows this type of pile. The hole is made by driving with an ordinary pile-driving apparatus, a sheet steel tube tapering from 20 inches at the top to 6 inches at the bottom, the tube is driven by means of a collapsible core which is withdrawn. When the desired depth is reached, the tube is then filled with concrete. Fig. 10 shows another method of forming this type of pile. A sheet steel shell is formed by telescopic sections, each section is 8 ft. long and has at its upper end projections which engage with projections on the lower end of the next section. To the bottom section is attached a cast iron point with a $\frac{3}{4}$-in. jet hole or nozzle, to which is fitted a $2\frac{1}{2}$-in. pipe, this pipe is held in place by spreaders and remains in place in the finished pile, to which it adds lateral strength. The shell is sunk by water jet and filled with concrete.

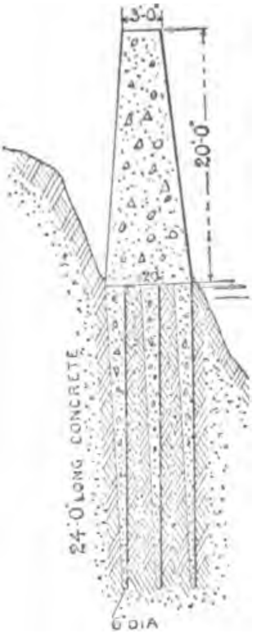


Fig. 9. Concrete Pile.

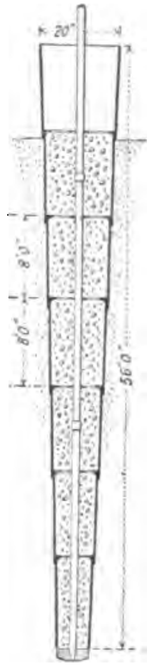


Fig. 10. Concrete Pile.

PILE-DRIVING.

Timber piles are driven either point or butt end down; the latter is considered the better method. When piles are directed to be sharpened the points should have a length of from one and a half times to twice the diameter.

To prevent the head of the pile from being broomed or split by the blows of the driving-ram it is bound with a wrought-iron hoop, 2 to 3 inches wide and ½ to 1 inch thick. Instead of the wrought-iron band a cast-iron cap is sometimes used. It consists of a block with a tapering recess above and below, the chamfered head of the pile fitting into the one below, and a cushion piece of hard wood upon which the hammer falls fitting into the one above.

When brooming occurs the broomed part should be cut off, because a broomed head cushions the blow and dissipates it without any useful effect. Piles that split or broom excessively or are otherwise injured during the driving must be drawn out.

Bouncing of the hammer occurs when the pile refuses to drive further, or it may be caused by the hammer being too light, or its striking velocity being too great, or both. The remedy for bouncing is to diminish the fall.

Excessive hammering on piles which refuse to move should be avoided, as they are liable to be crippled, split, or broken below the ground. Such injury will pass unnoticed and may be the cause of future failure.

As a general rule, a heavy hammer with a low fall drives more satisfactorily than a light one with a high fall. More blows can be made in the same time with a low fall, and this gives less time for the soil to compact itself around the piles between the blows. At times a pile may resist the hammer after sinking some distance, but start again after a short rest; or it may refuse a heavy hammer and start under a light one. It may drive slowly at first, and more rapidly afterwards, from causes difficult to discover.

The driving of a pile sometimes causes adjacent ones previously driven to spring upwards several feet. The driving of piles in soft ground or mud will generally cause adjacent ones previously driven to lean outwards unless means of prevention be taken.

A pile may rest upon rock and yet be very weak, for if driven through very soft soil all the pressure is borne by the sharp point, and the pile becomes merely a column in a worse condition than a pillar with one rounded end. In such soils the piles need very little sharpening; indeed, they had better be driven butt end down without any point. Solid metal piles are usually of uniform diameter and are driven with either blunt or sharpened points.

Piles are driven by machines called pile drivers. A pile driver consists essentially of two upright guides or leads, often of great height, erected upon a platform, or on a barge when used in water. These guides serve to hold the pile vertical while being driven, and also hold and guide the hammer used in driving. This is a block of iron called a ram, monkey, or hammer, weighing anywhere from 800 to 4,000 pounds, usually about 2,000 to 3,000 pounds. The accessories are a hoisting engine for raising the hammer and the devices for allowing it to drop freely on the heads of the piles.

The steam hammer is also employed for driving piles, and has certain advantages over the ordinary form, the chief of which lies in the great rapidity with which the blows follow one another, allowing no time for the disturbed earth, sand, etc., to recompact itself around the sides and under the foot of the pile. It is less liable than other methods to split and broom the piles, so that these may be of softer and cheaper wood, and the piles are not so liable to "dodge" or get out of line.

When piles have to be driven below the end of the leaders of the pile driver a follower is used. This is made from a pile of suitable length placed on top of the pile to be driven. To prevent its bouncing off caps of cast iron are used, one end being bolted to the follower and the other end fitting over the head of the pile.

Piles are also driven by the "water jet." This process consists of an iron pipe fastened by staples to the side of the pile, its lower end placed near the point of the pile and its upper end connected by a hose to a force pump. The pile can be sunk through almost any material, except hardpan and rock, by forcing water through the pipe. It seems to make very little difference, either in the rapidity of sinking or in the accuracy with which the pile preserves its position, whether the nozzle is exactly under the middle of the pile or not.

The efficiency of the jet depends upon the increased fluidity given the material into which the piles are sunk, the actual displacement of material being small. Hence the efficiency of the jet is greatest in clear sand, mud, or soft clay. In gravel or in sand containing a large percentage of gravel, or in hard clay the jet is almost useless. For these reasons the engine, pump, hose, and nozzle should be arranged to deliver large quantities of water with a moderate force rather than smaller quantities with high initial velocity. In gravel, or in sand containing gravel, some benefit might result from a velocity sufficient to displace the pebbles and drive them from the vicinity of the pile. The error most frequently made in the application of the water jet is in using pumps with insufficient capacity.

The approximate volume of water required per minute, per inch of average diameter of pile, for penetrations under 40 feet is 16 gallons; for greater depths the increase in the volume of water is approximately at the rate of 4 gallons per inch of diameter of pile per minute, for each additional 10 feet of penetration.

The number and size of pipes required for various depths are about as follows:

TABLE 7.

Depth of penetration, feet.	Diameter of pipe, inches.	Number of pipes.	Diameter of nozzle, inches.
20	2	1	1
30	2½	1	1¼
40	2½	2	1⅛
50	2½	2	1
60	2½	2	⅞

When the descent of the pile becomes slow, or it sticks or "brings up" in some tenacious material, it can usually be started by striking a few blows with the pile-driving hammer, or by raising the pile about 6 inches and allowing it to drop suddenly, with the jet in operation. By repeating the operation as rapidly as possible the obstruction will generally be overcome.

It is an advantage to use an ordinary pile-driving machine for sinking piles with the water jet. The hammer being allowed to rest upon the head of the pile aids in accelerating the descent, and light blows can be struck as often as may appear necessary. The efficiency of the jet can also be greatly increased by bringing the weight of the pontoon upon which the machinery is placed to bear upon the pile by means of a block and tackle.

Splicing Piles. It frequently happens in driving piles in swampy places, for false works, etc., that a single pile is not long enough, in which case two are spliced together. A common method of doing this is as follows. After the first pile is driven its head is cut off square, a hole 2 inches in diameter and 12 inches deep is bored in its head, and an oak treenail or dowel-pin 23 inches long is driven into the hole; another pile similarly squared and bored is placed upon the lower pile, and the driving continued. Spliced in this way the pile is deficient in lateral stiffness, and the upper section is liable to bounce off while driving. It is better to reinforce the splice by flattening the sides of the piles and nailing on with, say, 8-inch spikes four or more pieces 2 or 3 inches thick, 4 or 5 inches wide, and 4 to 6 feet long.

CONCRETE WITH STEEL BEAMS.

The foundation is prepared by first laying a bed of concrete to a depth of from 4 to 12 inches and then placing upon it a row of I-beams at right angles to the face of the wall. In the case of heavy piers, the beams may be crossed in two directions. Their distance apart, from center to center, may vary from 9 to 24 inches, according

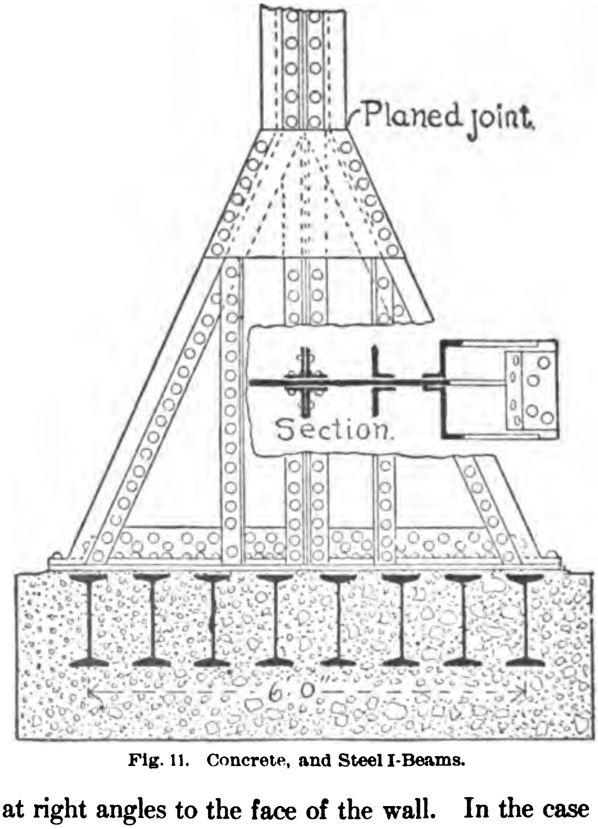


Fig. 11. Concrete, and Steel I-Beams.

to circumstances, *i.e.*, length of their projection beyond the masonry, thickness of concrete, estimated pressure per square foot, etc. They should be placed far enough apart to permit the introduction of the concrete filling and its proper tamping.

Hollow Cylinders of cast iron or plate steel, commonly called *caissons*, are frequently used with advantage. The cylinders are made in short lengths with internal flanges and are bolted together as each preceding length is lowered. They are sunk by excavating the natural soil from the interior. When the stratum on which they are to rest has been reached they are filled with concrete.

Cofferdams. There are many circumstances under which it becomes necessary to expose the bottom and have it dry before commencing operations. This is done by enclosing the site of the foundation with a water-tight wall. The great difficulties in the construction of a cofferdam in deep water are, first, to keep it water-tight, and, second, to support the sides against the pressure of the water outside. Fig. 12 shows the simplest form; it consists of two rows of piles driven closely and filled with clay puddle. In shallow water and on land sheet piling is sometimes sufficient.

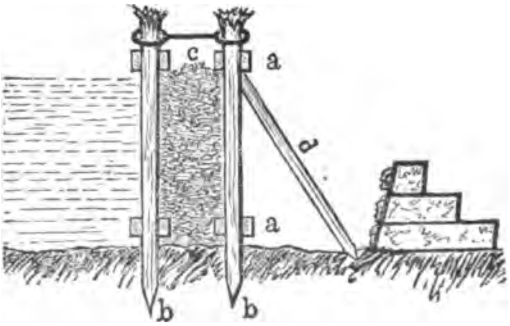


Fig. 12. Cofferdam,

Sheet Piles are flat piles, usually of plank, either tongued and grooved or grooved only, into which a strip of tongue is driven; or they may be of squared timber, in which case they are called "close piles," or of sheet iron. The timber ones are of any breadth that can be procured, and from 2 to 10 inches thick, and are shaped at the lower end to an edge wholly from one side; this point being placed next to the last pile driven tends to crowd them together and make tighter joints (the angle formed at the point should be 30°). In stony ground they are shod with iron.

When a space is to be enclosed with sheet piling two rows of guide piles are first driven at regular intervals of from 6 to 10 feet, and to opposite sides of these near the top are notched or bolted a pair of parallel string pieces or "wales," from 5 to 10 inches square, so fastened to the guide piles as to leave between the wales equal to the thickness of the sheet piles.

If the sheeting is to stand more than 8 or 10 feet above the ground, a second pair of wales is required near the level of the ground. The sheet piles are driven between the wales, working from each end towards the middle of the space between a pair of guide piles, so that the last or central pile acts as a wedge to tighten the whole.

Sheet piles are driven either by mauls wielded by men or by a pile-driving machine. Ordinary planks are also used for sheet piling, being driven with a lap; such piling is designated as "single lap," "double lap," and "triple lap." The latter is also known as the "Wakefield" triple-lap sheet piling, shown in Fig. 13.

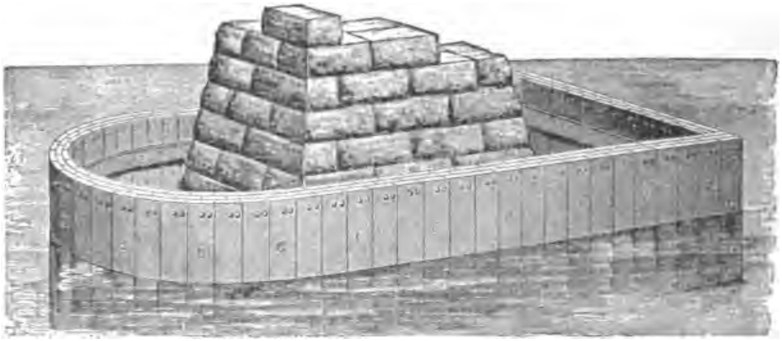


Fig. 13. Sheet Piling.

Cribs are boxes constructed of round or square timber, divided by partitions of solid timber into square or rectangular cells. The cells are floored with planks, placed a little above the lower edge so as not to prevent the crib from settling slightly into the soil, and thus coming to a full bearing on the bottom. After it has been sunk the cells are filled with sand and stone. On uneven rock bottom it may be necessary to scribe the bottom of the crib to fit the rock. In some cases rip-rap is deposited outside around the crib to prevent undermining by the current. A crib with only an outside row of cells for sinking it is sometimes used, with an interior chamber in which concrete is laid under water and the masonry started thereon. Cribs are sometimes sunk into place and then piles are driven in the cells, which are afterward filled with concrete or broken stone. The masonry may then rest on the piles only, which in turn will be protected by the crib. If the bottom is liable to scour, sheet piles or rip-rap may be placed outside around the base of the crib. Cribs with only an outer row of cells for puddling may be used as a cofferdam, the joints between the outer timbers being well calked, and care taken by means of outside pile planks to prevent water from entering beneath it.

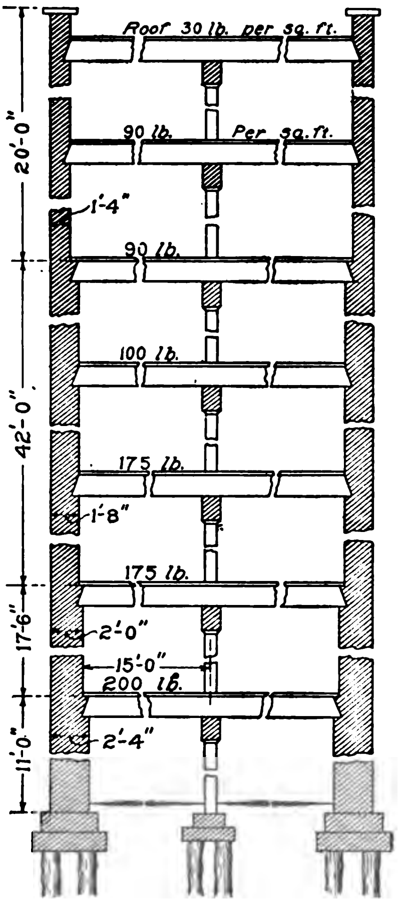


Fig. 14. Building on Pile Foundation.

Caissons are of two forms, the "erect" or "open" and the "inverted." The former is a strong water-tight timber box, which is floated over the site of the work, and being kept in place by guide piles, is loaded with stone until it rests firmly on the ground. In some cases the stone is merely thrown in, the regular masonry commencing with the top of the caisson; which is sunk a little below the level of low water, so that the whole of the timber is always covered, and the caisson remains as part of the structure. In others, the masonry is built on the bottom of the caisson, and when the work reaches the level of the water the sides of the caisson are removed. The site is prepared to receive the caisson by dredging and depositing a layer of concrete, or by driving piles, or a combination of both.

The *inverted caisson* is also a strong water-tight box, open at the bottom and closed at the top, upon which the structure is built, and which sinks as the masonry is added. This type of caisson is usually aided in sinking by excavation made in the interior. The processes employed to aid the sinking of the inverted caissons are called the "vacuum" and the "plenum."

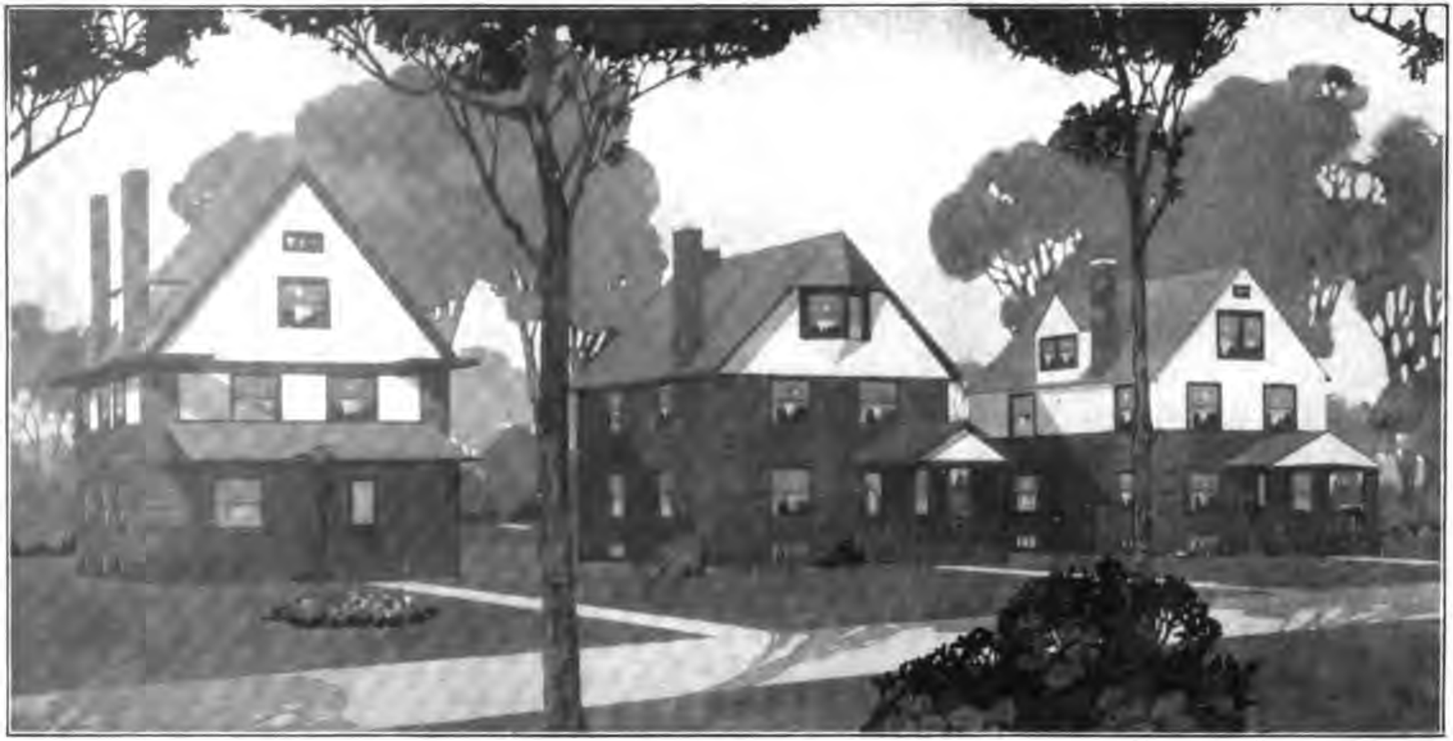
The vacuum process consists in exhausting the air from the interior of the caisson, and using the pressure of the atmosphere upon the top of it to force it down. Exhausting the air allows the water to flow past the lower edge into the interior, thus loosening the soil.

The plenum or compressed-air process consists in pumping air into the chamber of the caisson, which by its pressure excludes the water. An air lock or entrance provided with suitable doors is arranged in the top of the caisson, by which workmen can enter to loosen up the soil and otherwise aid in the sinking of the caisson vertically by removing and loosening the material at the sides. If the loosened material is of a suitable character it is removed with a sand pump; if not, hoisting apparatus is provided and, being loaded into buckets by the workmen, it is hoisted out through the air lock.

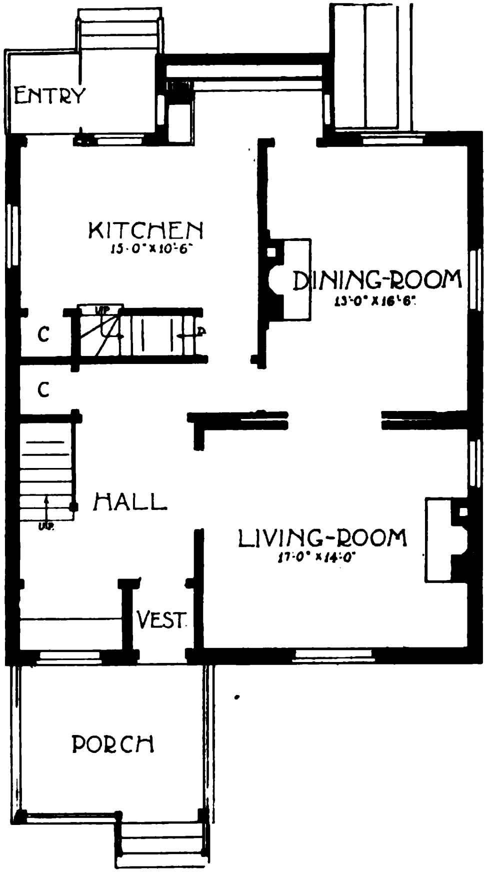
Freezing Process. This process is employed in sinking foundation pits through quicksand and soils saturated with water. The Poetsch-Sooysmith process is to sink a series of pipes 10 inches in diameter through the earth to the rock; these are sunk in a circle around the proposed shaft. Inside of the 10-inch pipes 8-inch pipes closed at the bottom are placed, and inside of these are placed smaller pipes open at the bottom. Each set of the small pipes is connected in a series. A freezing mixture is then allowed to flow downwards through one set of the smaller pipes and return upwards through the other. The freezing mixture flows from a tank placed at a sufficient height to cause the liquid to flow with the desired velocity through the pipes. The effect of this process is to freeze the earth into a solid wall.

DESIGNING THE FOUNDATION.

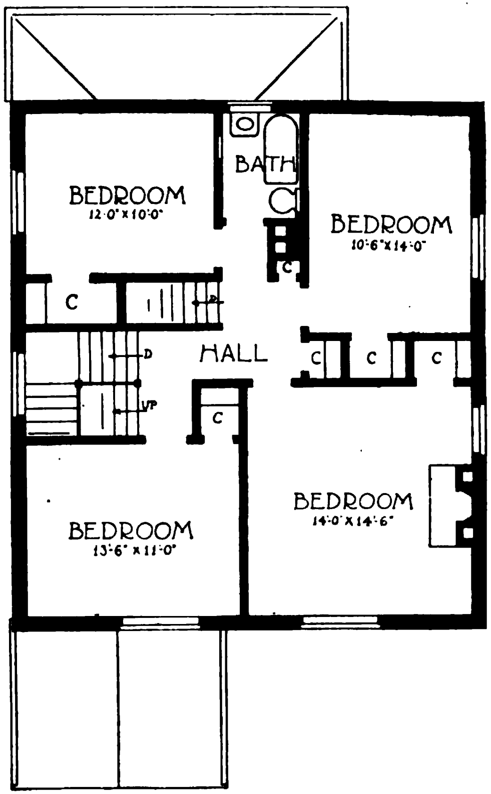
Load to be Supported. The first step is to ascertain the load to be supported by the foundation. This load consists of three parts: (1) The structure itself, (2) the movable loads on the floors and the snow on the roof, and (3) the part of the load that may be transferred from one part of the foundation to the other by the force of the wind.



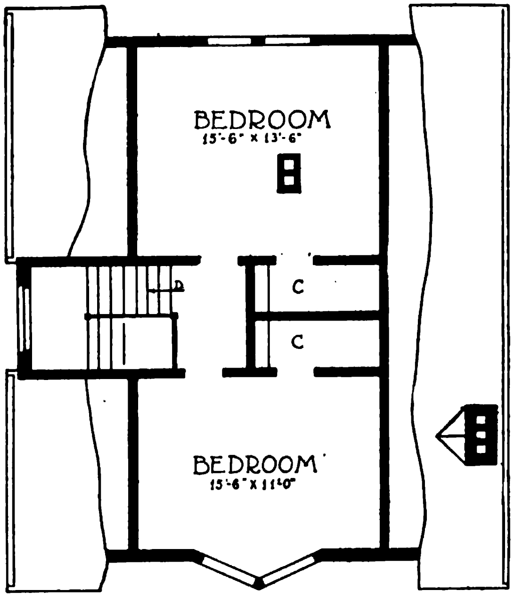
THREE HOUSES FOR INDIANA STEEL COMPANY, GARY, IND.
Dean & Dean, Architects, Chicago, Ill.
Paving Brick Veneer and Plaster; Tile Roofs.



First-Story Plan.



Second-Story Plan.



Third-Story Plan.

THREE HOUSES FOR INDIANA STEEL COMPANY, GARY, IND.

Dean & Dean, Architects, Chicago, Ill.
Exteriors Shown on Opposite Page.

The weight of the building is easily ascertained by calculating the cubical contents of all the various materials in the structure. The following data will be useful in determining the weight of the structure.

TABLE 8.

Weight of Masonry.

Kind of Masonry.	Weight in lb. per cu. ft.
Brickwork, pressed brick, thin joints	145
" ordinary quality	125
" soft brick, thick joints	100
Concrete	130 to 160
Granite or limestone, well dressed throughout	165
" rubble, well dressed with mortar	155
" roughly dressed with mortar	150
" well dressed, dry	140
" roughly dressed, dry	125
Mortar dried	100
Sandstone, $\frac{1}{10}$ less than granite	....

Ordinary lathing and plastering weighs about 10 lb. per sq. ft.

Floors weigh approximately:

Dwellings	10 lb. per sq. ft.
Public buildings	25 lb. per sq. ft.
Warehouses	40 to 50 lb. per sq. ft.

Roofs vary according to the kind of covering, span, etc.

Shingle roof weighs about 10 lb. per sq. ft.

Slate or corrugated iron	25 lb per sq. ft.

The movable load on the floor depends upon the nature of the building. It is usually taken as follows:

Dwellings	10 lb. per sq. ft.
Office buildings	20 lb. per sq. ft.
Churches, theatres, etc.	100 lb. per sq. ft.
Warehouses, factories	100 to 400 lb. per sq. ft.

The weight of snow on the roof will vary from 0 in a warm climate to 20 lb. in the latitude of Michigan. The pressure of the wind is usually taken at 50 lb. per sq. ft. on a flat surface perpendicular to the wind, and on a cylinder at about 40 lbs. per sq. ft. over the vertical projection of the cylinder.

Bearing Power of Soils. The best method of determining the load which a particular soil will bear is by direct experiment and examination—particularly of its compactness and the amount of water it contains. The values given in the following table may be considered safe for good examples of the kind of soil quoted.

TABLE 9.

Bearing Power of Soils.

Kind of soil.	Bearing power, tons per square foot.	
	Min.	Max.
Rock, hard	25	30
" soft	5	10
Clay on thick bed, always dry	4	6
" " " " moderately dry	2	4
" soft	1	2
Gravel and coarse sand, well cemented	8	10
Sand, compact and well cemented	4	6
" clean, dry	2	4
Quicksand, alluvial soil, etc	0.5	1

Area Required. Having determined the pressure which may safely be brought upon the soil, and having ascertained the weight of each part of the structure, the area required for the foundation is easily determined by dividing the latter by the former. Then, having found the area required, the base of the structure must be extended by footings of concrete, masonry, timber, etc., so as to (1) cover that area and (2) distribute the pressure uniformly over it.

Bearing Power of Piles. Several formulas have been proposed and are in use for determining the safe working loads on piles. The three in general use are:

Sander's formula.

$$\text{Safe load in lb.} = \frac{\text{Weight of hammer in lb.} \times \text{fall in inches.}}{8 \times \text{Sinking at last blow.}}$$

Trautwine's formula.

Extreme load in tons of 2240 lbs. =

$$\frac{\text{Cube root of fall in feet} \times \text{Weight of hammer in lb.} \times 0.023}{\text{Last sinking in inches.}}$$

Safe load to be taken at one-half of extreme load when driven in firm soils, and at one-fourth when driven in river mud or marshy soil.

Engineering News formula is the latest and is considered reliable.

$$\text{Safe load in lb.} = \frac{2\,wh}{S + 1}$$

in which w = weight of hammer in lb., h = its fall in feet, S = average sinking under last blows in inches.

Example of Pile Foundation. As an example of the method of determining the number of piles required to support a given building, the side walls of a warehouse are selected, a vertical section of which is shown in Fig. 15. The walls are of brick, and the weight is taken at 110 pounds per cubic foot of masonry.

The piles are to be driven in two rows, spaced two feet between centers, and it has been found that a test pile 20 feet long and 10 inches at the top will sink one inch under a 1,200-pound hammer falling 20 feet after the pile has been entirely driven into the soil. What distance should the piles be placed center to center lengthwise of the wall?

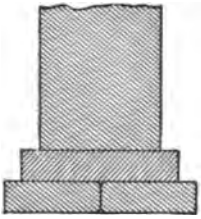


Fig. 15. Stone Footing.

By calculation it is found that the wall contains 157⅓ cubic feet of masonry per running foot, and hence weighs 17,306 pounds. The load from the floors which comes upon the wall is:

From the 1st floor	1500 lb.
" " 2nd "	1380 "
" " 3rd "	1380 "
" " 4th "	790 "
" " 5th "	720 "
" " 6th "	720 "
" " roof	240 "
Total	6730 lb.

Hence the total weight of the wall and its load per running foot is 24,036 pounds.

The load which one pile will support is, by Sander's rule

$$\frac{1200 \times 240}{8 \times 1} = 36{,}000 \text{ pounds.}$$

By Trautwine's rule, using a factor of safety of 2.5, the safe load would be

$$\frac{\sqrt[3]{20} \times 1200 \times 0.023}{2.5 \times (1 + 1)} = 15 \text{ tons or } 33{,}600 \text{ lb.}$$

Then one pair of piles would support 72,000 or 67,200 pounds according to which rule we take.

Dividing these numbers by the weight of one foot of the wall and its load, it is found, that, by Sander's rule, one pair of piles will support 3 feet of the wall, and, by Trautwine's rule, 2.8 feet of wall; hence the pile should be placed 2 feet 9 inches or 3 feet between centers.

DESIGNING THE FOOTING.

The term *footing* is usually understood as meaning the bottom course or courses of concrete, timber, iron, or masonry employed to increase the area of the base of the walls, piers, etc. Whatever the character of the soil, footings should extend beyond the fall of the wall (1) to add to the stability of the structure and lessen the danger of its being thrown out of plumb, and (2) to distribute the weight of the structure over a larger area and thus decrease the settlement due to compression of the ground.

Offsets of Footings. The area of the foundation having been determined and its center having been located with reference to the axis of the load, the next step is to determine how much narrower each footing course may be than the one next below it.

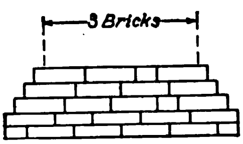


Fig. 16. Brick Footing.

The proper offset for each course will depend upon the vertical pressure, the transverse strength of the material, and the thickness of the course. Each footing may be regarded as a beam fixed at one end and uniformly loaded. The part of the footing course that projects beyond the one above it, is a cantilever beam uniformly loaded. From the formulas for such beams, the safe projection may be calculated.

Stone Footings. Table 10 gives the safe offset for masonry footing courses, in terms of the thickness of the course, computed for a factor of safety of 10.

TABLE 10.

Kind of stone.	R* in lb. per sq. in.	Offset for a pressure in tons per sq. ft. on the bottom of the course of 0.5	1.0	2.0
Bluestone flag	2,700	3.6	2.6	1.8
Granite	1,800	2.9	2.1	1.5
Limestone	1,500	2.7	1.9	1.3
Sandstone	1,200	2.6	1.8	1.3
Slate	5,400	5.0	3.6	2.5
Best hard brick	1,500	2.7	1.9	1.3
Hard brick	800	1.9	1.4	0.8
Concrete 1 Portland 2 Sand 3 Pebbles } 10 days old	150	0.8	0.6	0.4
Concrete 1 Rosendale 2 Sand 3 Pebbles } 10 days old	80	0.6	0.4	0.3

* Modulus of rupture.

To illustrate the method of using the preceding table, assume that it is desired to determine the offset for a limestone footing course when the pressure on the bed of the foundation is 1 ton per square foot, using a factor of safety of 10. On the table, opposite limestone, in next to the last column, we find the quantity 1.9. This shows that under the conditions stated, the offset may be 1.9 times the thickness of the course.

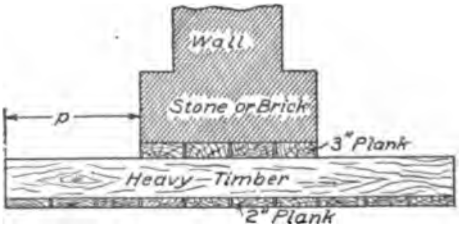


Fig. 17. Timber Footing.

Timber Footing. The rise of the transverse timbers (Fig. 17) may be calculated by the following formula:

$$\text{Breadth in inches} = \frac{2 \times w \times p^2 \times s}{D^2 \times A}$$

in which w = the bearing power in lb. per sq. ft.;

p = the projection of the beam in feet;

s = the distance between centers of beams in feet;

D = the assumed depth of the beam in inches;

A = the constant for strength which is taken for Georgia pine at 90, oak 65, Norway pine 60, white pine or spruce 55.

Steel I-Beam Footings. The dimensions of the I-beams, Fig. 18, can be calculated by the usual formulas, by means of the strain to which the part of the beam in cantilever is submitted. The safe load per running foot is given by the expression

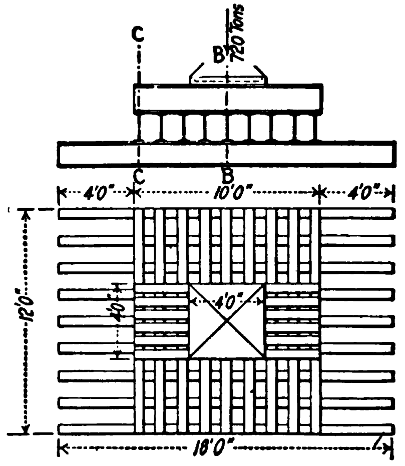


Fig. 18. Steel I-Beam Footing.

$$W = \frac{S\,I}{6\,m} \times \frac{1}{z^2}$$

in which W = load in pounds per running foot;

S = 16,000 lb. per sq. in., extreme fibre strain of beams;

m = distance from center of gravity of sections to top or bottom;

I = moment of inertia of section, neutral axis through center of gravity;

z = span in feet.

A ready method of determining the size of the beams is by computing the required *coefficient of strength,* and finding in the tables furnished by the manufacturers of steel beams the size of the beam which has a coefficient equal to, or next above, the value obtained by the formula. C, the coefficient, is found by the following expression:

$$C = 4 \times w \times p^2 \times s$$

in which w = bearing power in pounds per sq. ft.;
p = the projection of the beam in feet;
s = the spacing of the beam, center to center, in feet.

Table 11 gives the safe projection of steel I-beams spaced on I foot centers and for loads varying from 1 to 5 tons per sq. ft.

TABLE 11.

Safe Projection of I-Beam Footings.

Depth of beam, in.	Weight per foot, lb.	b (Tons per Square Foot).										
		1	1¼	1½	2	2¼	2½	3	3½	4	4½	5
20	80	14.0	12.5	11.5	10.0	9.0	9.0	8.0	7.5	7.0	6.5	6.0
20	64	12.5	11.0	10.0	8.5	8.0	8.0	7.0	6.5	6.0	6.0	5.5
15	75	11.5	10.5	9.5	8.0	7.5	7.5	6.5	6.0	6.0	5.5	5.0
15	60	10.5	9.5	8.5	7.5	7.0	6.5	6.0	5.5	5.5	5.0	5.0
15	50	9.5	8.5	8.0	7.0	6.5	6.0	5.5	5.0	5.0	4.5	4.5
15	41	8.5	8.0	7.0	6.0	6.0	5.5	5.0	4.5	4.5	4.0	4.0
12	40	8.0	7.0	6.5	5.5	5.5	5.0	4.5	4.0	4.0	3.5	3.5
12	32	7.0	6.5	5.5	5.0	4.5	4.5	4.0	4.0	3.5	3.5	3.0
10	33	6.5	6.0	5.5	4.5	4.5	4.0	4.0	3.5	3.5	3.0	3.0
10	25.5	5.5	5.0	4.5	4.0	4.0	3.5	3.5	3.0	3.0	2.5	2.5
9	27	5.5	5.0	4.5	4.0	4.0	3.5	3.5	3.0	3.0	2.5	2.5
9	21	5.0	4.5	4.0	3.5	3.5	3.0	3.0	2.5	2.5	2.5	2.0
8	22	5.0	4.5	4.0	3.5	3.5	3.0	3.0	2.5	2.5	2.5	2.0
8	18	4.5	4.0	3.5	3.0	3.0	3.0	2.5	2.5	2.0	2.0	2.0
7	20	4.5	4.0	3.5	3.0	3.0	3.0	2.5	2.5	2.0	2.0	2.0
7	15.5	4.0	3.5	3.0	2.5	2.5	2.5	2.0	2.0	2.0	2.0	1.5
6	16	3.5	3.0	3.0	2.5	2.5	2.0	2.0	2.0	1.5	1.5	1.5
6	13	3.0	3.0	2.5	2.5	2.0	2.0	2.0	1.5	1.5	1.5	1.5
5	13	3.0	2.5	2.5	2.0	2.0	2.0	1.5	1.5	1.5	1.5	1.5
5	10	2.5	2.5	2.0	2.0	1.5	1.5	1.5	1.5	1.5	....	....
4	10	2.5	2.0	2.0	1.5	1.5	1.5	1.5	....	....	....	....
4	7.5	2.0	2.0	1.5	1.5	1.5	1.5	....	....	....	....	....

SAFE WORKING LOADS FOR MASONRY.

Brick Masonry in Walls or Piers.

	Tons per sq. ft.
Hard brick in lime mortar	5 to 7
Hard brick in Rosendale cement 1 to 3	8 to 10

	Tons per sq. ft.
Pressed brick in lime mortar	6 to 8
Pressed brick in Rosendale cement	9 to 12
Pressed brick in Portland cement	12 to 15

Piers exceeding in height six times their least dimension should be increased 4 inches in size for each additional 6 feet.

According to the New York building laws, brickwork in good lime mortar 8 tons per sq. ft., 11½ tons when good lime and cement mortar is used, and 15 tons when good cement mortar is used.

According to the Boston building laws:

Best hard-burned brick (height less than six times least dimension) with

	Lb. per sq. ft.
Mortar, 1 cement, 2 sand	30,000
Mortar, 1 cement, 1 lime, 3 sand	24,000
Mortar, lime	16,000

Best hard-turned brick (height six to twelve times least dimension) with

Mortar, 1 cement, 2 sand	26,000
Mortar, 1 cement, 1 lime, 3 sand	20,000
Mortar, lime	14,000

For light hard-burned brick use ⅔ the above amounts.

STONE MASONRY.

	Tons per sq. ft.
Rubble walls, irregular stone	3
Rubble walls, coursed, soft stone	2½
Rubble walls, coursed, hard stone	5 to 16

Dimension stone in cement:

Sandstone and limestone	10 to 20
Granite	20 to 40

Dressed stone, with ⅜-inch dressed joints, in cement:

Granite	60
Marble or limestone	40
Sandstone	30

Height of columns not to exceed eight times least diameter.

MORTARS.

In ½ inch joints 3 months old:	Tons per sq. ft.
Portland cement 1 to 4	40

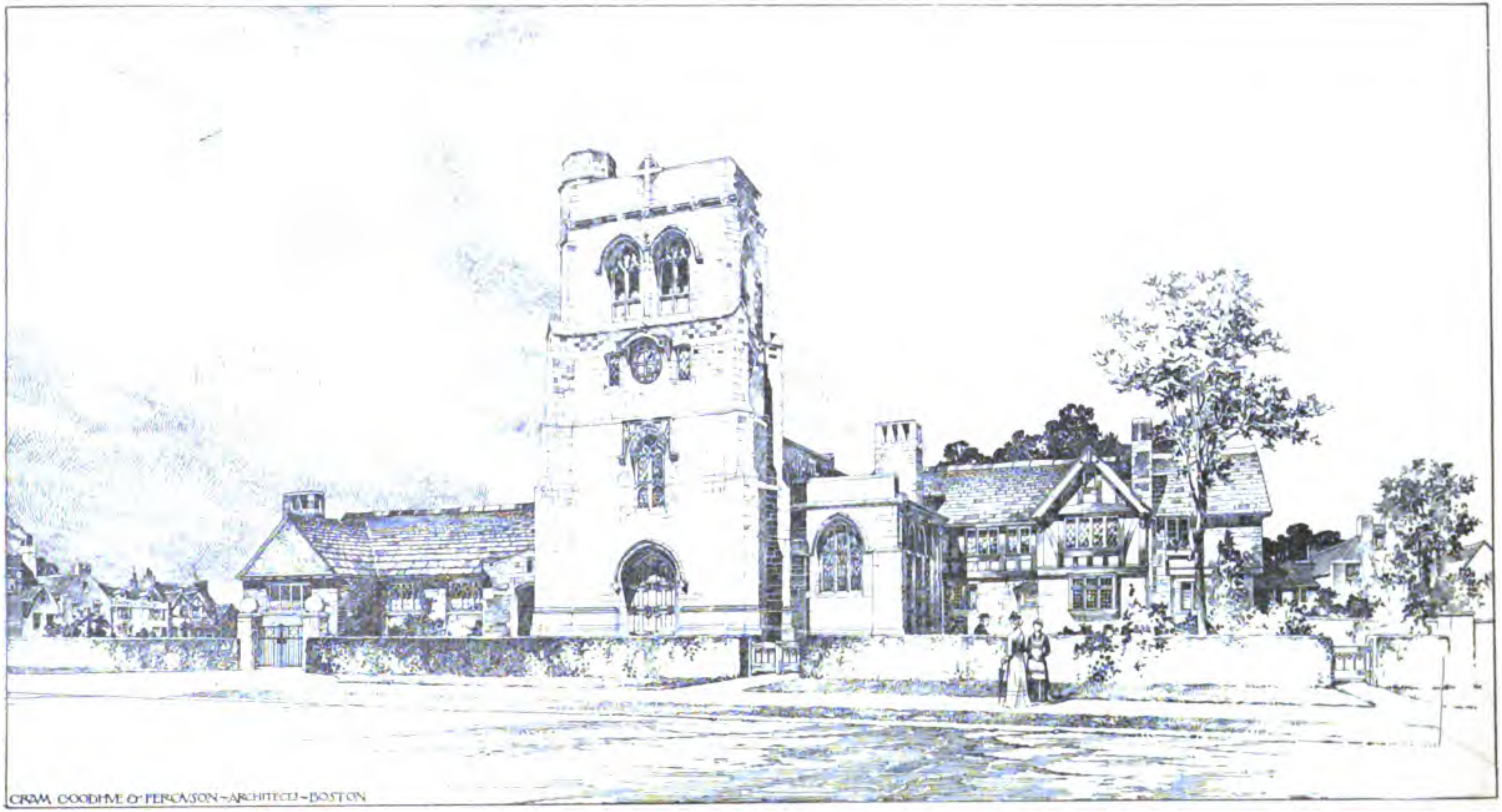
	Tons per sq. ft.
Rosendale cement 1 to 3	13
Lime mortar	8 to 10
Portland 1 to 2 in ¼-inch joints for bedding iron plates	70

CONCRETE.

	Tons per sq. ft.
Portland cement 1 to 8	8 to 20
Rosendale cement 1 to 6	5 to 10
Lime, best, 1 to 6	5

HOLLOW TILE.

	Pounds per sq. ft.
Hard fire-clay tiles	80
Hard ordinary clay tiles	60
Porous terra-cotta tiles	40
Terra-cotta blocks, unfilled	10,000
Terra-cotta blocks, filled solid with brick or cement	20,000


CRAM GOODHUE & FERGUSON - ARCHITECTS - BOSTON

ST. MARY'S CHURCH, WALKERVILLE, ONT.

Cram, Goodhue & Ferguson, Architects, Boston and New York.

MASONRY CONSTRUCTION.

PART II.

CLASSIFICATION OF MASONRY.

Masonry is classified according to the nature of the material used, as "stone masonry," "brick masonry," "mixed masonry," composed of stones and bricks, and "concrete masonry."

Stone masonry is classified (1) according to the manner in which the material is prepared, as "rubble masonry," "squared stone masonry," "ashlar masonry," "broken ashlar," and the combinations of these four kinds; and (2) according to the manner in which the work is executed, as "uncoursed rubble," "coursed rubble," "dry rubble," "regular-coursed ashlar,"' 'broken or irregular-coursed ashlar," "ranged work," "random ranged," etc.

DEFINITIONS OF THE TERMS USED IN MASONRY.

Abutment: (1) That portion of the masonry of a bridge or dam upon which the ends rest, and which connects the superstructure with the adjacent banks. (2) A structure that receives the lateral thrust of an arch.

Arris: The external angle or edge formed by the meeting of two plane or curved surfaces, whether walls or the sides of a stick or stone.

Backed: Built on the rear face.

Backing: The rough masonry of a wall faced with cut stone.

Batter: The slope or inclination given to the face of a wall. It is expressed by dividing the height by the horizontal distance. It is described by stating the extent of the deviation from the vertical, as one in twelve, or one inch to the foot.

Bats: Broken bricks.

Bearing Blocks or Templets: Small blocks of stone built in the wall to support the ends of particular beams.

Belt Stones or Courses: Horizontal bands or zones of stone encircling a building or extending through a wall.

Blocking Course: A course of stone placed on the top of a cornice, crowning the walls.

Bond: The disposing of the blocks of stone or bricks in the walls so as to form the whole into a firm structure by a judicious overlapping of each other so as to break joint.

A stone or brick which is laid with its length across the wall, or extends through the facing course into that behind, so as to bind the facing to the backing, is called a "header" or "bond." Bonds are described by various names, as:

Binders, when they extend only a part of the distance across the wall.

Through Bonds, when they extend clear across from face to back.

Heart Bonds, when two headers meet in the middle of the wall and the joint between them is covered by another header.

Perpend Bond signifies that a header extends through the whole thickness of the wall.

Chain Bond is the building into the masonry of an iron bar, chain, or heavy timber.

Cross Bond, in which the joints of the second stretcher course come in the middle of the first; a course composed of headers and stretchers intervening.

Block and Cross Bond, when the face of the wall is put up in cross bond and the backing in block bond.

English Bond (brick masonry) consists of alternate courses of headers and stretchers.

Flemish Bond (brick masonry) consists of alternate headers and stretchers in the same course.

Blind Bond is used to tie the front course to the wall in pressed brick work where it is not desirable that any headers should be seen in the face work.

To form this bond the face brick is trimmed or clipped off at both ends, so that it will admit a binder to set in transversely from the face of the wall, and every layer of these binders should be tied with a header course the whole length of the wall. The binder should be put in every fifth course, and the backing should be done in a most substantial manner, with hard brick laid in close joints, for the reason that the face work is laid in a fine putty mortar, and the joints consequently close and tight; and if the backing is not the same the pressure upon the wall will make it settle and draw the wall inward. The common form of bond in brickwork is to lay three or five courses as stretchers, then a header course.

Breast Wall: One built to prevent the falling of a vertical face cut into the natural soil; in distinction to a retaining wall, etc.

Brick Ashlar: Walls with ashlar facing backed with bricks.

Build or Rise: That dimension of the stone which is perpendicular to the quarry bed.

Buttress: A vertical projecting piece of stone or brick masonry built in front of a wall to strengthen it.

Closers are pieces of brick or stone inserted in alternate courses of brick and broken ashlar masonry to obtain a bond.

Cleaning Down consists in washing and scrubbing the stonework with muriatic acid and water. Wire brushes are generally used for marble and sometimes for sandstone. Stiff bristle brushes are ordinarily used. The stones should be scrubbed until all mortar stains and dirt are entirely removed.

For cleaning old stonework the sand blast operated either by steam or compressed air is used. Brick masonry is cleaned in the same manner as stone masonry. During the process of cleaning all open joints under window sills and elsewhere should be pointed.

Coping: The coping of a wall consists of large and heavy stones, slightly projecting over it at both sides, accurately bedded on the wall, and jointed to each other with cement mortar. Its use is to shelter the mortar in the interior of the wall from the weather, and to protect by its weight the smaller stones below it from being knocked off or picked out. Coping stones should be so shaped that water may rapidly run off from them.

For coping stones the objections with regard to excess of length do not apply; this excess may, on the contrary, prove favorable, because, the number of top joints being thus diminished, the mass beneath the coping will be better protected.

Additional stability is given to a coping by so connecting the coping stones together that it is impossible to lift one of them without at the same time lifting the ends of the two next it. This is done either by means of iron cramps inserted into holes in the stone and fixed there with lead, or, better still, by means of dowels of wrought iron, cast iron, copper, or hard stone. The metal dowels are inferior in durability to those of hard stone, though superior in strength. Copper is strong and durable, but expensive. The stone dowels are small prismatic or cylindrical blocks, each of which fits into a pair of opposite holes in the contiguous ends of a pair of coping stones and fixed with cement mortar.

The under edge should be throated or dipped, that is, grooved, so that the drip will not run back on the wall, but drop from the edge. Coping is divided into three kinds:

Parallel coping, level on top. *Feather-edged coping,* bedded level and sloping on top. *Saddle-back coping* has a curved or doubly inclined top.

Corbell: A horizontal projecting piece, or course, of masonry which assists in supporting one resting upon it which projects still further.

Cornice: The ornamental projection at the eaves of a building or at the top of a pier or any other structure.

Counterfort: Vertical projections of stone or brick masonry built at intervals along the back of a wall to strengthen it, and generally of very little use.

Course: The term course is applied to each horizontal row or layer of stones or bricks in a wall; some of the courses have particular names, as:

Plinth Course, a lower, projecting, square-faced course; also called the water table.

Blocking Course, laid on top of the cornice.

Bonding Course, one in which the stones or bricks lie with their length across the wall; also called heading course.

Stretching Course, consisting of stretchers.

Springing Course, the course from which an arch springs.

String Course, a projecting course.

Rowlock Course, bricks set on edge.

Cramps: Bars of iron having the ends turned at right angles to the body of the bar, and inserted in holes and trenches cut in the upper sides of adjacent stones to hold them together (see Coping).

Cutwater or Starling: The projecting ends of a bridge-pier, etc., usually so shaped as to allow water, ice, etc., to strike them with but little injury.

Dowels: Straight bars of iron, copper, or stone which are placed in holes cut in the upper bed of one stone and in the lower bed of the next stone above. They are also placed horizontally in the adjacent ends of coping stones (see under Coping). Cramps and dowels are fastened in place by pouring melted lead, sulphur, or cement grout around them.

Dry Stone Walls may be of any of the classes of masonry previously described, with the single exception that the mortar is omitted. They should be built according to the principles laid down for the class to which they belong.

Face: The front surface of the wall.

Facing: The stone which forms the face or outside of the wall exposed to view.

Footing: The projecting courses at the base of a wall for the purpose of distributing the weight over an increased area, and thereby diminishing the liability to vertical settlement from compression of the ground.

Footings, to have any useful effect, must be securely bonded into the body of the work, and have sufficient strength to resist the cross strains to which they are exposed. The beds should be dressed true and parallel. Too much care cannot be bestowed upon the footing courses of any building, as upon them depends much of the stability of the work. If the bottom course be not solidly bedded, if any rents or vacuities are left in the beds of the masonry, or if the materials be unsound or badly put together, the effects of such carelessness will show themselves sooner or later, and always at a period when remedial efforts are useless.

Gauged Work: Bricks cut and rubbed to the exact shape required.

Grout is a thin or fluid mortar made in the proportion of 1 of cement to 1 or 2 of sand. It is used to fill up the voids in walls of rubble masonry and brick. Sometimes the interior of a wall is built up dry and grout poured in to fill the voids. Unless specifically instructed to permit its use, grout should not be used unless in the presence of the inspector. When used by masons without instructions it is usually for the purpose of concealing bad work. Grout is used for solidifying quicksand.

Grouting is pouring fluid mortar over last course for the purpose of filling all vacuities.

Header. Also called a bond. A stone or brick whose greatest dimension lies perpendicular to the face of the wall, and used for the purpose of tying the face to the backing (see Bond). A trick of masons is to use "blind headers," or short stones that look like headers on the face, but do not go deeper into the wall than the adjacent stretchers. When a course has been put on top of these they are completely covered up, and, if not suspected, the fraud will never be discovered unless the weakness of the wall reveals it.

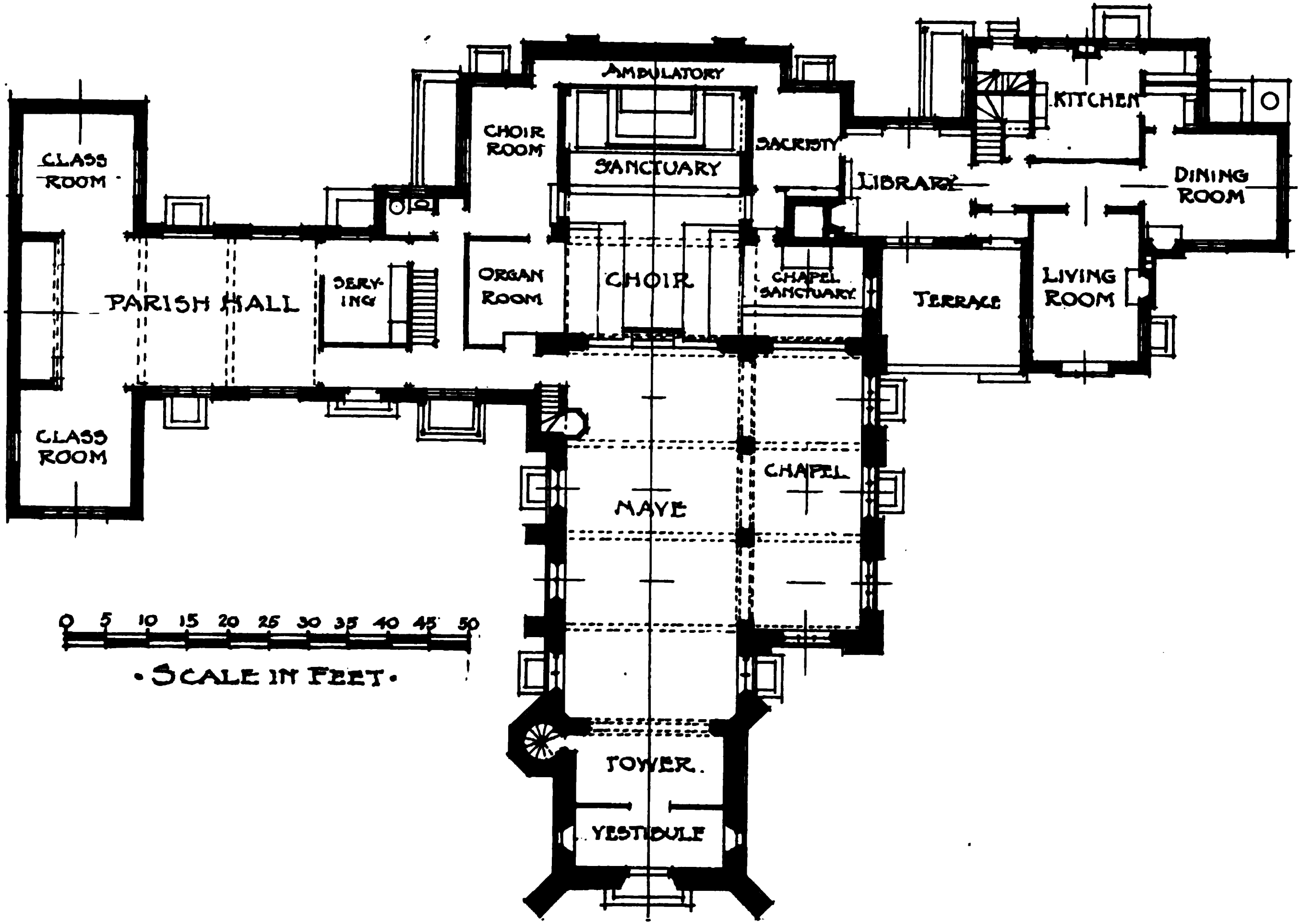
In facing brick walls with pressed brick the bricklayer will frequently cut the headers for the purpose of economizing the more expensive material; thus great watchfulness is necessary to secure a good bond between the facing and common brick. "All stone foundation walls 24 inches or less in thickness shall have at least one header extending through the wall in every 3 feet in height from the bottom of the wall, and in every 3 feet in length, and if over 24 inches in thickness shall have one header for every 6 superficial feet on both sides of the wall, and running into the wall at least 2 feet. All headers shall be at least 12 inches in width and 8 inches in thickness, and consist of good, flat stone.

"In all brick walls every sixth course shall be a heading course, except where walls are faced with brick in running bond, in which latter case every sixth course shall be bonded into the backing by cutting the course of the face brick and putting in diagonal headers behind the same, or by splitting the face brick in half and backing the same with a continuous row of headers."

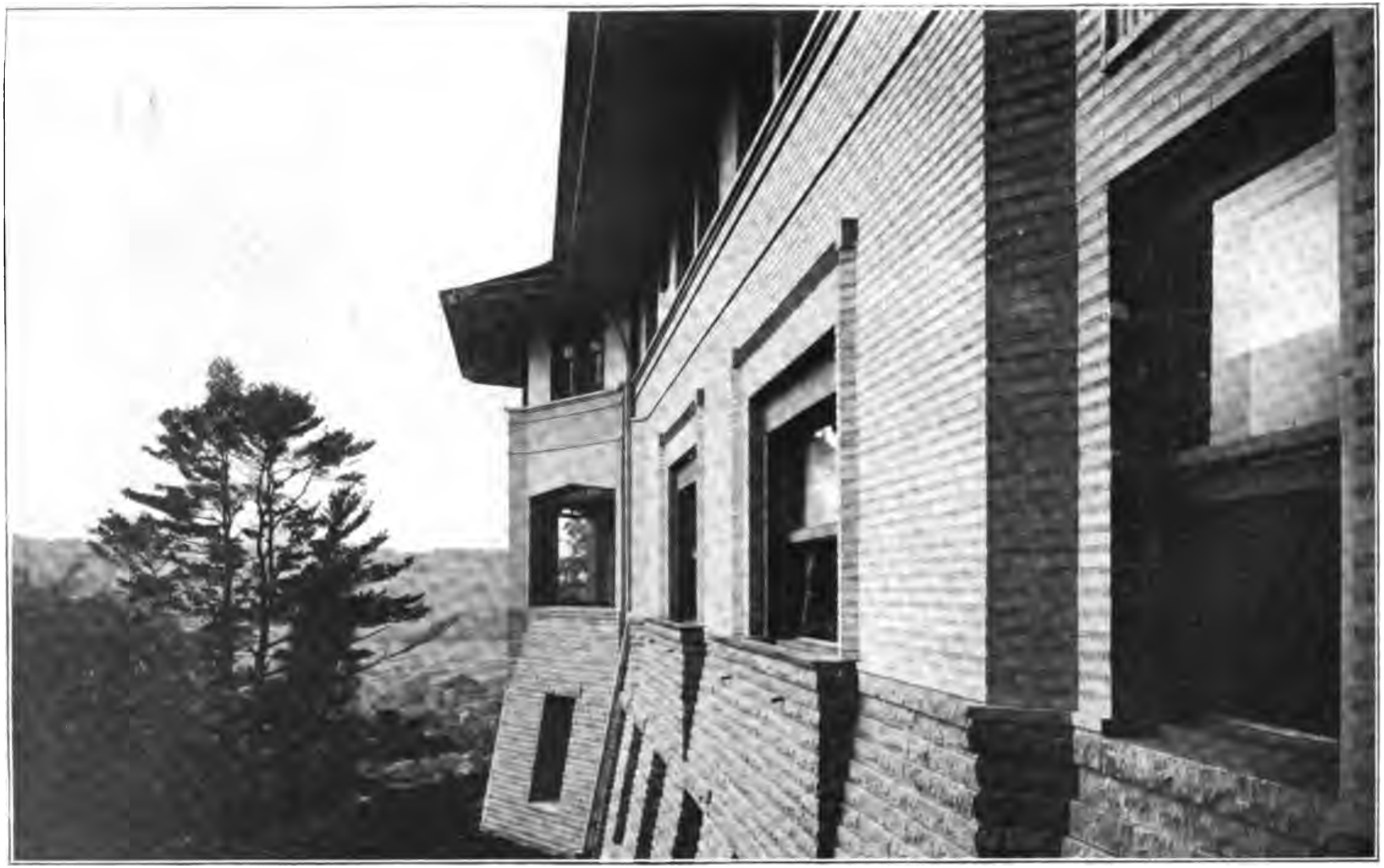
Joints. The mortar layers between the stone or bricks are called the joints. The horizontal joints are called "bed joints;" the end joints are called the vertical joints, or simply the "joints."

Excessively thick joints should be avoided. In good brickwork they should be about $\frac{1}{4}$ to $\frac{3}{8}$ inch thick; for ashlar masonry and pressed brickwork, about $\frac{1}{8}$ to $\frac{1}{16}$ inch thick; for rubble masonry they vary according to the character of the work.

The joints of both stone and brick masonry are finished in different ways, with the object of presenting a neat appearance and of throwing the rainwater away from the joint.



ST. MARY'S CHURCH, WALKERVILLE, ONT.
Cram, Goodhue & Ferguson, Architects, Boston and New York.
For Exterior, See Page 276.



ALPHA DELTA PHI CHAPTER-HOUSE AT CORNELL UNIVERSITY, ITHACA, N. Y.

Dean & Dean, Architects, Chicago, Ill.

Ground Floor Split-Faced Bedford Stone; Main Floor of Roman Brick with ⅞-Inch Mortar Joints; Second Floor of Plaster.
For Plans, See Page 298; for Interiors, See Vol. II, Page 152, and Vol. IV, Pages 282 and 298.

Flush Joints. In these the mortar is pressed flat with the trowel and the surface of the joint is flush with the face of the wall.

Struck Joints are formed by pressing or striking back with the trowel the upper portion of the joint while the mortar is moist, so as to form an outward sloping surface from the bottom of the upper course to the top of the lower course. This joint is also designated by the name "weather joint." Masons generally form this joint so that it slopes inwards, thus leaving the upper arris of the lower course bare and exposed to the action of the weather. The reason for forming it in this improper manner is that it is easier to perform.

Key Joints are formed by drawing a curved iron key or jointer along the center of the flushed joint, pressing it hard, so that the mortar is driven in beyond the face of the wall; a groove of curved section is thus formed, having its surface hardened by the pressure.

White Skate or Groove Joint is employed in front brickwork. It is about $\frac{3}{16}$-inch thick. It is formed with a jointer having the width of the intended joint. It is guided along the joint by a straight edge and leaves its impress upon the material.

Joggle: A joint piece or dowel pin let into adjacent faces of two stones to hold them in position. It may vary in form and approach in its shape either the dowel or clamp.

Jamb: The sides of an opening left in a wall.

Lintel: The stone, wood, or iron beam used to cover a narrow opening in a wall.

One-Man Stone: A stone of such size as to be readily lifted by one man.

Parapet Wall is a low wall running along the edge of a terrace or roof to prevent people from falling over.

Pointing a piece of masonry consists in scraping out the mortar in which the stones were laid from the face of the joints for a depth of from ½ to 2 inches, and filling the groove so made with clear Portland-cement mortar, or with mortar made of 1 part of cement and 1 part of sand.

The object of pointing is that the exposed edges of the joints are always deficient in density and hardness, and the mortar near the surface of the joint is specially subject to dislodgment, since the contraction and expansion of the masonry are liable either to separate the stone from the mortar or to crack the mortar in the joint, thus permitting the entrance of rainwater, which freezing forces the mortar from the joints.

The pointing mortar, when ready for use, should be rather incoherent and quite deficient in plasticity.

Before applying the pointing, the joint must be well cleansed by scraping and brushing out the loose matter, then thoroughly saturated with water, and maintained in such a condition of dampness that the stones will neither absorb water from the mortar nor impart any to it. Walls should not be allowed to dry too rapidly after pointing.

Pointing should not be prosecuted either during freezing or excessively hot weather.

The pointing mortar is applied with a mason's trowel, and the joint well calked with a calking iron and hammer. In the very best work the surface of the mortar is rubbed smooth with a steel polishing tool. The form given to the finish joint is the same as described under joints.

Pointing with colored mortar is frequently employed to improve the appearance of the work. Various colors are used, as white, black, red, brown, etc., different colored pigments being added to the mortar to produce the required color.

Tuck Pointing, used chiefly for brickwork, consists of a projecting ridge with the edges neatly pared to an uniform breadth of about ⅛-inch. White mortar is usually employed for this class of pointing.

Many authorities consider that pointing is not advisable for new work, as the joints so formed are not as enduring as those which are finished at the time the masonry is built. Pointing is, moreover, often resorted to when it is intended to give the work a superior appearance, and also to conceal defects in inferior work.

Pallets, Plugs: Wooden bricks inserted in walls for fastening trim, etc.

Plinth: A projecting base to a wall; also called "water table."

Pitched-Face Masonry: That in which the face of the stone is roughly dressed with the pitching chisel so as to give edges that are approximately true.

Quarry-Faced or Rock-Faced Masonry: That in which the face of the stone is left untouched as it comes from the quarry.

Quoin: A cornerstone. A quoin is a header for one face and a stretcher for the other.

Rip-Rap. Rip-rap is composed of rough undressed stone as it comes from the quarry, laid dry about the base of piers, abutments, slopes of embankments, etc., to prevent scour and wash. When used for the protection of piers the stones are dumped in promiscuously, their size depending upon the material and the velocity of the current. Stones of 15 to 25 cubic feet are frequently employed. When used for the protection of banks the stones are laid by hand to a uniform thickness.

Rise: That dimension of a stone which is perpendicular to its quarry bed (see Build).

Retaining Wall or Revetment: A wall built to retain earth deposited behind it (see Breast Wall).

Reveal: The exposed portion of the sides of openings in walls in front of the recesses for doors, window frames, etc.

Slope-Wall Masonry: A slope wall is a thin layer of masonry used to protect the slopes of embankments, excavations, canals, river banks, etc., from rain, waves, weather, etc.

Slips: See Wood Bricks.

Spall: A piece of stone chipped off by the stroke of a hammer.

Sill: The stone, iron, or wood on which the window or door of a building rests. In setting stone sills the mason beds the ends only; the middle is pointed up after the building is enclosed. They should be set perfectly level lengthwise, and have an inclination crosswise, so the water may flow from the frame.

Stone Paving consists of roughly squared or unsquared blocks of stone used for paving the waterway of culverts, etc.; it is laid both dry and in mortar.

Starling: See Cutwater.

Stretcher: A stone or brick whose greatest dimension lies parallel to the face of the wall.

String Course: A horizontal course of brick or stone masonry projecting a little beyond the face of the wall. Usually introduced for ornament.

Two-Men Stone: Stone of such size as to be conveniently lifted by two men.

Toothing: Unfinished brickwork so arranged that every alternate brick projects half its length.

Water-Table: See Plinth.

Wood Bricks, Pallets, Plugs, or Slips are pieces of wood laid in a wall in order the better to secure any woodwork that it may be necessary to fasten to it. Great injury is often done to walls by driving wood plugs into the joints, as they are apt to shake the work. Hollow porous terra-cotta bricks are frequently used instead of wood bricks, etc.

PREPARATION OF THE MATERIALS.

STONE CUTTING.

Dressing the Stones. The stonecutter examines the rough blocks as they come from the quarry in order to determine whether the blocks will work to better advantage as a header, a stretcher, or a cornerstone. Having decided for which purpose the stone is suited, he prepares to dress the bottom bed. The stone is placed with bottom bed up, all the rough projections are removed with the hammer and pitching tool, and approximately straight lines are pitched off around its edges; then a chisel draft is cut on all the edges. These drafts are brought to the same plane as nearly as practicable by the use of two straight edges having parallel sides and equal widths, and the enclosed rough portion is then dressed down with the pitching tool or point to the plane of the drafts. The entire bed is then pointed down to a surface true to the straight edge when applied in any direction—crosswise, lengthwise, and diagonally.

Lines are then marked on this dressed surface parallel and perpendicular to the face of the stone, enclosing as large a rectangle as the stone will admit of being worked to, or of such dimensions as may be directed by the plan.

The faces and sides are pitched off to these lines. A chisel draft is then cut along all four edges of the face, and the face either dressed as required, or left rock faced. The sides are then pointed down to true surfaces at right angles to the bed. The stone is turned over bottom bed down, and the top bed dressed in the same manner as the bottom. It is important that the top bed be exactly parallel to the bottom bed in order that the stone may be of uniform thickness.

Stones having the beds inclined to each other, as skewbacks, or stones having the sides inclined to the beds, are dressed by using a bevelled straight edge set to the required inclination.

Arch stones have two plane surfaces inclined to each other; these are called the beds. The upper surface or extrados is usually left rough; the lower surface or intrados is cut to the curve of the arch. This surface and the beds are cut true by the use of a wooden or metal templet which is made according to the drawings furnished by the engineer or architect.

TOOLS USED IN STONE CUTTING.

The Double-Face Hammer is a heavy tool, weighing from 20 to 30 pounds, used for roughly shaping stones as they come from the quarry and for knocking off projections. This is used for only the roughest work.

The Face Hammer has one blunt and one cutting end, and is used for the same purpose as the double-face hammer where less weight is required. The cutting end is used for roughly squaring stones preparatory to the use of the finer tools.

The Cavil has one blunt and one pyramidal or pointed end, and weighs from 15 to 20 pounds. It is used in quarries for roughly shaping stone for transportation.

The Pick somewhat resembles the pick used in digging, and is used for rough dressing, mostly on limestone and sandstone. Its length varies from 15 to 24 inches, the thickness at the eye being about 2 inches.

The Axe or Pean Hammer has two opposite cutting edges. It is used for making drafts around the arris or edge of stones, and in reducing faces, and sometimes joints, to a level. Its length is about 10 inches and the cutting edge about 4 inches. It is used after the point and before the patent hammer.

The Tooth Axe is like the axe, except that its cutting edges are divided into teeth, the number of which varies with the kind of work required. This tool is not used in cutting granite or gneiss.

The Bush Hammer is a square prism of steel, whose ends are cut into a number of pyramidal points. The length of the hammer is from 4 to 8 inches and the cutting face from 2 to 4 inches square. The points vary in number and in size with the work to be done. One end is sometimes made with a cutting edge like that of the axe.

The Crandall is a malleable-iron bar about 2 feet long slightly flattened at one end. In this end is a slot 3 inches long and $\frac{3}{8}$-inch wide. Through this slot are passed ten double-headed points of $\frac{1}{4}$-inch square steel 9 inches long, which are held in place by a key.

The Patent Hammer is a double-headed tool so formed as to hold at each end a set of wide thin chisels. The tool is in two parts, which are held together by the bolts which hold the chisels. Lateral motion is prevented by four guards on one of the pieces. The tool without the teeth is $5\frac{1}{2} \times 2\frac{3}{4} \times 1\frac{1}{2}$ inches. The teeth are $2\frac{3}{4}$ inches wide; their thickness varies from $\frac{1}{12}$ to $\frac{1}{6}$ of an inch. This tool is used for giving a finish to the surface of stones.

The Hand Hammer, weighing from 2 to 5 pounds, is used in drilling holes and in pointing and chiselling the harder rocks.

The Mallet is used where the softer limestones and sandstones are cut.

The Pitching Chisel is usually of $1\frac{1}{8}$-inch octagonal steel, spread on the cutting edge to a rectangle of $\frac{1}{8} \times 2\frac{1}{2}$ inches. It is used to make a well-defined edge to the face of a stone, a line being marked on the joint surface, to which the chisel is applied and the portion of the stone outside of the line broken off by a blow with the hand hammer on the head of the chisel.

The Point is made of round or octagonal steel from $\frac{1}{4}$ to 1 inch in diameter. It is made about 12 inches long, with one end brought to a point. It is used until its length is reduced to about 5 inches. It is employed for dressing off the irregular surface of stones, either for a permanent finish or preparatory to the use of the axe. According to the hardness of the stone, either the hand hammer or the mallet is used with it.

The Chisel is of round steel of $\frac{1}{4}$ to $\frac{3}{4}$-inch diameter and about 10 inches long, with one end brought to a cutting edge from $\frac{1}{4}$ inch to 2 inches wide; is used for cutting drafts or margins on the face of stones.

The Tooth Chisel is the same as the chisel, except that the cutting edge is divided into teeth. It is used only on marbles and sandstones.

The Splitting Chisel is used chiefly on the softer stratified stones, and sometimes on fine architectural carvings in granite.

The Plug, a truncated wedge of steel, and the *feathers* of half-round malleable iron, are used for splitting unstratified stone. A row of holes is made with the drill on the line on which the fracture is to be made; in each of these two feathers are inserted, and the plugs lightly driven in between them. The plugs are then gradually driven home by light blows of the hand hammer on each in succession until the stone splits.

Machine Tools. In all large stone yards machines are used to prepare the stone. There is a great variety in their form, but since the kind of dressing never takes its name from the machine which forms it, it will be neither necessary nor profitable to attempt a description of individual machines. They include stone saws, stone cutters, stone grinders, stone polishers, etc.

DEFINITION OF TERMS USED IN STONE CUTTING.

Axed : Dressed to a plane surface with an axe.

Boasted or Chiselled : Having face wrought with a chisel or narrow tool.

Broached : Dressed with a "punch" after being droved.

Bush Hammered : Dressed with a bush hammer.

Crandalled : Wrought to a plane with a crandall.

Deadening : The crushing or crumbling of a soft stone under the tools while being dressed.

Dressed Work : That which is wrought on the face; also applied to stones having the joints wrought to a plane surface, but not "squared."

Drafted : Having a narrow chisel draft cut around the face or margin.

Droved, Stroked : Wrought with a broad chisel or hammer in parallel flutings across the stone from end to end.

Hammer Dressed : Worked with the hammer.

Herring Bone : Dressed in angular flutings.

Nigged or Nidged : Picked with a pointed hammer or cavil to the desired form.

Patent Hammered : Dressed with a patent hammer.

Picked : Reduced to an approximate plane with a pick.

Pitched : Dressed to the neat lines or edges with a pitching chisel.

Plain : Rubbed smooth to remove tool marks.

Pointed : Dressed with a point or very narrow tool.

Polished : Rubbed down to a reflecting surface.

Prison: Having surfaces wrought into holes.

Random Tooled or Droved: Cut with a broad tool into irregular flutings.

Rock Faced, Quarry Faced, Rough: Left as it comes from the quarry. It may be drafted or pitched to reduce projecting points on the face to give limits.

Rubbed: See Plain.

Rustic, Rusticated: Having the faces of stones projecting beyond the arrises, which are bevelled or drafted. The face may be dressed in any desired manner.

Scabble: To dress off the angular projections of stones for rubble masonry with a stone axe or hammer.

Smooth: See Plain.

Square Droved: Having the flutings perpendicular to the lower edge of the stone.

Striped: Wrought into parallel grooves with a point or punch.

Stroked: See Droved.

Tooled: Wrought to a plane with an inch tool. See Droved.

Toothed: Dressed with a tooth chisel.

Vermiculated Worm Work: Wrought into veins by cutting away portions of the face.

METHODS OF FINISHING THE FACES OF CUT STONE.

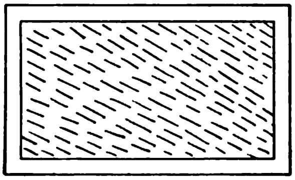
In architecture there are a great many ways in which the faces of cut stone may be dressed, but the following are those that will be usually met in engineering work.

Rough Pointed. When it is necessary to remove an inch or more from the face of a stone it is done by the pick or heavy point until the projections vary from ½ to 1 inch. The stone is said to be rough pointed. In dressing limestone and granite this operation precedes all others.

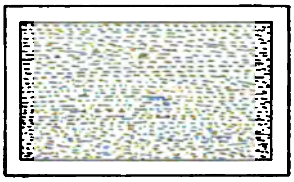
Fine Pointed. If a smoother finish is desired rough pointing is followed by fine pointing, which is done with a fine point. Fine pointing is used only where the finish made by it is to be final, and never as a preparation for a final finish by another tool.

Crandalled. This is only a speedy method of pointing, the effect being the same as fine pointing, except that the dots on the stone are more regular. The variations of level are about ⅛ inch and the rows are made parallel. When other rows at right angles to the first are introduced the stone is said to be cross-crandalled.

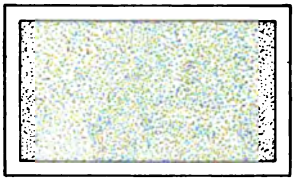
Axed or Pean Hammered, and Patent Hammered. These two vary only in the degree of smoothness of the surface which is produced. The number of blades in a patent hammer varies from 6 to 12 to the inch; and in precise specifications the number of cuts to the inch must be stated, such as 6-cut, 8-cut, 10-cut, 12-cut. The effect of axing is to cover the surface with chisel marks, which are made parallel as far as practicable. Axing is a final finish.



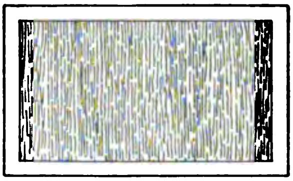
Rough Pointed



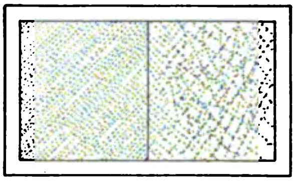
Fine Pointed



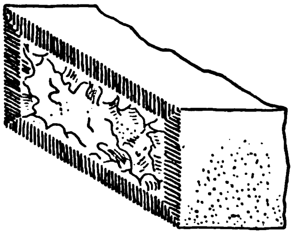
Bush-hammered



Patent-hammered



Crandalled



Rock-face with Draft Line

Fig. 19. Methods of Finishing the Faces of Cut Stone.

Tooth Axed. The tooth axe is practically a number of points, and it leaves the surface of a stone in the same condition as fine pointing. It is usually, however, only a preparation for bush hammering, and the work is then done without regard to effect, so long as the surface of the stone is sufficiently levelled.

Bush Hammered. The roughnesses of a stone are pounded off by the bush hammer, and the stone is then said to be "bushed." This kind of finish is dangerous on sandstone, as experience has shown that sandstone thus treated is very apt to scale. In dressing limestone which is to have a bush hammered finish the usual sequence of operation is (1) rough pointing, (2) tooth axing, and (3) bush hammering.

CLASSIFICATION OF THE STONES.

All the stones used in building are divided into three classes according to the finish of the surface, viz.: 1. Rough stones that are used as they come from the quarry. 2. Stones roughly squared and dressed. 3. Stones accurately squared and finely dressed.

Unsquared Stones. This class covers all stones which are used as they come from the quarry without other preparation than the removal of very acute angles and excessive projections from the general figure.

Squared Stones. This class covers all stones that are roughly squared and roughly dressed on beds and joints. The dressing is usually done with the face hammer or axe, or in soft stones with the tooth hammer. In gneiss, hard limestones, etc., it may be necessary to use the point. The distinction between this class and the third lies in the degree of closeness of the joints. Where the dressing on the joints is such that the distance between the general planes of the surfaces of adjoining stones is one-half inch or more, the stones properly belong to this class.

Three subdivisions of this class may be made, depending on the character of the face of the stones.

(*a*) *Quarry-faced* or *Rock-faced* stones are those whose faces are left untouched as they come from the quarry.

(*b*) *Pitched-faced* stones are those on which the arris is clearly defined by a line beyond which the rock is cut away by the pitching chisel, so as to give edges that are approximately true.

(*c*) *Drafted* stones are those on which the face is surrounded by a chisel draft, the space inside the draft being left rough. Ordinarily, however, this is done only on stones in which the cutting of the joints is such as to exclude them from this class.

In ordering stones of this class the specifications should always state the width of the bed and end joints which are expected, and also how far the surface of the face may project beyond the plane of the edge. In practice the projection varies between 1 inch and 6 inches. It should also be specified whether or not the faces are to be drafted.

Cut Stones. This class covers all squared stones with smoothly dressed beds and joints. As a rule, all the edges of cut stones are drafted, and between the drafts the stone is smoothly dressed. The face, however, is often left rough where construction is massive. The stones of this class are frequently termed "dimension" stone or "dimension" work.

ASHLAR MASONRY.

Ashlar masonry consists of blocks of stone cut to regular figures, generally rectangular, and built in courses of uniform height or rise, which is seldom less than a foot.

Size of the Stones. In order that the stones may not be liable to be broken across, no stone of a soft material, such as the weaker kinds of sandstone and granular limestone, should have a length greater than 3 times its depth or rise; in harder materials the length may be 4 to 5 times the depth. The breadth in soft materials, may range from $1\frac{1}{2}$ to double the depth; in hard materials it may be 3 times the depth.

Laying the Stone. The bed on which the stone is to be laid should be thoroughly cleansed from dust and well moistened with water. A thin bed of mortar should then be spread evenly over it, and the stone, the lower bed of which has been cleaned and moistened, raised into position, and lowered first upon one or two strips of wood laid upon the mortar bed; then, by the aid of the pinch bar, moved exactly into its place, truly plumbed, the strips of wood removed, and the stone settled in its place and levelled by striking it with wooden mallets. In using bars and rollers in handling cut stone, the mason must be careful to protect the stone from injury by a piece of old bagging, carpet, etc.

In laying "rock-faced" work, the line should be carried above it, and care must be taken that the work is kept plumb with the cut margins of the corners and angles.

The Thickness of Mortar in the joints of well executed ashlar masonry should be about $\frac{1}{8}$ of an inch, but it is usually about $\frac{3}{8}$.

Amount of Mortar. The amount of mortar required for ashlar masonry varies with the size of the blocks, and also with the closeness of the dressing. With ⅜ to ½-inch joints and 12 to 20-inch courses will be about 2 cubic feet of mortar per cubic yard; with larger blocks and closer joints, there will be about 1 cubic foot of mortar per yard of masonry. Laid in 1 to 2 mortar, ordinary ashlar will require ¼ to ⅓ of a barrel of cement per cubic yard of masonry.

Bond of Ashlar Masonry. No side joint in any course should be directly above a side joint in the course below; but the stones should overlap or break joint to an extent of from once to once and a half the depth or rise of the course. This is called the bond of the masonry; its effect is to cause each stone to be supported by at least two stones of the course below, and assist in supporting at least two stones of the course above; and its objects are twofold: first, to distribute the pressure, so that inequalities of load on the upper part of the structure, or of resistance at the foundation, may be transmitted to and spread over an increasing area of bed in proceeding downwards or upwards, as the case may be; and second, to tie the structure together, or give it a sort of tenacity, both lengthwise and from face to back, by means of the friction of the stones where they overlap. The strongest bond in ashlar masonry is that in which each course at the face of the wall contains a header and a stretcher alternately, the outer end of each header resting on the middle of a stretcher of the course below, so that rather more than one-third of the area of the face consists of ends of headers. This proportion may be deviated from when circumstances require it; but in every case it is advisable that the ends of headers should not form less than one-fourth of the whole area of the face of the wall.

SQUARED-STONE MASONRY.

The distinction between squared-stone masonry and ashlar lies in the character of the dressing and the closeness of the joints. In this class of masonry the stones are roughly squared and roughly dressed on beds and joints, so that the width of the joints is half an inch or more. The same rules apply to breaking joint, and to the proportions which the lengths and breadths of the stones should bear to their depths, as in ashlar; and as in ashlar, also, at least one-fourth of the face should consist of headers, whose length should be from three to five times the depth of the course.

Amount of Mortar. The amount of mortar required for squared-stone masonry varies with the size of the stones and with the quality of the masonry; as a rough average, one-sixth to one-quarter of the mass is mortar. When laid in 1 to 2 mortar, from ½ to ¾ of a barrel of cement will be required per cubic yard of masonry.

BROKEN ASHLAR.

Broken ashlar consists of cut stones of unequal depths, laid in the wall without any attempt at maintaining courses of equal rise, or the stones in the same course of equal depth. The character of the dressing and closeness of the joints may be the same as in ashlar or squared-stone masonry, depending upon the quality desired. The same rules apply to breaking joint, and to the proportions which the lengths and breadths of the stones should bear to their depths, as in ashlar; and as in ashlar, also, at least one-fourth of the face of the wall should consist of headers.

Amount of Mortar. The amount of mortar required when laid in 1 to 2 mortar, will be from ¾ to 1 barrel per cubic yard of masonry, depending upon the closeness of the joints.

RUBBLE MASONRY.

Masonry composed of unsquared stones is called rubble. This class of masonry covers a wide range of construction, from the commonest kind of dry-stone work to a class of work composed of large stones laid in mortar. It comprises two classes: (1) uncoursed rubble, in which irregular-shaped stones are laid without any attempt at regular courses, and (2) coursed rubble, in which the blocks of unsquared stones are levelled off at specified heights to an approximately horizontal surface. Coursed rubble is often built in random courses; that is to say, each course rests on a plane bed, but is not necessarily of the same depth or at the same level throughout, so that the beds occasionally rise or fall by steps. Sometimes it is required that the stone shall be roughly shaped with the hammer.

In building rubble masonry of any of the classes above mentioned the stone should be prepared by knocking off all the weak angles of the block. It should be cleansed from dust, etc., and moistened before being placed on its bed. Each stone should be firmly imbedded in the mortar. Care should be taken not only that each stone shall rest on its natural bed, but that the sides parallel to that natural bed shall be the largest, so that the stone may lie flat, and not be set on edge or on end. However small and irregular the stones, care should be taken to break joints. Side joints should not form an angle with the bed joint sharper than 60°. The hollows or interstices between the larger stones must be filled with smaller stones and carefully bedded in mortar.

One-fourth part at least of the face of the wall should consist of bond stones extending into the wall a length of at least 3 to 5 times their depth, as in ashlar.

Amount of Mortar. If rubble masonry is composed of small and irregular stones, about $\frac{1}{3}$ of the mass will consist of mortar; if the stones are larger and more regular $\frac{1}{5}$ to $\frac{1}{4}$ will be mortar. Laid in 1 to 2 mortar, ordinary rubble requires from $\frac{1}{2}$ to 1 barrel of cement per cubic yard of masonry.

ASHLAR BACKED WITH RUBBLE.

In this class of masonry the stones of the ashlar face should have their beds and joints accurately squared and dressed with the hammer or the points, according to the quality desired, for a breadth of from once to twice (or on an average, once and a half), the depth or rise of the course, inwards from the face; but the backs of these stones may be rough. The proportion and length of the headers should be the same as in ashlar, and the "tails" of these headers, or parts which extend into the rubble backing, may be left rough at the back and sides; but their upper and lower beds should be hammer dressed to the general plane of the beds of the course. These tails may taper slightly in breadth, but should not taper in depth.

The rubble backing, built in the manner described under Rubble Masonry, should be carried up at the same time with the face work, and in courses of the same rise, the bed of each course being carefully formed to the same plane with that of the facing.

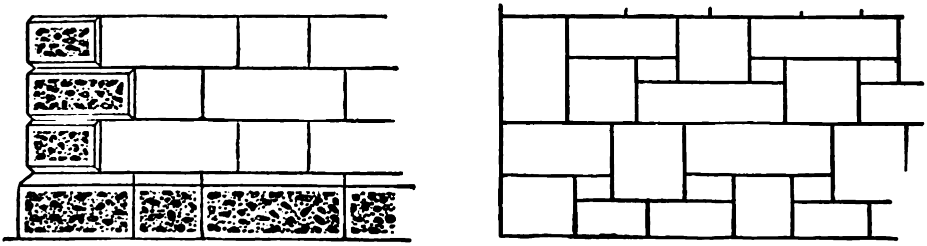
GENERAL RULES FOR LAYING ALL CLASSES OF STONE MASONRY.

1. Build the masonry, as far as possible, in a series of courses, perpendicular, or as nearly so as possible, to the direction of the pressure which they have to bear, and by breaking joints avoid all long continuous joints parallel to that pressure.

2. Use the largest stones for the foundation course.

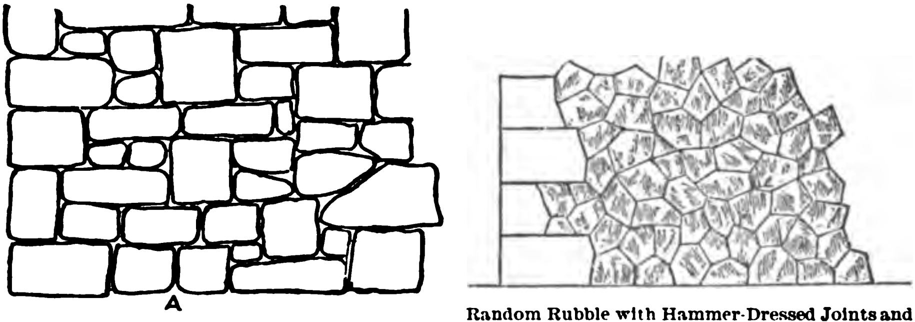
3. Lay all stones which consist of layers in such a manner that the principal pressure which they have to bear shall act in a direction perpendicular, or as nearly so as possible, to the direction of the layers. This is called laying the stone on its natural bed, and is of primary importance for strength and durability.

4. Moisten the surface of dry and porous stones before bedding them, in order that the mortar may not be dried too fast and reduced to powder by the stone absorbing its moisture.



Regular Coursed Ashlar.

Random Coursed Ashlar.



Rubble, Undressed, Laid at Random.

Random Rubble with Hammer-Dressed Joints and no Spalls on Face.

Fig. 20. Types of Masonry.

5. Fill every part of every joint and all spaces between the stones with mortar, taking care at the same time that such spaces shall be as small as possible.

6. The rougher the stones, the better the mortar should be. The principal object of the mortar is to equalize the pressure; and the more nearly the stones are dressed to closely fitting surfaces, the less important is the mortar. Not infrequently this rule is exactly reversed; *i.e.*, the finer the dressing the better the quality of the mortar used.

All projecting courses, such as sills, lintels, etc., should be covered with boards, bagging, etc., as the work progresses, to protect them from injury and mortar stains.

When setting cut stone a pailful of clean water should be kept at hand, and when any fresh mortar comes in contact with the face of the work it should be immediately washed off.

GENERAL RULES FOR BUILDING BRICK MASONRY.

1. Reject all misshapen and unsound bricks.

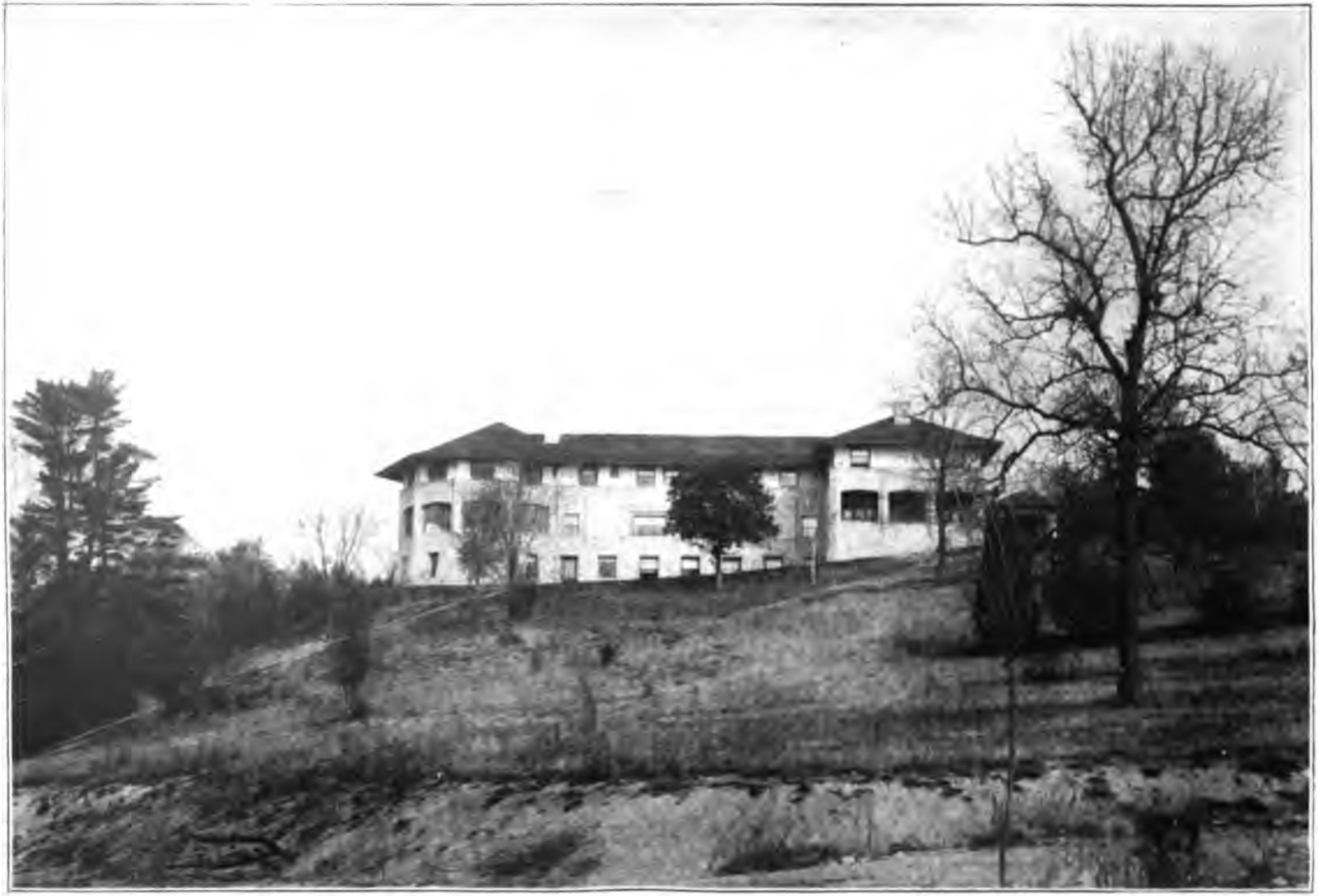
2. Cleanse the surface of each brick, and wet it thoroughly before laying it, in order that it may not absorb the moisture of the mortar too quickly.

3. Place the beds of the courses perpendicular, or as nearly perpendicular as possible, to the direction of the pressure which they have to bear; and make the bricks in each course break joint with those of the courses above and below by overlapping to the extent of from one-quarter to one-half of the length of a brick. (For the style of bond used in brick masonry, see under Bond in list of definitions.)

4. Fill every joint thoroughly with mortar.

Brick should not be merely laid, but every one should be rubbed and pressed down in such a manner as to force the mortar into the pores of the bricks and produce the maximum adhesion; with quick-setting cement, this is still more important than with lime mortar. For the best work it is specified that the brick shall be laid with a "shove joint," that is, that the brick shall first be laid so as to project over the one below, and be pressed into the mortar, and then be shoved into its final position.

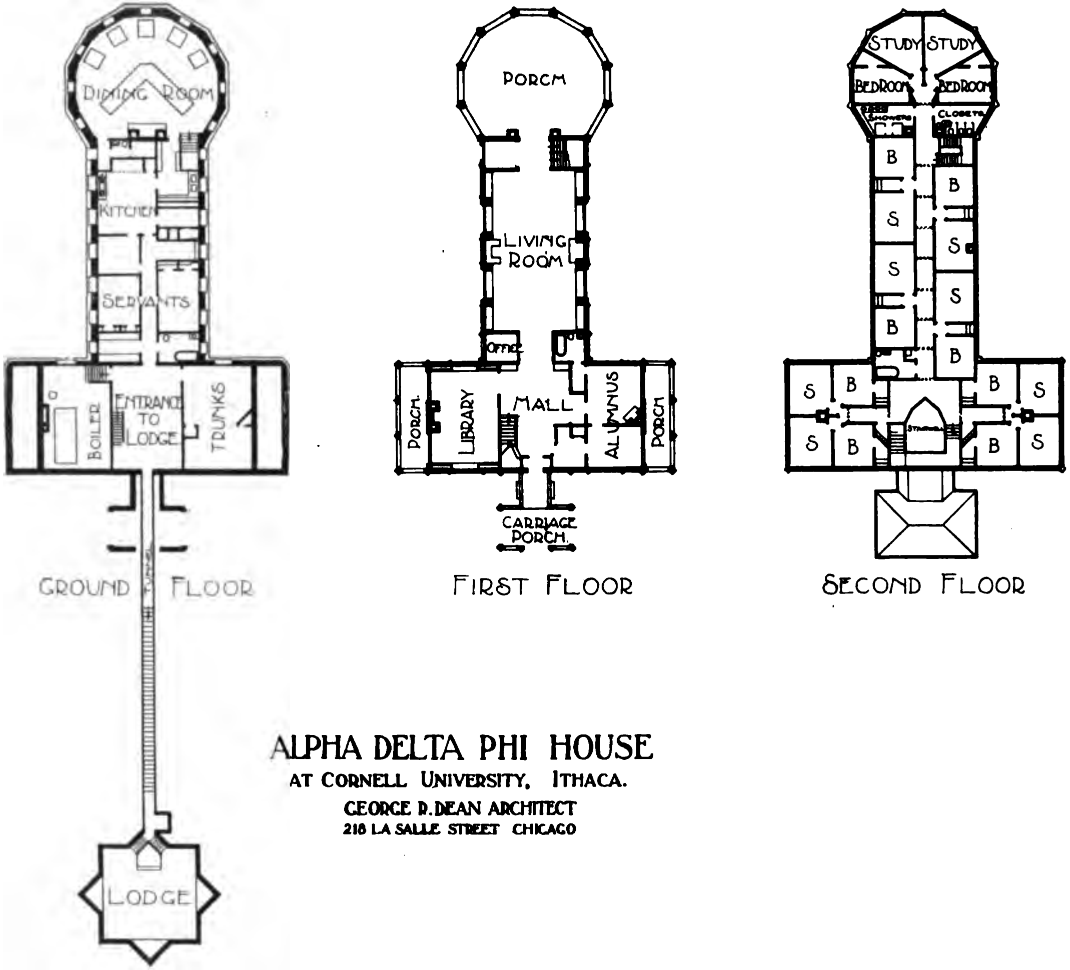
Bricks should be laid in full beds of mortar, filling end and side joints in one operation. This operation is simple and easy with skilful masons—if they will do it—but it requires persistence to get it accomplished. Masons have a habit of laying brick in a bed of mortar, leaving the vertical joints to take care of themselves, throwing a little mortar over the top beds and giving a sweep with the trowel which more or less disguises the open joint below. They also have a way after mortar has been sufficiently applied to the top bed of brick to draw the point of their trowel through it, making an open channel with only a sharp ridge of mortar on each side (and generally throwing some of it overboard), so that if the succeeding brick is



ALPHA DELTA PHI CHAPTER-HOUSE AT CORNELL UNIVERSITY, ITHACA, N. Y.

Dean & Dean, Architects, Chicago, Ill.

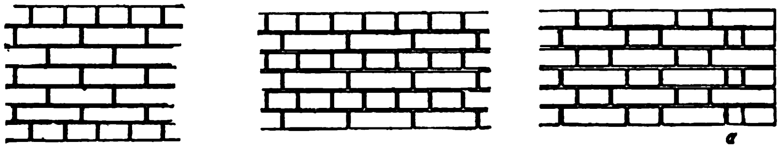
For Detail of Exterior, See Page 282. For Interiors, See Vol. II, Page 152, and Vol. IV, Pages 282 and 208. Built in 1903.



PLANS OF ALPHA DELTA PHI CHAPTER-HOUSE AT CORNELL UNIVERSITY, ITHACA, N. Y

Dean & Dean, Architects, Chicago, Ill.
Exterior Shown on Opposite Page.

taken up it will show a clear hollow, free from mortar through the bed. This enables them to bed the next brick with more facility and avoid pressure upon it to obtain the requisite thickness of joint.



Common Bond. English Bond. Flemish Bond.

Fig. 21. Bond Used in Brick Masonry.

With ordinary interior work a common practice is to lay brick with ½ and ¾-inch mortar joints; an inspector whose duty is to keep joints down to ¼ or ⅜ inch will not have an enviable task.

Neglect in wetting the brick before use is the cause of most of the failures of brickwork. Bricks have a great avidity for water, and if the mortar is stiff and the bricks dry, they will absorb the water so rapidly that the mortar will not set properly, and will crumble in the fingers when dry. Mortar is sometimes made so thin that the brick will not absorb all the water. This practice is objectionable; it interferes with the setting of the mortar, and particularly with the adhesion of the mortar to the brick. Watery mortar also contracts excessively in drying (if it ever does dry), which causes undue settlement and, possibly, cracks or distortion.

The bricks should not be wetted to the point of saturation, or they will be incapable of absorbing any of the moisture from the mortar, and the adhesion between the brick and mortar will be weak.

The common method of wetting brick by throwing water from buckets or spraying with a hose over a large pile is deceptive, the water reaches a few brick on one or more sides and escapes many. Immersion of the brick for from 3 to 8 minutes, depending upon its quality, is the only sure method to avert the evil consequences of using dry or partially wetted brick.

Strict attention must be paid to have the starting course level, for the brick being of equal thickness throughout, the slightest irregularity or incorrectness in it will be carried into the superposed courses, and can only be rectified by using a greater or less quantity of mortar in one part or another, a course which is injurious to the work.

A common but improper method of building thick brick walls is to lay up the outer stretcher courses between the header courses, and then to throw mortar into the trough thus formed, making it semi-fluid by the addition of a large dose of water, then throwing in the brick (bats, sand, and rubbish are often substituted for bricks), allowing them to find their own bearing; when the trough is filled it is plastered over with stiff mortar and the header course laid and the operation repeated This practice may have some advantage in celerity in executing work, but none in strength or security.

Amount of Mortar. The thickness of the mortar joints should be about ¼ to ⅜ of an inch. Thicker joints are very common, but should be avoided. If the bricks are even fairly good the mortar is the weaker part of the wall; hence the less mortar the better. Besides, a thin layer of mortar is stronger under compression than a thick one. The joints should be as thin as is consistent with their insuring a uniform bearing and allowing rapid work in spreading the mortar. The joints of outside walls should be thin in order to decrease the disintegration by weathering. The joints of inside walls are usually made from ⅜ to ½-inch thick.

The proportion of mortar to brick will vary with the size of the brick and with the thickness of the joint. With the standard brick (8¼ × 4 × 2¼ inches), the amount of mortar required will be as follows:

Thickness of Joints.	Mortar required.	
	Per Cubic Yard. Cubic Yards.	Per 1,000 Brick. Cubic Yards.
½ to ⅝ inch	0.30 to 0.40	0.80 to 0.90
¼ " ⅜ "	0.20 " 0.30	0.40 " 0.60
⅛ " "	0.10 " 0.15	0.15 " 0.20

Face or Pressed Brick Work. This term is applied to the facing of walls with better bricks and thinner joints than the backing. The bricks are pressed, of various colors, and are laid in colored mortar. The bricks are laid in close joints, usually ⅛-inch thick, and set with an imperceptible batter in themselves, which may not be seen when looking at the work direct, but which makes the joint a prominent feature and gives the work a good appearance. The brick of each course must be gauged with care and exactness, so that the joints may appear all alike. The bond used for the face of the wall is called the "running bond," the bricks are clipped on the back, and a binder placed transversely therein to bond the facing to the backing. The joints in the backing being thicker than those of the face work, it is only in every six or seven courses that they come to the same level, so as to permit headers being put in. This class of work requires careful watching to see that the binders or headers are put in; it frequently happens that the face work is laid up without having any bond with the backing.

In white-joint work the mortar is composed of white sand and fine lime putty. The mason when using this mortar spreads it carefully on the bed of the brick which is to be laid in such a way that when the brick is set the mortar will protrude about an inch from the face of the wall. When there are a number laid, and before the mortar becomes too hard, the mortar that protrudes is cut off flush with the wall, the joint struck downwards, and the upper and lower edges cut with a knife guided by a small straight edge. When the front is built, the whole is cleaned down with a solution of muriatic acid and water, not too strong, and sometimes oiled with linseed oil cut with turpentine, and applied with a flat brush. After the front is thoroughly cleaned with the muriatic acid solution, it should be washed with clean water to remove all remains of the acid.

When colored mortars are required, the lime and sand should be mixed at least 10 days before the colored pigments are added to it, and they should be well soaked in water before being added to the mortar.

BRICK MASONRY IMPERVIOUS TO WATER.

It sometimes becomes necessary to prevent the percolation of water through brick walls. A cheap and effective process has not yet been discovered, and many expensive trials have proved failures. Laying the bricks in asphaltic mortar and coating the walls with asphalt or coal tar are successful. "Sylvester's Process for Repelling Moisture from External Walls," has proved entirely successful. The process consists in using two washes for covering the surface of the walls, one composed of Castile soap and water, and one of alum and water. These solutions are applied alternately until the walls are made impervious to water.

EFFLORESCENCE.

Masonry, particularly in moist climates or damp places, is frequently disfigured by the formation of a white efflorescence on the surface. This deposit generally originates with the mortar. The water which is absorbed by the mortar dissolves the salts of soda, potash, magnesia, etc., contained in the lime or cement, and on evaporating deposits these salts as a white efflorescence on the surface. With lime mortar the deposit is frequently very heavy, and, usually, it is heavier with Rosendale than with Portland cement. The efflorescence sometimes originates in the brick, particularly if the brick was burned with sulphurous coal or was made from clay containing iron pyrites; and when the brick gets wet the water dissolves the sulphates of lime and magnesia, and on evaporating leaves the crystals of these salts on the surface. The crystallization of these salts within the pores of the mortar and of the brick or stone causes disintegration, and acts in many respects like frost.

The efflorescence may be entirely prevented by applying "Sylvester's" washes, composed of the same ingredients and applied in the same manner as for rendering masonry impervious to moisture. If can be much diminished by using impervious mortar for the face of the joints.

REPAIR OF MASONRY.

In effecting repairs in masonry, when new work is to be connected with old, the mortar of the old must be thoroughly cleaned off along the surface where the junction is to be made and the surface thoroughly wet. The bond and other arrangements will depend upon the circumstances of the case. The surfaces connected should be fitted as accurately as practicable, so that by using but little mortar no disunion may take place from settling.

As a rule, it is better that new work should butt against the old, either with a straight joint visible on the face, or let into a chase, sometimes called a "slip-joint," so that the straight joint may not show; but if it is necessary to bond them together the new work should be built in a quick-setting cement mortar and each part of it allowed to set before being loaded.

In pointing old masonry all the decayed mortar must be completely raked out with a hooked iron point and the surfaces well wetted before the fresh mortar is applied.

MASONRY STRUCTURES.

The component parts of masonry structures may be divided into several classes according to the efforts they sustain, their forms and dimensions depending on these efforts.

1. Those which sustain only their own weight, and are not liable to any cross strain upon the blocks of which they are composed, as the walls of enclosures.
2. Those which, besides their own weight, sustain a vertical pressure arising from a weight borne by them, as the walls of edifices, columns, the piers of arches, bridges, etc.
3. Those which sustain lateral pressures and cross strains, arising from the action of earth, water, frames, arches, etc.
4. Those which sustain a vertical upward or downward pressure, and a cross strain, as lintels, etc.
5. Those which transfer the pressure they directly receive to lateral points of support, as arches.

WALLS.

Walls are constructions of stone, brick, or other materials, and serve to retain earth or water, or in buildings to support the roof and floors and to keep out the weather. The following points should be attended to in the construction of walls:

The whole of the walling of a building should be carried up simultaneously; no part should be allowed to rise more than about 3 feet above the rest; otherwise the portion first built will settle down to its bearings before the other is attached to it, and then the settlement which takes place in the newer portion will cause a rupture, and cracks will appear in the structure. If it should be necessary to carry up one part of a wall before the other, the end of that portion first built should be *racked back*, that is, left in steps, each course projecting farther than the one above it.

Work should not be hurried along unless done in cement mortar, but given time to settle to its bearings.

Thickness of Walls. The thickness necessary to be given walls depends upon the height, length, and pressure of the load, wind, etc., and may be determined from that section of applied mechanics termed "Stability of Structures." In practice, however, these calculations are rarely made except for the most important structures, for the reason that if a vertical wall be properly constructed upon a sufficient foundation, the combined mass will retain its position, and bear pressure acting in the direction of gravity, to any extent that the ground on which it stands and the component materials will sustain. But pressure acting laterally has a tendency to overturn the wall, and therefore it must be the aim of the constructor to compel as far as possible, all forces that can act upon an upright wall to act in the direction of gravity.

In determining thickness of walls the following general principles must be recognized:

1. That the center of pressure (a vertical line through the center of gravity of the weight), shall pass through the center of the area of the foundation. If the axis of pressure does not coincide exactly with the axis of the base, the ground will yield most on the side which is pressed most; and as the ground yields, the base assumes an inclined position, and carries the lower part of the structure with it, producing cracks, if nothing more.

2. That the length of a wall is a source of weakness and that the thickness should be increased at least 4 inches for every 25 feet over 100 feet in length.

3. That high stories and clear spans exceeding 25 feet require thick walls.

4. That walls of warehouses and factories require a greater thickness than those used for dwellings or offices.

5. That walls containing openings to the extent of 33 per cent of the area should be increased in thickness.

6. That a wall should never be bonded into another wall either much heavier or lighter than itself.

In nearly all of the larger cities the minimum thickness of walls is prescribed by ordinance.

The accompanying table gives the more usual dimensions:

HEIGHT OF WALLS. Measured from the curb opposite center of building.	FOUNDATIONS.		OUTSIDE AND BEARING WALLS.	REMARKS.
	Stone.	Brick.		
Not exceeding 35 feet......	20″	16″	12″ in basement, 8″ above	8″ partition walls may be built not exceeding 50 ft. in height
35 feet and not over 50 ft..	20″	16″	12″ above foundation.	
50 feet and not over 60 ft..	24″	20″	16″ in first story if level with ground, 12″ above	The height in all cases to be taken to the nearest tier of floor beams.
60 feet and not over 75 ft..	24″	20″	16″ for 25 ft. and 12″ above	
75 feet and not over 85 ft..	28″	24″	20″ for 20 ft., 16″ to 60 ft., 12″ above.	Non-bearing walls may be 4″ less in thickness, but not less than 12″.
85 feet and not over 100 ft..	32″	28″	24″ for 35 ft., 20″ to 75 ft, 16″ above.	
100 feet and not over 115 ft..	36″	32″	28″ for 25 ft., 24″ to 50 ft., 20″ to 90 ft., 16″ above.	

Walls exceeding 115 feet in height to be increased at the bottom 4″ for every additional 25 ft. in height or part thereof, the upper 115 feet remaining the same as specified for walls of that height.

WALLS FOR WAREHOUSES.

Not exceeding 40 feet......	20″	16″	12 inches.	If there is to be a clear span of over 25 ft. between walls, the bearing walls shall be 4″ more in thickness than here specified for every 12½ ft. or fraction thereof, that said walls are more than 25 ft. apart.
40 feet and not over 60 ft..	24″	20″	16″ for 40 ft., 12″ above.	
60 feet and not over 75 ft..	28″	24″	20″ for 25 ft., 16″ above.	
75 feet and not over 85 ft..	32″	28″	24″ for 25 ft., 20″ to 60 ft., 16″ above.	
85 feet and not over 100 ft..	36″	32″	28″ for 25 ft., 24″ to 50 ft., 20″ to 75 ft., 16″ above.	

Walls exceeding 100 feet in height to be increased at the bottom 4″ for every additional 25 feet in height or part thereof, the upper 100 feet remaining the same as specified for walls of that height.

RETAINING WALLS.

A retaining wall is a wall built for the purpose of "retaining" or holding up earth or water. In engineering practice such walls attain frequently large proportions, being used in the construction of railroads, docks, waterworks, etc.

The form of cross-section varies considerably according to circumstances, and often according to the fancy of the designer. The more usual forms are shown in Figs. 22 to 25. The triangular section is the one which is theoretically the most economical, and the nearer that practical consideration will allow of its being conformed to the better.

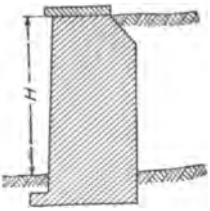


Fig. 22.

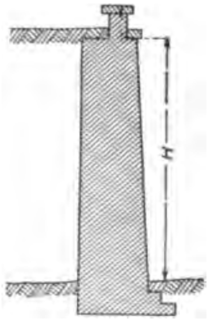


Fig. 23.

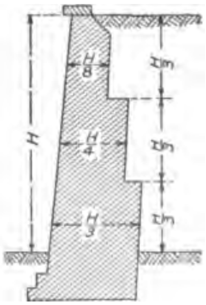


Fig. 24.

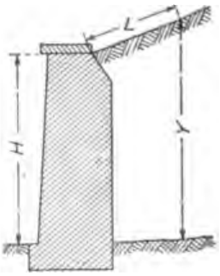


Fig. 25.

All other things being equal, the greater the face batter the greater will be the stability of the wall; but considerations connected with the functions of the wall limit the full application of this condition, and walls are usually constructed with only a moderate batter on the face, the diminution towards the top being obtained by a back batter worked out in a series of offsets. Walls so designed contain no more material and present greater resistance to overturning than walls with vertical backs.

Dry stone retaining walls are best suited for roads on account of their self-draining properties and their cheapness. If these dry walls are properly filled in behind with stones and chips, they are, if well constructed, seldom injured or overthrown by pressure from behind. If the stone is stratified with a flat cleavage, the construction of retaining and parapet walls is much facilitated. If the stone has no natural cleavage, great care is necessary to obtain a proper bond. If walls built of such stone are of coursed rubble, care is required that the masons do not sacrifice the strength of the walls to the face appearance. The practice of building walls with square or rectangular-faced stones, tailing off behind, laid in rows, one course upon the other, the rear portions of the walls being of chips and rough stones, set anyhow, cannot be condemned too strongly. Such a construction, which is very common, has little transverse and no longitudinal strength.

Little or no earth should be used for back filling if stone is available. Where earth filling is used, it should only be thrown in and left to settle itself; on no account should it be wetted and rammed.

Thickness of Walls. Retaining walls require a certain thickness to enable them to resist being overthrown by the thrust of the material which they sustain. The amount of this thrust depends upon the height of the mass to be supported and upon the quality of the material.

Surcharged Walls. A retaining wall is said to be surcharged when the bank it retains slopes backwards to a higher level than the top of the wall; the slope of the bank may be either equal to or less, but cannot be greater, than the angle of repose of the earth of the bank.

Proportions of Retaining Walls. In determining the proportions of retaining walls experience, rather than theory, must be our guide. The proportions will depend upon the character of the material to be retained. If the material be stratified rock with interposed beds of clay, earth, or sand, and if the strata incline toward the wall, it may require to be of far greater thickness than any ordinary retaining wall; because when the thin seams of earth become softened by infiltrating rain, they act as lubricants, like soap or tallow, to facilitate the sliding of the rock strata; and thus bring an enormous pressure against the wall. Or the rock may be set in motion by the action of frost on the clay seams. Even if there be no rock, still if the strata of soil dip toward the wall, there will always be danger of a similar result; and additional precautions must be adopted, especially when the strata reach to a much greater height than the wall.

The foundation of retaining walls should be particularly secure; the majority of failures which have occurred in such walls have been due to defective foundations.

Failure of Retaining Walls. Retaining walls generally fail (1) by overturning or by sliding, or (2) by bulging out of the body of the masonry. Sliding may be prevented by inclining the courses inward. An objection to this inclination of the joints in dry walls is that rainwater, falling on the battered face, is thereby carried inwards to the earth backing, which thus becomes soft and settles. This objection may be overcome by using mortar in the face joints to the depth of a foot, or by making the face of the wall nearly vertical.

Protection of Retaining Walls. The top of the walls should be protected with a coping of large heavy stones laid as headers.

Where springs occur behind or below the wall, they must be carried away by piping or otherwise got rid of.

The back of the wall should be left as rough as possible, so as to increase the friction of the earth against it.

Weep Holes. In masonry walls, weep holes must be left at frequent intervals, in very wet localities as close as 4 feet, so as to permit the free escape of any water which may find its way to the back of the wall. These holes should be about 2 inches wide and should be backed with some permeable material, such as gravel, broken stone, etc.

Formula for Calculating Thickness of Retaining Walls.

E = weight of earthwork per cubic yard.
W = weight of wall per cubic yard.
H = height of wall.
T = thickness of wall at top.
$T = H \times$ tabular number (Table 12).

TABLE 12.

Coefficients for Retaining Walls.

Batter of Wall.	E : W :: 1 : 5		E : W :: 1 : 4	
	Clay.	Sand.	Clay.	Sand.
1 in 4	.083	.029	.115	.054
1 in 5	.122	.065	.155	.092
1 in 6	.149	.092	.183	.118
1 in 8	.184	.125	.218	.153
1 in 12	.221	.160	.256	.189
Vertical	.300	.239	336	.267

Retaining walls of dry stone should not be less than 3 feet thick at top, with a face batter of 1 in 4 and back perpendicular, the courses laid perpendicular to the face batter. Weep holes are unnecessary unless the walls are in very wet situations.

Retaining walls of masonry should be at least 2 feet thick at top, back perpendicular and face battered at the rate of 1 in 6.

Surcharged Walls. In calculating the strength of surcharged walls substitute Y for H, Y being the perpendicular at the end of a line, L = H measured along the slope to be retained (Fig. 26).

Y = 1.71H in slopes of 1 : 1;
= 1.55H " " " 1½ : 1;
= 1.35H " " " 2 : 1;
= 1.31H " " " 3 : 1;
= 1.24H " " " 4 : 1.

DESCRIPTION OF ARCHES.

Basket-Handle Arch : One in which the intrados resembles a semi-ellipse, but is composed of arcs of circles tangent to each other

Circular Arch : One in which the intrados is a part of a circle.

Discharging Arch : An arch built above a lintel to take the superincumbent pressure therefrom.

Elliptical Arch : One in which the intrados is a part of an ellipse.

Geostatic Arch : An arch in equilibrium under the vertical pressure of an earth embankment.

Hydrostatic Arch : An arch in equilibrium under the vertical pressure of water

Inverted Arches are like ordinary arches, but are built with the crown downwards. They are generally semicircular or segmental in section, and are used chiefly in connection with foundations.

Plain or Rough Arches are those in which none of the bricks are cut to fit the splay. Hence the joints are quite close to each other at the soffit, and wider towards the outer curve of the arch; they are generally used as *relieving and trimer arches*, *for tunnel lining,* and all arches where strength is essential and appearance no particular object. In constructing arches of this kind it is usual to form them of two or more four-inch concentric rings until the required thickness is obtained. Each of the successive rings is built independently, having no connection with the others beyond the adhesion of the mortar in the ring joint. It is necessary that each ring should be finished before the next is commenced; also that each course be bonded throughout the length of the arch, and that the ring joint should be of a regular thickness. For if one ring is built with a thin joint and another with a thick one the one having the most mortar will shrink, causing a fracture and depriving the arch of much of its strength.

Pointed Arch : One in which the intrados consists of two arcs of equal circles intersecting over the middle of the span.

Relieving Arch : See Discharging Arch.

Right Arch : A cylindrical arch either circular or elliptical, terminated by two planes, termed *heads* of the arch, at right angles to the axis of the arch.

Segmental Arch: One whose intrados is less than a semicircle.

Semicircular Arch : One whose intrados is a semicircle; also called a *full-centered* arch.

Skew Arch : One whose heads are oblique to the axis. Skew arches are quite common in Europe, but are rarely employed in the United States; and in the latter when an oblique arch is employed it is usually made, not after the European method with spiral joints, but by building a number of short right arches or ribs in contact with each other, each successive rib being placed a little to one side of its neighbor.

DEFINITIONS OF PARTS OF ARCHES.

Abutment : The outer wall that supports the arch, and which connects it to the adjacent banks.

Arch Sheeting: The voussoirs which do not show at the end of the arch.

Camber is a slight rise of an arch, as $\frac{1}{8}$ to $\frac{1}{4}$ inch per foot of span.

Crown: The highest point of the arch.

Extrados: The upper and outer surface of the arch.

Haunches: The sides of the arch from the springing line half way up to the crown.

Heading Joint: A joint in a plane at right angles to the axis of the arch. It is not continuous.

Intrados or Soffit: The under or lower surface of the arch.

Invert: An inverted arch, one with its intrados below the axis or springing line; *e.g.*, the lower half of a circular sewer.

Keystone: The center voussoir at the crown.

Length: The distance between face stones of the arch.

Pier: The intermediate support for two or more arches.

Ring Course: A course parallel to the face of the arch.

Ring Stones: The voussoirs or arch stones which show at the ends of the arch.

Rise: The height from the springing line to under side of the arch at the keystone.

Skew Back: The upper surface of an abutment or pier from which an arch springs; its face is on a line radiating from the center of the arch.

Span: The horizontal distance from springing to springing of the arch.

Spandrel: The space contained between a horizontal line drawn through the crown of the arch and a vertical line drawn through the upper end of the skew back.

Springing: The point from which the arch begins or springs.

Springer: The lowest voussoir or arch stone.

String Course: A course of voussoirs extending from one end of the arch to the other.

Voussoirs: The blocks forming the arch.

Arches: The arch is a combination of wedge-shaped blocks, termed arch stones, or voussoirs, truncated towards the angle of the wedges by a curved surface which is usually normal to the surfaces of the joints between the blocks. This inferior surface of the arch is termed the soffit. The upper or outer surface of the arch is termed the back.

The extreme blocks of the arch rest against lateral supports, termed abutments, which sustain both the vertical pressure arising from the weight of arch stones, and the weight of whatever lies upon them; also the lateral pressure caused by the action of the arch.

The *forms* of an arch may be the semicircle, the segment, or a compound curve formed of a number of circular curves of different radii. Full center arches, or entire semicircles, offer the advantages of simplicity of form, great strength, and small lateral thrust; but if the span is large they require a correspondingly great rise, which is often objectionable. The flat or segmental arch enables us to reduce the rise, but it throws a great lateral strain upon the abutments. The compound curve gives, when properly proportioned, a strong arch with a moderate lateral action, is easily adjustable to different ratios between the span and the rise, and is unsurpassed in its general appearance. In striking the compound curve, the following conditions are to be observed: The tangents at the springing must be vertical, the tangent at the crown horizontal, and the number of centers must be uneven, curves of 3 and 5 centers will be found to fulfil all requirements.

In designing an arch the first step is to determine the thickness at the crown, *i.e.*, the depth of the keystone. This depth depends upon the form, and rise of the arch, the character of the masonry, and the quality of the stone; and is usually determined by Trautwine's formula, which is as follows for a first-class cut stone arch whether circular or elliptical.

$$D = \frac{\sqrt{R + \frac{1}{2} S}}{4} + 0.2.$$

in which

D = the depth at the crown in feet.
R = the radius of curvature of the intrados in feet.
S = the span in feet.

For second-class work, the depth found by this formula may be increased about one-eighth part; and for *brickwork* or *fair rubble*, about one-third.

Table 13 gives the depth of keystone for semicircular arches, the second column being for hammer-dressed beds, the third for beds roughly dressed with the chisel, and the fourth for brick masonry.

TABLE 13.

Span in feet.	Thickness of Arch in inches.		
	First-class Masonry.	Second-class Masonry	Brick Masonry.
6	12	15	12
8	13	16	16
10	14	17	20
12	15	19	20
14	16	20	24
16	17	21	24
18	18	23	24
20	19	24	24
25	20	25	28
30	21	26	28
35	22	28	28
40	23	29	32
45	24	30	32
50	25	31	32

Thickness of Arch at the Springing. Generally the thickness of the arch at the springing is found by an application of theory.

If the loads are vertical, the horizontal component of the compression on the arch is constant; and hence, to have the mean pressure on the joints uniform, the vertical projection of the joints should be constant. This principle leads to the following formula:

The length measured radially of each joint between the joint of rupture and the crown should be such that its vertical projection is equal to the depth of the keystone.

The length of the joint of rupture, *i.e.*, the thickness of the arch at the practical springing line, can be computed by the formula

$$z = d \sec a$$

in which z is the length of the joint,
d the depth of the crown,
a the angle the joint makes with the vertical.

The following are the values for circular and segmental arches:

$$\text{If } \frac{R}{S} > \frac{1}{4},\ l = 2.00\,d$$
$$\text{“ } \frac{R}{S} = \frac{1}{6},\ l = 1.40\,d$$
$$\text{“ } \frac{R}{S} = \frac{1}{8},\ l = 1.24\,d$$
$$\text{“ } \frac{R}{S} = \frac{1}{10},\ l = 1.18\,d$$
$$\text{“ } \frac{R}{S} = \frac{1}{12},\ l = 1.10\,d$$

in which R = the rise, in feet
S = the span, in feet.

Thickness of the Abutments. The thickness of the abutment is determined by the following formula:

$$t = 0.2\,p + 0.1\,R + 2.0$$

in which t is the thickness of the abutment at the springing, p the radius, and R the rise—all in feet.

The above formula applies equally to the smallest culvert or the largest bridge—whether circular or elliptical, and whatever the proportions of rise and span—and to any height of abutment.

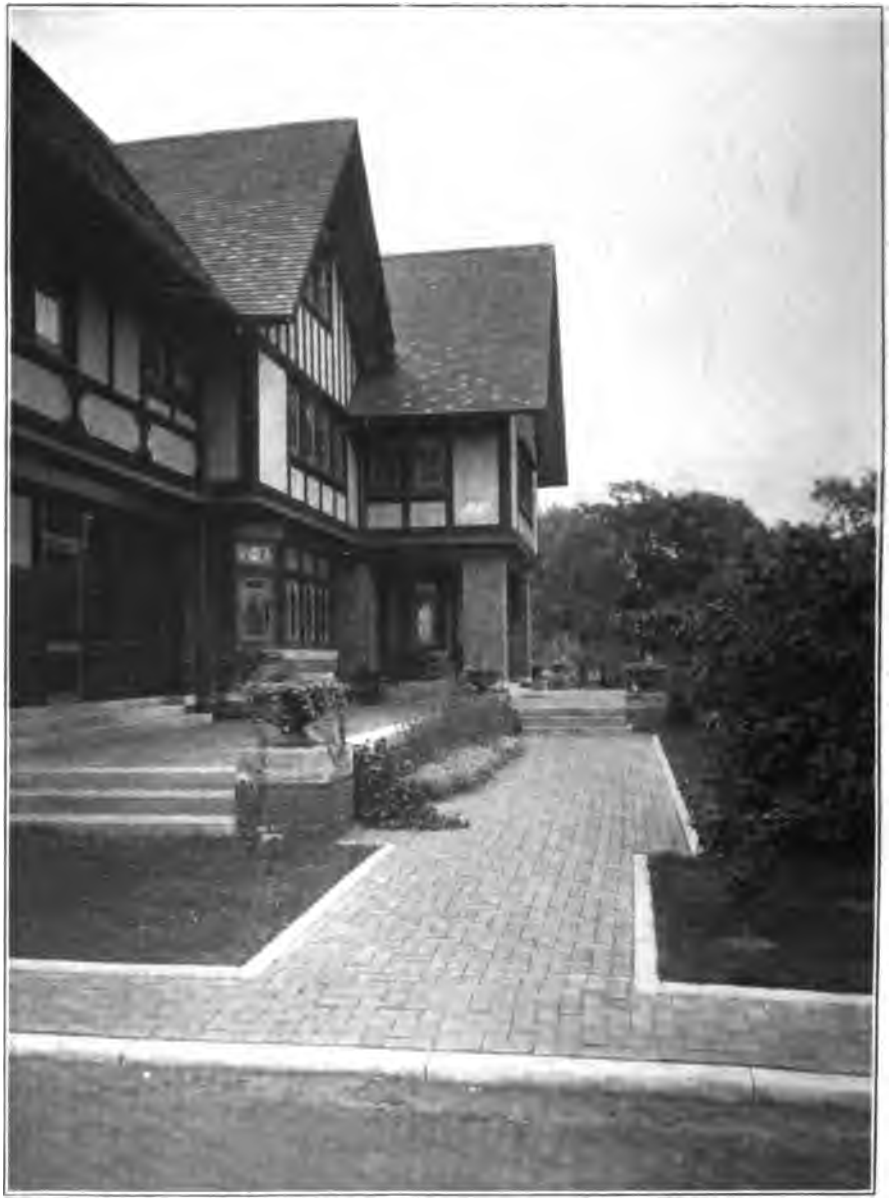
Table 14 gives the minimum thickness of abutments for arches of 120 degrees where the depth of crown does not exceed 3 feet.

Calculated from the formula

$$T = \sqrt{6R + \left(\frac{3R}{2H}\right)^2} - \frac{3R}{2H},$$

in which D = depth or thickness of crown in feet;
H = height of abutment to springing in feet;
R = radius of arch at crown in feet;
T = thickness of abutment in feet.

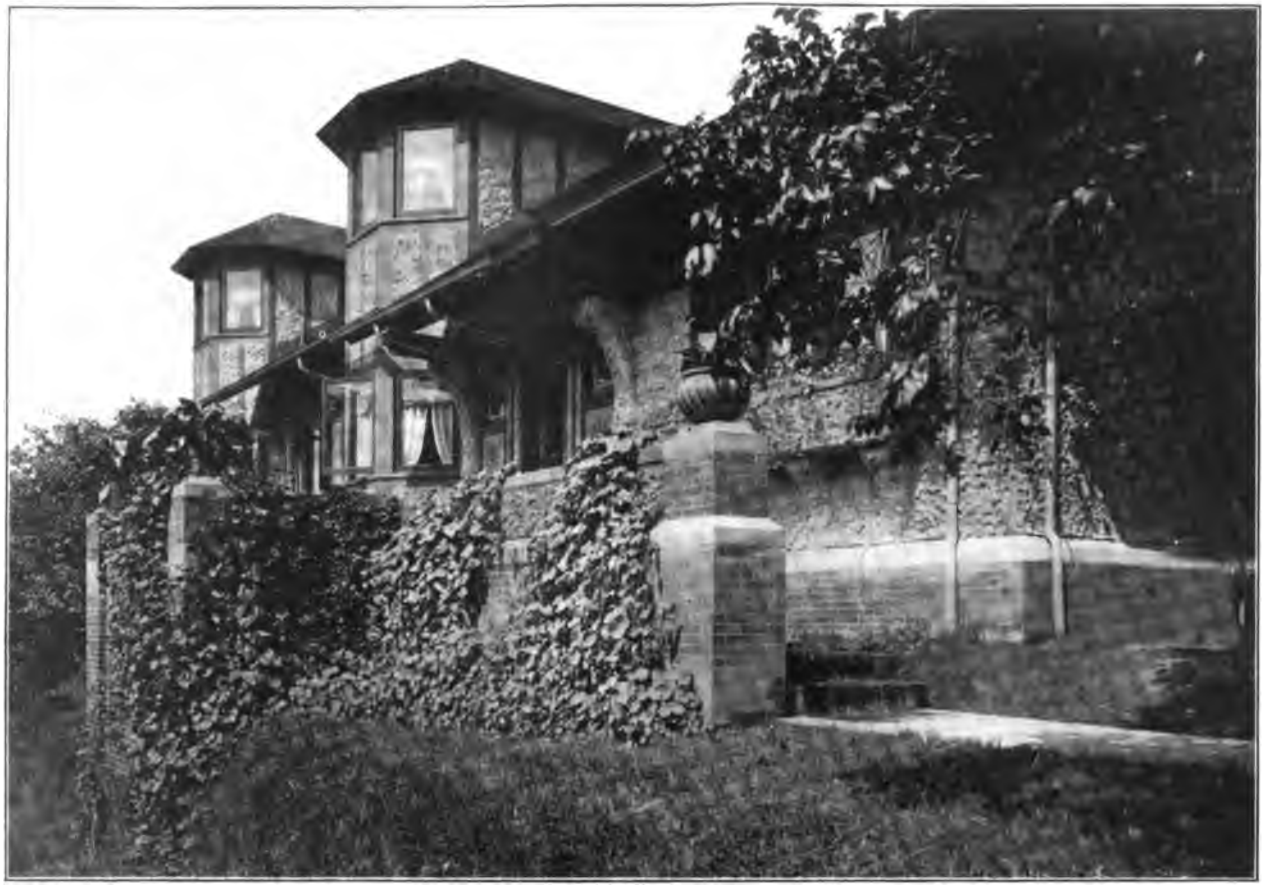
Arches fail by the crown falling inward, and thrusting outward the lower portions, presenting five points of rupture, one at the keystone, one on each side of it which limit the portions that fall inward, and one on each side near the springing lines which limit the parts thrust outward. In pointed arches, or those in which the rise is greater than half the span, the tending to yielding is, in some cases, different; and thrust upward and outward the parts near the crown.



HOUSE AT CANTON, ILL.

R. C. Spencer, Jr., Architect, Chicago, Ill

Paving Brick Walls; Stud Frame Above, Plaster on Metal Lath, Paneled by Undressed and Stained Boards; Red Shingle Tile Roof.



HOUSE AT PARKERSBURG, W. VA.

R. C. Spencer, Jr., Architect, Chicago, Ill.

Red Brick to Water Table. Rough-Cast Plaster on Metal Lath on Stud Frame above Water Table. Woodwork is Undressed. Stained Shingle Roofs.

TABLE 14.

Minimum Thickness of Abutments for Arches of 120 Degrees Where the Depth of Crown Does Not Exceed 3 Feet.

Span of Arch.	Height of Abutment to Springing, in feet.				
	5	7.5	10	20	30
8 feet	3.7	4.2	4.3	4.6	4.7
9 "	3.9	4.4	4.6	4.9	5.0
10 "	4.2	4.6	4.8	5.1	5.2
12 "	4.5	4.7	5.2	5.6	5.7
14 "	4.7	5.2	5.5	6.0	6.1
16 "	4.9	5.5	5.8	6.4	6.5
18 "	5.1	5.8	6.1	6.7	6.9
20 "	5.3	6.0	6.4	7.1	7.2
22 "	5.5	6.2	6.6	7.3	7.6
24 "	5.6	6.4	6.9	7.6	7.9
30 "	6.0	7.0	7.5	8.4	8.8
40 "	6.5	7.7	8.4	9.6	10.0
50 "	6.9	8.2	9.1	10.5	11.1
60 "	7.2	8.7	9.7	11.4	12.0
70 "	7.4	9.1	10.2	11.8	12.9
80 "	7.6	9.4	10.6	12.8	13.6
90 "	7.8	9.7	11.0	13.4	14.3
100 "	7.9	10.0	11.4	14.0	15.0

NOTE. The thickness of abutment for a semicircular arch may be taken from the above table by considering it as approximately equal to that for an arch of 120 degrees having the same radius of curvature; therefore by dividing the span of the semicircular arch by 1.155 it will give the span of the 120-degree arch requiring the same thickness of abutment.

The angle which a line drawn from the center of the arch to the joint of rupture makes with a vertical line is called the *angle of rupture.* This term is also used when the arch is stable, or where there is no joint of rupture, in which case it refers to that point about which there is the greatest tendency to rotate. It may also be defined as including that portion of the arch near the crown which will cause the greatest *thrust* or horizontal pressure at the crown. This thrust tends to crush the *voussoirs* at the crown, and also to overturn the abutments about some outer joint. In very thick arches rupture may take place from *slipping* of the joints.

In order to avoid any tendency of the joints to open, the arch should be so designed that the actual resistance line shall everywhere be within the middle third of the depth of the arch ring.

In general the design of an arch is reached by a series of approximations. Thus, a form of arch and spandrel must be assumed in advance in order to find their common center of gravity for the purpose of determining the horizontal thrust at the crown, and the reaction at the skewback.

Backing. The backing is masonry of inferior quality or concrete, laid outside and above the arch stones proper, to give additional security. Ordinarily, the backing has a zero thickness at or near the crown, and gradually increases to the upringing line.

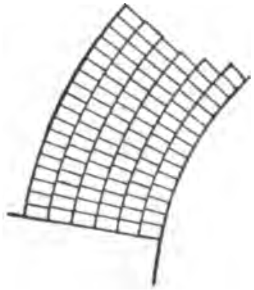


Fig. 26. Rowlock Bond.

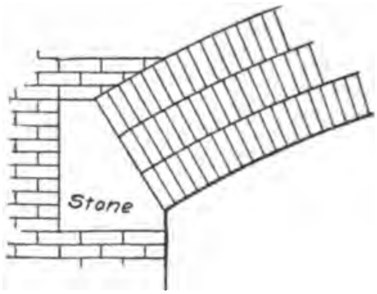


Fig. 27. Rowlock with Skewback.

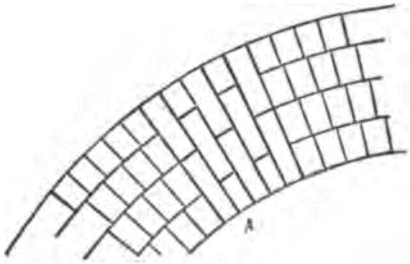


Fig. 28. Block in Course Bond.

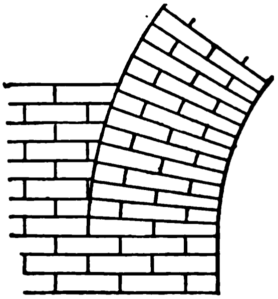


Fig. 29. Header and Stretcher.

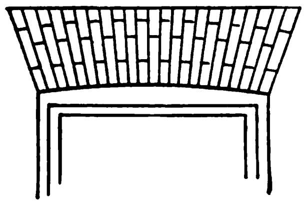


Fig. 30. Flat Arch.

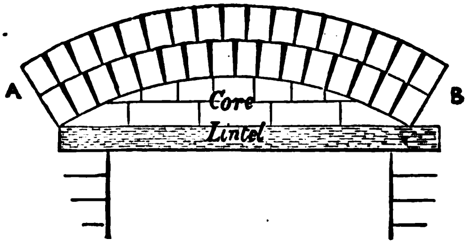


Fig. 31. Relieving Arch.

Spandrel Filling. Since the surface of the roadway must not deviate from a horizontal line, a considerable quantity of material is required above the backing to bring the roadway level. Ordinarily this space is filled with earth, gravel, broken stone, cinders, etc. Sometimes to save filling small arches are built over the haunches of the main arch.

Drainage. The drainage of arch bridges of more than one span is generally effected by giving the top surface of the backing a slight inclination from each side toward the center of the width of the bridge and also from the center toward the end of the span. The water is thus collected over the piers, from whence it is discharged through pipes laid in the masonry.

To prevent leakage through the backing and through the arch sheeting, the top of the former should be covered with a layer of puddle, or plastered with a coat of cement mortar, or painted with coal tar or asphaltum.

Brick Arches. The only matter requiring special mention in connection with brick arches is the bond to be employed. When the thickness of the arch exceeds a brick and a half, the bond from the soffit outward is a very important matter. There are three principal methods employed in bonding brick arches: (1) The arch may be built in concentric rings; *i.e.*, all the brick may be laid as stretchers, with only the tenacity of the mortar to unite the several rings. This method is called *rowlock bond*: (2) Part of the brick may be laid as stretchers and part as headers, by thickening the outer ends of the joints—either by using more mortar or by driving in thin pieces of slate, so that there shall be the same number of brick in each ring. This form of construction is called *header and stretcher bond:* (3) *Block in course bond* is formed by dividing the arch into sections similar in shape to the voussoirs of stone arches, and laying the brick in each section with any desired bond.

Skewback. In brick arches of large span a stone skewback is used for the arch to spring from. The stone should be cut so as to bond into the abutment, and the springing surface should be cut to a true plane, radiating from the center from which the arch is struck.

Flat Arches are often built over door or window openings; they are always liable to settle and should be supported by an angle bar, the vertical flange of which may be concealed behind the arch.

Relieving Arches. This term is applied to arches turned over openings in walls to support the wall above; beams called lintels are usually used in connection with this type of arch, the lintel should not have a bearing on the wall of more than 4 inches, and the arch should spring from beyond the ends of the lintel as shown in A, Fig. 31, and not as at B.

CONSTRUCTION OF ARCHES.

In constructing ornamental arches of small span the bricks should be cut and rubbed with great care to the proper splay or wedge like form necessary, and according to the gauges or regularly measured dimensions.

This is not always done, the external course only being rubbed, so that the work may have a pleasing appearance to the eye, while the interior, which is hidden from view, is slurred over, and in order to save time many of the interior bricks are apt to be so cut away as to deprive the arch of its strength. This class of work produces cracks and causes the arch to bulge forward, and may cause one of the bricks of a straight arch to drop down lower than the soffit.

In setting arches the mason should be sure that the centers are set *level* and *plumb*, that the arch brick or stone may rest upon them *square*. When the brick or stone are properly cut beforehand the courses can be gauged upon the center from the key downwards. The soffit of each course should fit the center perfectly.

The mortar joints should be as thin as possible and well flushed up.

In setting the face stones it is necessary to have a radius line, and draw it up and test the setting of each stone as it is laid.

The framing, setting up, and striking of the centers are very important parts of the construction of any arch, particularly one of long span. A change in the shape of the center, due to insufficient strength or improper bracing, will be followed by a change in the curve of the intrados, and consequently of the line of resistance, which may endanger the safety of the arch itself.

CENTERING FOR ARCHES.

No arch becomes self-supporting until keyed up, that is, until the crown or keystone course is laid. Until that time the arch ring, which should be built up simultaneously from both abutments, has to be supported by frames called centers. These consist of a series of ribs placed from 3 to 6 or more feet apart, supported from below. The upper surface of these ribs is cut to the form of the arch, and over these a series of planks called *laggings* are placed, upon which the arch stones directly rest. The ribs may be of timber or iron. They should be strong and stiff. Any deformation that occurs in the rib will distort the arch, and may even result in its collapse.

Striking the Center. The ends of the ribs or center frames usually rest upon a timber lying parallel to, and near, the springing line of the arch. This timber is supported by wedges, preferably of hardwood, resting upon a second stick, which is in turn supported by wooden posts, usually one under each end of each rib. The wedges between the two timbers, as above, are used in removing the center after the arch is completed, and are known as *striking wedges.* They consist of a pair of folding wedges, 1 to 2 feet long, 6 inches wide, and having a slope of from 1 to 5 to 1 to 10, placed under each end of each rib. It is necessary to remove the centers slowly, particularly for large arches; and hence the striking wedges should have a very slight taper, the larger the span the smaller the taper.

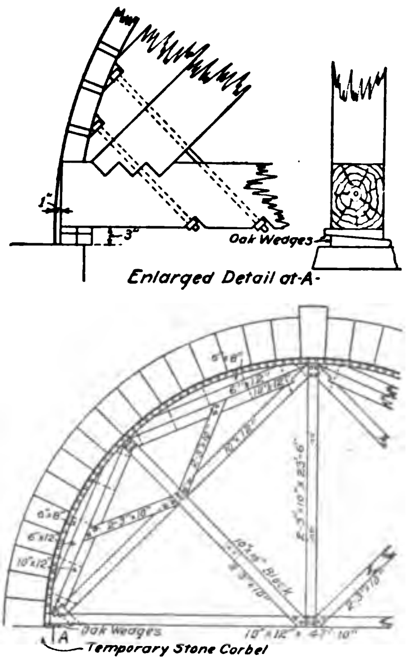


Fig. 32. Arch Center.

The center is lowered by driving back the wedges. To lower the center uniformly the wedges must be driven back uniformly. This is most easily accomplished by making a mark on the side of each pair of wedges before commencing to drive, and then *moving* each the same amount.

The inclined surfaces of the wedges should be lubricated when the center is set up, so as to facilitate the striking.

Screws may be used instead of wedges for lowering centers.

Sand is also employed for the same purpose. The method followed is to support the center frames by wooden pistons or plungers resting on sand confined in plate-iron cylinders. Near the bottom of each cylinder there is a plug which can be withdrawn and replaced at pleasure, thus regulating the outflow of the sand and the descent of the center.

There is great difference of opinion as to the proper time for striking centers. Some hold that the center should be struck as soon as the arch is completed and the spandrel filling is in place; while others contend that the mortar should be given time to harden. It is probably best to slacken the centers as soon as the keystone course is in place, so as to bring all the joints under pressure. The length of time which should elapse before the centers are finally removed should vary with the kind of mortar employed and also with its amount. In brick and rubble arches a large proportion of the arch ring consists of mortar, and if the center is removed too soon the compression of this mortar might cause a serious or even dangerous deformation of the arch. Hence the centers of such arches should remain until the mortar has not only set, but has attained a considerable part of its ultimate strength.

Frequently the centers of bridge arches are not removed for three or four months after the arch is completed, but usually the centers for the arches of tunnels, sewers, and culverts are removed as soon as the arch is turned and, say, half of the spandrel filling is in place.

BRIDGE ABUTMENTS.

Form. There are four forms of abutment in use, they are named according to their form as the *straight* abutment, the *wing* abutment, the U abutment and the T abutment.

The form to be adopted for any particular case will depend upon the location—whether the banks are low and flat, or steep and rocky, whether the current is swift or slow, and also upon the relative cost of earthwork and masonry. Where a river acts dangerously upon a shore, wing walls will be necessary. These wings may be curved or straight. The slope of the wings may be finished with an inclined coping, or offset at each course. Wing walls subjected to

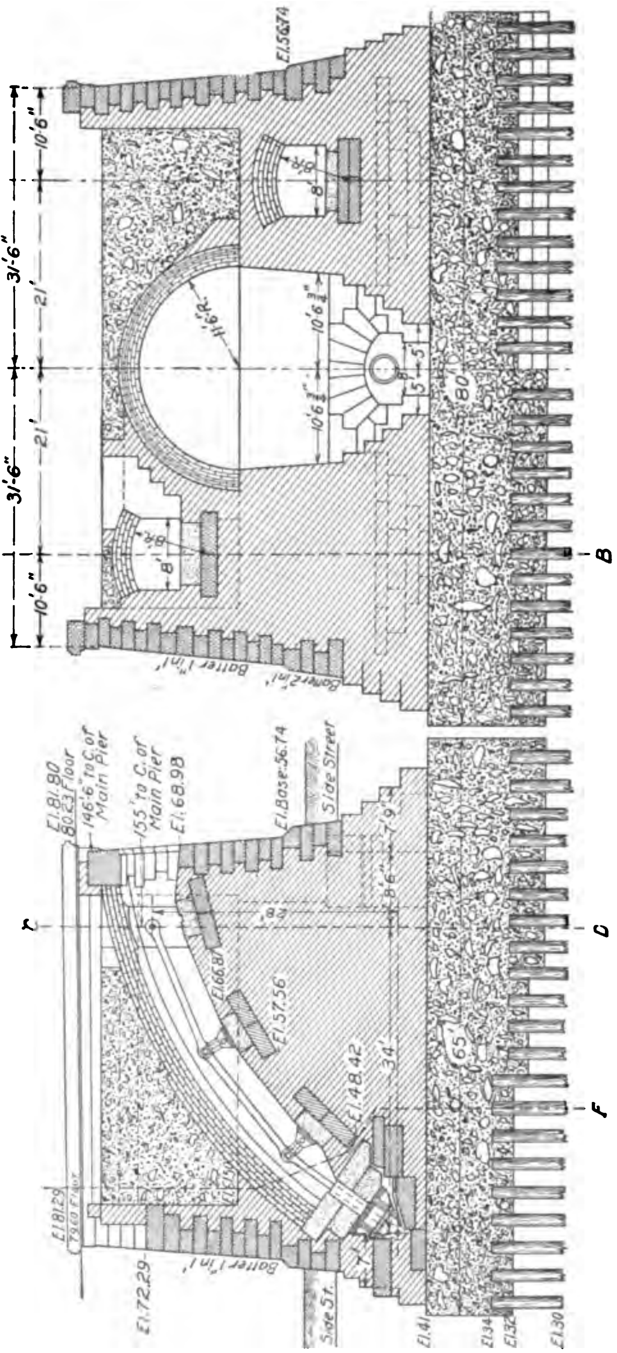
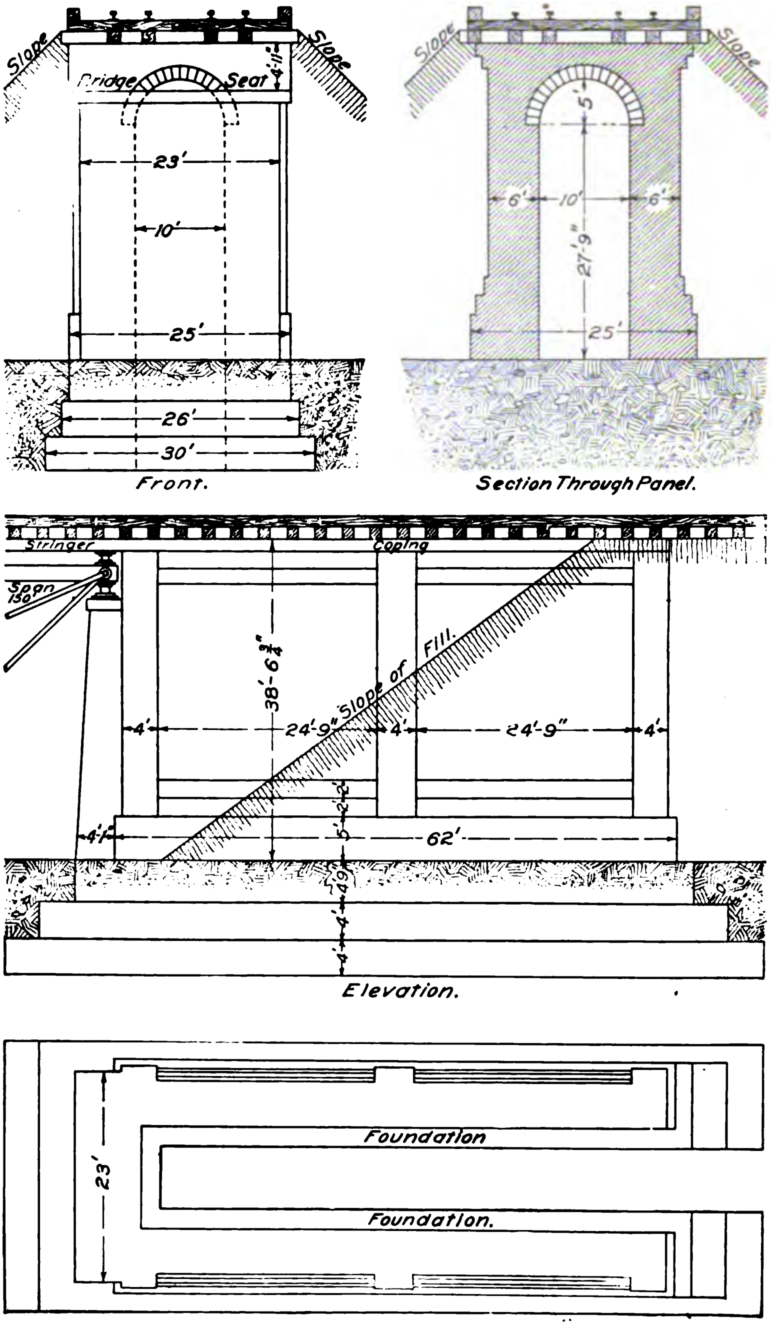


Fig. 83. Bridge Abutment and Anchorage.



Front.

Section Through Panel.

Elevation.

Plan

Fig. 34. Abutment for Railroad Bridge.

special strains, or to particular currents of water require positions and forms accordingly.

The abutment of a bridge has two offices to perform; (1) to support one end of the bridge, and (2) to keep the earth embankment from sliding into the water.

The abutment may fail (1) by sliding forward, (2) by bulging, or (3) by crushing.

The dimensions of abutments will vary with each case, with the form and size of the bridge and with the pressure to be sustained; the dimensions may be determined by the same formulas as used for retaining walls.

For railroad bridges the top dimensions are usually 5 feet wide by 20 feet long. The usual batter is 1 in 12, for heights under 20 feet the top dimensions and the batter determine the thickness at the bottom. For greater heights, the uniform rule is to make the thickness four-tenths the height.

Bridge abutments are built of first or second-class masonry or of concrete alone or faced with stone masonry, according to the importance and location of the structure.

BRIDGE PIERS.

The thickness of a pier for simply supporting the weight of the superstructure need be but very little at the top, care being taken to secure a sufficient bearing at the foundation. Piers should be thick enough, however, to resist shocks and lateral strains, not only from a passing load, but from floating ice and ice jams; and in rivers where a sandy bottom is liable to deep scouring, so that the bottom may work out much deeper on one side of a pier than on the other, regard should be paid to the lateral pressure thus thrown on the pier. For mere bearing purposes the following widths are ample for first-class masonry—span 50 feet, width 4 feet, span 200 feet, width 7 feet. Theoretically the dimensions at the bottom are determined by the area necessary for stability; but the top dimensions required for the bridge seat, together with the batter, 1 in 12 or 1 in 24, generally make the dimensions of the base sufficient for stability.

The up-stream end of a pier, and to a considerable extent the down-stream end also, should be rounded or pointed to serve as a cutwater to turn the current aside and to prevent the formation of whirls which act upon the bed of the stream around the foundation, and also to form a fender to protect the pier proper from being damaged by ice, tugs, boats, etc. This rounding or pointing is designated by the name *starling*, the best form appears to be a semi-ellipse. The distance to which they should extend from the pier depends upon local circumstances.

A bridge pier may fail in any one of these ways; (1) by sliding on any section on account of the action of the wind against the exposed part of the pier; (2) by overturning at any section where the moment of the horizontal forces above the section exceeds the moment of the weight of the section; or (3) by crushing at any section under the combined weight of the pier, the bridge and the load. Bridge piers are usually constructed of quarry-faced ashlar backed with

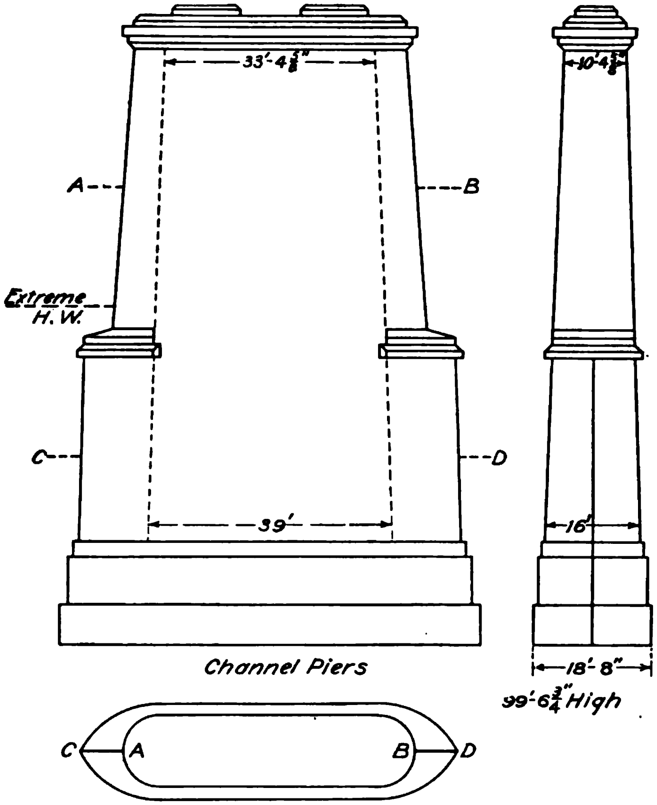


Fig. 35. Type of Bridge Pier.

rubble or concrete. Occasionally, for economy, piers, particularly pivot-piers, are built hollow—sometimes with and sometimes without cross walls.

CULVERTS.

Culverts are employed for conveying under a railroad, highway, or canal the small streams crossed. They may be of stone, brick, concrete, earthenware, or iron pipe or any of these in combination. Two general forms of masonry culverts are in use, the *box* and the *arch.*

Box Culverts. The box consists of vertical side walls of masonry with flagstones on top extending from one wall to another.

The foundation consists of large stones and the side walls may be laid dry or in mortar.

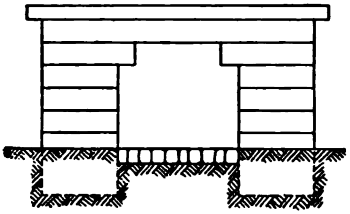


Fig. 36. End Elevation.

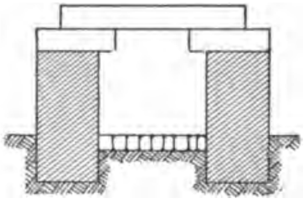


Fig. 37. Section AB.

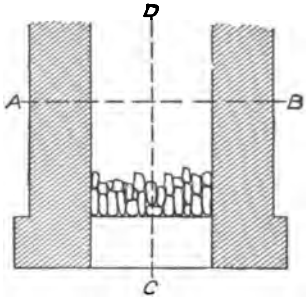


Fig. 38. Plan.

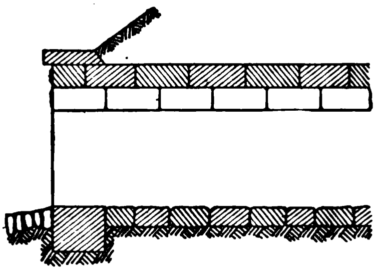


Fig. 39. Section CD.

Types of Box Culverts.

The paving should be laid independent of the walls and should be set in cement mortar. The end walls are finished either with a plain wall perpendicular to the axis of the culvert and may be stepped, or provided with wing walls as the circumstances of each case may require.

The thickness of the cover stone may be determined by considering it as a beam supported at the ends and loaded uniformly.

Figs. 36 to 39 show the form of this class of culverts and the dimensions given in Table 15 will serve as an approximate guide for general use.

TABLE 15.

Dimensions for Box Culverts.

Area.	Opening.	Side Wall.	Depth of Cover.	Length of Cover.
4 feet	2′ × 2′	2′ × 2′	12 inches	5 feet
9 "	3 × 3	3 × 2½	16 "	6 "
16 "	4 × 4	4 × 3	20 "	7 "
25 "	5 × 5	5 × 3½	22 "	8 "
36 "	6 × 6	6 × 4	24 "	9 "

Arch Culverts. The dimensions of arch culverts are determined in the manner described herein under arches, attention, however, being given to the following points:

Wing Walls. There are three common ways of arranging the wing walls at the end of arch culverts: (1) The culvert is finished with straight walls at right angles to the axis of the culvert. (2) The wings are placed at an angle of 30 degrees with the axis of the culvert. (3) The wing walls are built parallel to the axis of the culvert, the back of the wing and the abutment being in a straight line and the only splay being derived from thinning the wings at their outer edge. The most economical and better form for hydraulic considerations is the second form.

Designing Culverts. In the design of culverts care is required to provide an ample way for the water to be passed. If the culvert is too small, it is liable to cause a washout, entailing interruption of traffic and cost of repairs, and possibly may cause accidents that will require the payment of large sums for damages. On the other hand, if the culvert is made unnecessarily large, the cost of construction is needlessly increased.

The area of waterway required, depends (1) upon the rate of rainfall; (2) the kind and condition of the soil; (3) the character and inclination of the surface; (4) the condition and inclination of the bed of the stream; (5) the shape of the area to be drained, and the position of the branches of the stream; (6) the form of the mouth and the inclination of the bed of the culvert; and (7) whether it is permissible to back the water up above the culvert, thereby causing it to discharge under a head.

(1) The maximum rainfall as shown by statistics is about one

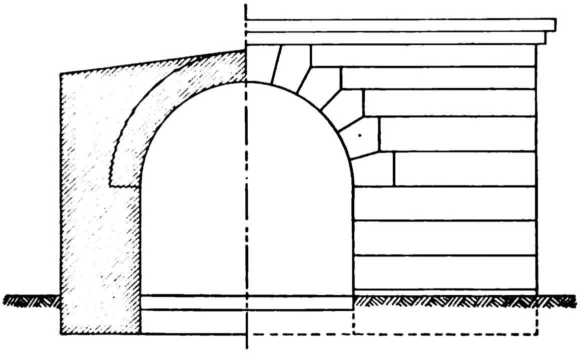


Fig. 40. Sectional Elevation.

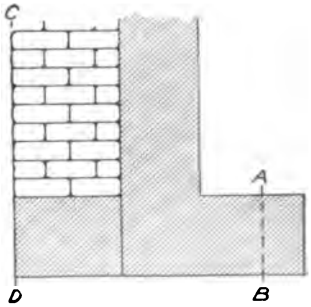


Fig. 41. Plan.

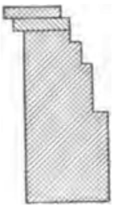


Fig. 42. Section AB.

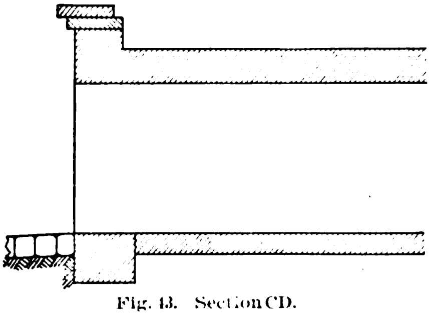


Fig. 43. Section CD.

Type of Arch Culvert.

inch per hour (except during heavy storms), equal to 3,630 cubic feet per acre. Owing to various causes, not more than 50 to 75 per cent of this amount will reach the culvert within the same hour.

Inches of rainfall × 3,630 = cubic feet per acre.

Inches of rainfall × 2,323,200 = cubic feet per square mile.

(2) The amount of water to be drained off will depend upon the permeability of the surface of the ground, which will vary greatly with the kind of soil, the degree of saturation, the condition of the cultivation, the amount of vegetation, etc.

(3) The rapidity with which the water will reach the watercourse depends upon whether the surface is rough or smooth, steep or flat, barren or covered with vegetation, etc.

(4) The rapidity with which the water will reach the culvert depends upon whether there is a well-defined and unobstructed channel, or whether the water finds its way in a broad thin sheet. If the watercourse is unobstructed and has a considerable inclination, the water may arrive at the culvert nearly as rapidly as it falls; but if the channel is obstructed, the water may be much longer in passing the culvert than in falling.

(5) The area of the waterway depends upon the amount of the area to be drained; but in many cases the shape of this area and the position of the branches of the stream are of more importance than the amount of the territory. For example, if the area is long and narrow, the water from the lower portion may pass through the culvert before that from the upper end arrives; or, on the other hand, if the upper end of the area is steeper than the lower, the water from the former may arrive simultaneously with that from the latter. Again, if the lower part of the area is better supplied with branches than the upper portion, the water from the former will be carried past the culvert before the arrival of that from the latter; or, on the other hand, if the upper portion is better supplied with branch watercourses than the lower, the water from the whole area may arrive at the culvert at nearly the same time. In large areas the shape of the area and the position of the watercourses are very important considerations.

(6) The efficiency of a culvert may be materially increased by so arranging the upper end that the water may enter it without being retarded. The discharging capacity of a culvert can also be increased by increasing the inclination of its bed, provided the channel below will allow the water to flow away freely after having passed the culvert.

(7) The discharging capacity of a culvert can be greatly increased by allowing the water to dam up above it. A culvert will

discharge twice as much under a head of four feet as under a head of one foot. This can be done safely only with a well-constructed culvert.

The determination of the values of the different factors entering into the problem is almost wholly a matter of judgment. An estimate for any one of the above factors is liable to be in error from 100 to 200 per cent, or even more, and of course any result deduced from such data must be very uncertain. Fortunately, mathematical exactness is not required by the problem nor warranted by the data. The question is not one of 10 or 20 per cent of increase; for if a 2-foot pipe is insufficient, a 3-foot pipe will probably be the next size, an increase of 225 per cent; and if a 6-foot arch culvert is too small, an 8-foot will be used, an increase of 180 per cent. The real question is whether a 2-foot pipe or an 8-foot arch culvert is needed.

Calculating Area of Waterway. Numerous empirical formulas have been proposed for this and similar problems; but at best they are all only approximate, since no formula can give accurate results with inaccurate data.

The size of waterway may be determined approximately by the following formula:

$$Q = Cr\sqrt[4]{\frac{S}{A}},$$

in which

Q = the number of cubic feet per acre per second reaching the mouth of the culvert or drain.

C = a coefficient ranging from .31 to .75, depending upon the nature of the surface; .62 is recommended for general use.

r = average intensity of rainfall in cubic feet per acre per second.

S = the general grade of the area per thousand feet.

A = the area drained, in acres.

CONCRETE STEEL MASONRY.

Concrete in the form of blocks made at a factory, and concrete formed in place and reinforced by steel rods and bars of differing shapes is being substituted in many situations for stone and brick masonry. For the construction of bridges and floors it is extensively employed. Several systems are in use, each known by the name of the inventor. Fig. 44 shows the different types which are more or less popular.

The Monier type consists of a mesh work of longitudinal and transverse rods of steel, usually placed near the center line of the arch rib. This type rests on the theory that the steel rods will resist the compressive stresses of the rib, while the concrete acts merely as a stiffener to prevent the steel from buckling,

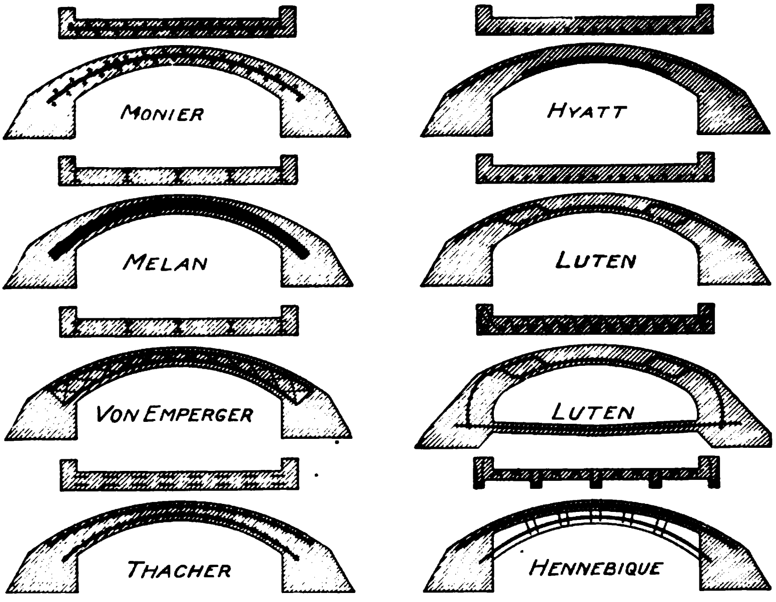
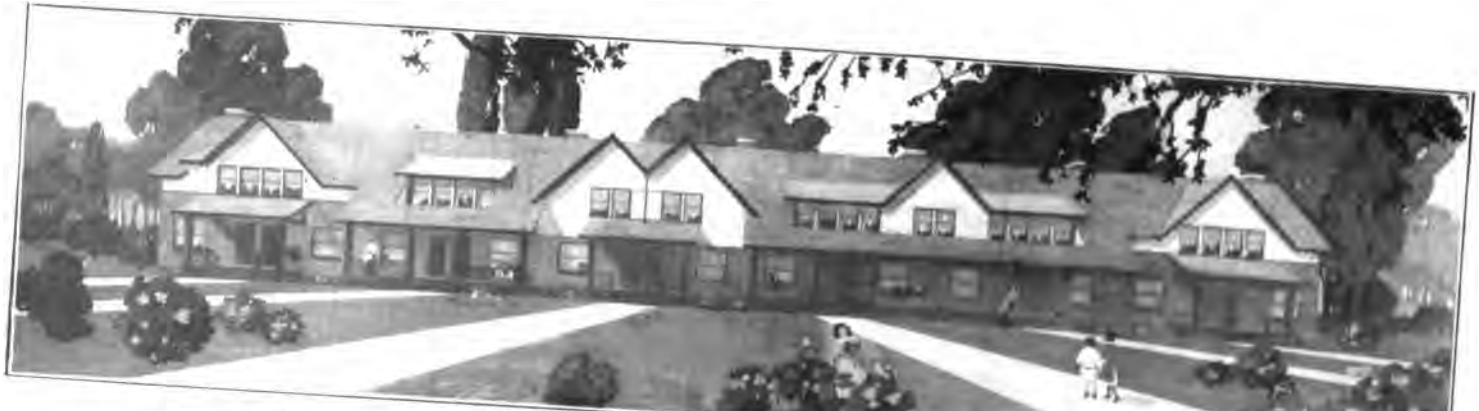
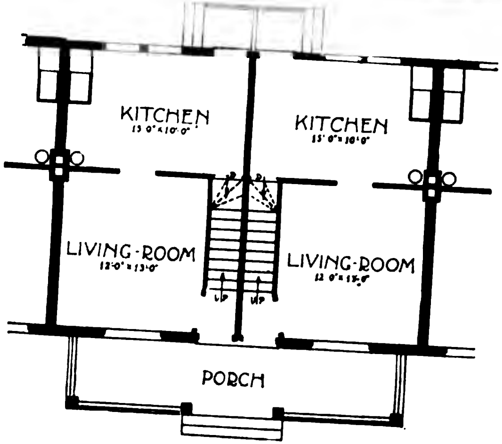


Fig. 44. Types of Concrete Steel Arches.

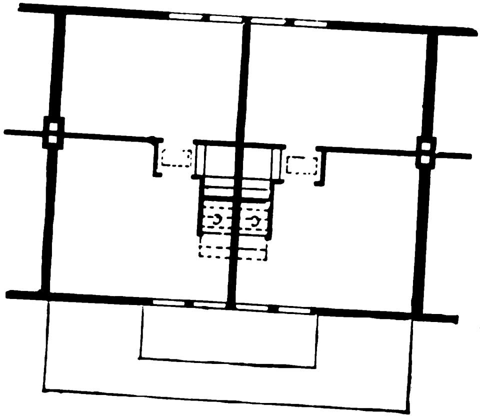
The Melan type consists of steel ribs embedded in the concrete and extending from abutment to abutment. The ribs are in the form of steel I-beams curved to follow the center line of the arch rib. The steel is assumed to be sufficient to resist the bending moments of the arch, while the concrete is relied upon to resist the thrust and to act as a preservative coating for the steel.

The Von Emperger arch is a modification of the Melan arch, the ribs are built up with angles for the flanges and diagonal lacing replaces the web, on the theory that the metal should be concentrated near the extrados of the arch to more effectually resist the bending moments.

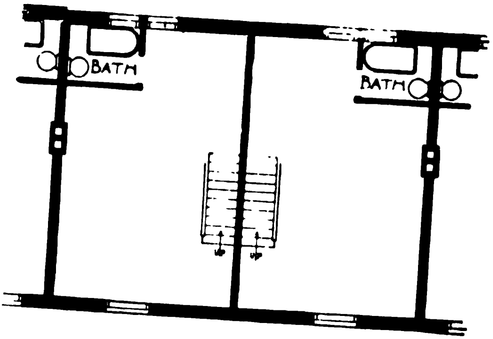




First-Story Plan.



Second-Story Plan.



Attic Plan.

BLOCK OF TWELVE WORKINGMEN'S HOUSES FOR INDIANA STEEL CO., AT GARY, IND.

Dean & Dean, Architects, Chicago, Ill.
Paving Brick Veneer and Plaster; Tile Roof.

The Thacher type is formed by omitting the web and reinforcing the concrete by steel bars in pairs one above the other, one near the extrados and one near the intrados, the steel being relied upon to resist the bending moments while the concrete is expected to resist the thrust of the arch.

In the Hyatt arch that portion of the steel bars or rods which in the Thacher arch is subjected to the greatest compression is omitted.

In the Luter arch the concrete rib is reinforced by tension members passing from one side of the arch rib to the other.

In the Hennebique system an arch barrel or drum, four to six inches in diameter, is supported by ribs of concrete below, the concrete of the drum being reinforced with steel rods placed near the extrados, and that of the ribs by steel rods near the intrados.

Numerous forms of steel shapes are advocated for the reinforcement of concrete when employed for arches, retaining walls, etc.; twisted bars, corrugated bars, expanded metal and lock woven steel are some of the names applied to the different shapes.

The method employed for constructing concrete walls is in brief as follows: A wooden form is erected, consisting of slotted standards made of 6-inch boards nailed together with spacing blocks between them at their ends, ⅝-inch bolts are used to join the standards on opposite sides of the wall. The standards are for the purpose of holding molding boards in position while the concrete is being deposited between them. These boards are of dressed pine 1½ inches thick. After the lower portions of the concrete has set the boards are removed and used above. Vertical rods of twisted or corrugated steel are built in the wall spaced about 12 inches apart. In some cases level horizontal bars of steel are also embedded in the walls.

REVIEW QUESTIONS.

PRACTICAL TEST QUESTIONS.

In the foregoing sections of this Cyclopedia numerous illustrative examples are worked out in detail in order to show the application of the various methods and principles. Accompanying these are examples for practice which will aid the reader in fixing the principles in mind.

In the following pages are given a large number of test questions and problems which afford a valuable means of testing the reader's knowledge of the subjects treated. They will be found excellent practice for those preparing for College, Civil Service, or Engineer's License. In some cases numerical answers are given as a further aid in this work.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRENGTH OF MATERIALS.

PART I.

---

1. When a ¾-inch round rod sustains a pull of 10,000 pounds, what is the value of the unit-tensile stress in the rod?

2. What do you understand by Hooke's Law?

3. What are the dimensions of a square white pine post, needed to support a steady load of 6,500 pounds with a factor of safety of 8?

4. How large a force is required to punch a 1-inch hole through a ¾-inch plate of wrought iron, if the ultimate shearing strength of the material is 40,000 pounds per square inch?

5. Compare the ultimate strengths of wood along and across the grain; also the ultimate tensile and compressive strengths of cast iron.

6. Make a sketch of a beam 20 feet long resting on end supports, and represent loads of 6,000, 3,000, 1,000, and 4,000 pounds at points 2, 5, 11, and 16 feet from the left end, respectively. What is the value and sign of the amount of each of these loads about the middle of the beam? Also about the left end?

7. A beam 15 feet long is supported at two points, 2 feet from the right end, and 3 feet from the left end. If the beam sustains a uniform load of 400 pounds per foot, what are the values of the reactions?

8. Compute the values of the external shear and bending moment for the loaded beam described in question 6, at sections 1 4, 10, and 15 feet from the left end.

9. Draw shear and moment diagrams to scale for the beam described in question 7.

10. Suppose a T-bar 2 inches deep, has a flange 3 inches wide, and is ¼ inch thick throughout. Locate the center of gravity by computation.

11. Cut a piece of stiff paper to the dimensions given in the preceding question, locate its center of gravity by balancing, and state its distance from the base as you find it by this method.

12. Draw a 1-inch, a 2-inch, and a 3-inch square touching each other, so that the 2-inch rests on the 3-inch square, and the 1-inch on the 2-inch square, three of the sides being in the same line. How far from the bottom and the side is the center of gravity of the three squares?

13. Compute the moment of inertia of a rectangle 2 × 16 inches with respect to its long side.

14. A square stick of red oak timber is to carry a compressive load of 15,000 pounds. What should be its size in order that the unit-stress may be one-half the ultimate strength along the grain?

15. A pressed brick $2 \times 4 \times 8\frac{1}{2}$ inches weighs about $5\frac{1}{2}$ pounds. What will be the height of a pile of brick, so that the unit-stress on the lower brick shall be one-half its ultimate strength? Use 8,000 pounds as the ultimate strength of the pressed brick.

16. From the diagram shown on page 11, determine the elastic limit of wrought iron, in tension.

17. A wrought-iron rod 2 inches in diameter sustains a load of 50,000 pounds. What is its working stress? If its ultimate strength is 60,000 pounds per square inch, find its factor of safety.

18. A wrought-iron bar 3 inches in diameter ruptures under a tension of 200,000 pounds. What is its ultimate strength?

19. Find the diameter of a cast-iron bar designed to carry a tension of 250,000 pounds with a factor of safety of 6. If the bar were of wrought iron, what would be its diameter?

20. A wrought-iron bar is to be under a stress of 50,000 pounds. Find its diameter when it is to be used in a building; also when it is to be used in a bridge.

21. Find the greatest steady load a short timber post can sustain with safety, when it is 8 × 8 inches in cross-section and its ultimate compressive strength is 10,000 pounds per square inch.

22. A wrought-iron bolt $1\frac{1}{2}$ inches in diameter has a head

$1\frac{1}{4}$ inches long. If a tension of 15,000 pounds is applied to the bolt, find the tensile unit-stress and the factor of safety for tension. Also find the unit-stress tending to shear off the head of the bolt, and the factor of safety against shear.

23. Find the reactions due to the loads in Fig. 9, when the beam is supported at its ends, and the loads are 2,000, 5,000, and 3,000 pounds respectively.

24. Compute the shear for sections one foot apart in the beam represented in Fig. 9, taking into consideration the weight of the beam, 500 pounds, and a distributed load of 400 pounds per foot, in addition to the loads as shown in Fig. 9.

25. Construct the shear diagram for the cantilever beam loaded as shown in Fig. 15, when the weight of the beam is 600 pounds, and the beam sustains a uniform load of 300 pounds per foot.

26. A cantilever beam has a load of 800 pounds at its end and is also uniformly loaded with 125 pounds per linear foot; its length is 5 feet. Compute the bending moments for five sections one foot apart, and construct the diagram of bending moments.

27. A deck beam used in building has a rectangular flange 3 inches × $\frac{1}{2}$ inch; a rectangular web 4 inches × $\frac{1}{2}$ inch; and an elliptical head which is $1\frac{1}{2}$ inches in depth, and whose area is 2.6 square inches. Find the distance of the center of gravity from the top of the head.

28. Compute the moment of inertia of a steel I-beam weighing 60 pounds per linear foot, it being 20 inches deep, with flange 5 inches wide, and mean thickness $\frac{3}{8}$ inch. The web is $\frac{3}{4}$ inch thick and has a moment of inertia of 446 inches.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRENGTH OF MATERIALS.

PART II.

---

1. A cantilever beam 6 feet in length projects from a wall and sustains an end load of 300 pounds. The cross-section being as in Fig. 38, find the greatest tensile and compressive unit-stresses, and state where they occur.

2. An I-beam weighing 30 pounds per foot, rests on end supports 25 feet apart. Its section modulus is 20.4 inches^3, and its working strength 16,000 pounds per square inch. Calculate weight of the beam.

3. A wooden beam 15 feet long, 4 × 14 inches in cross-section sustains a load of 4,000 pounds 5 feet from one end, and 2,000 pounds at the middle. Compute the greatest unit shearing stress.

4. What do you know about radius of gyration? Give an example.

5. Find the factor of safety of a 24-inch 80-pound steel I-beam 15 feet long, used as a flat-ended column to sustain a load of 150,000 pounds. Note.—Use "Rankine's Formula."

6. A steel Z-bar is 20 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches, and its area of cross-section is 24.5 square inches. Calculate the safe load with a factor of safety of 6. Note. Use "Rankine's Formula."

7. Make sketches of the following:

Lap joint single-riveted;
" " double-riveted;
Butt " single-riveted;
" " double-riveted.

8. The ends of a cast-iron rod 2 inches in diameter are secured to two heavy bodies which are to be drawn together, the

temperature of the rod being 300 degrees when fastened to the objects. A fall of 125 degrees has no effect on the objects. Compute the temperature stress, and state the pull exerted by the rod on each object.

9. Two half-inch plates 6 inches wide are connected by a lap joint with three $\frac{3}{4}$-inch rivets in a row. What is the safe strength of the joint?

10. A timber beam 6 × 14 inches and 20 feet long rests on end supports and sustains two loads of 3,000 pounds each five feet from one end. Compute the values of the greatest unit-tension, compression, and shear in the beam.

11. A 20-pound 7-inch I-beam 12 feet long is used as a cantilever beam supported in the middle. If its working strength is 16,000 pounds per square inch, find the greatest safe load that can be hung at each end considering the weight of the beam.

12. Compute the safe middle load for a 25-pound 10-inch I-beam 20 feet long, resting on end supports, if its working strength is 16,000 pounds per square inch. Consider the weight of the beam.

13. The width of the flanges of an 18-pound 8-inch I-beam is 4 inches. Compare the strengths of such a beam when used (1) with its web vertical (Fig. 24), and (2) when its web is horizontal.

14. A bar of steel 1 × 6 inches in section and 24 feet long is used as a beam on end supports, its load being 2,000 pounds uniformly distributed, and it also sustains end pulls of 20,000 pounds. Compute the greatest unit-tensile and compressive stresses in the bar by approximate methods. (See Art. 74.)

15. A timber beam 6 × 12 inches and 20 feet long on end supports bears a uniform load of 3,000 pounds, and end pushes of 15,000 pounds. Compute the greatest unit-tensile and compressive stresses in the timber by approximate methods. (See Art. 75.)

16. Answer the two preceding questions by the exact methods of Art. 76.

17. Two I-beams (instead of channels) are fastened together as represented in Fig. 46, *b*, to make a column. They are 40-pound 12-inch beams, the plates are $\frac{5}{8}$ inch thick and 16 inches wide; and from center to center of the webs of the beams is 10 inches. Compute the radii of gyration of the column section with

respect to two axes through the center of gravity parallel and perpendicular to the webs.

18. Compute the safe loads for two white pine columns 16×16 and 8 × 8 inches, each 14 feet long, using a factor of safety of 6.

19. What size of white oak column 16 feet long is needed to carry a load of 10,000 pounds with a factor of safety of 5? What size of steel I-beam will carry the same load if the ends of the column are flat? Note.—Solve for r, and use table C.

20. Compute the size of a circular, hollow, cast-iron column to carry a load of 150,000 pounds with a factor of safety of 10, its length being 18 feet and its ends flat. Note. Use "Rankine's Formula," and try an outside diameter of 10 inches.

21. If a solid shaft 4 inches in diameter is subjected to a twisting moment of 1,000 foot-pounds, compute the greatest unit-shearing stress in the shaft.

22. Compute the number of horse-power which a steel shaft 6 inches in diameter can safely transmit at 200 revolutions per minute, if its working strength in shear is 10,000 pounds per square inch.

23. What size of circular shaft is needed to transmit 1,600 horse-power at 100 revolutions per minute, if the working strength of the material in shear is 12,000 pounds per square inch?

24. How much will a round steel rod, 1 inch in diameter and 20 feet long, elongate under a pull of 10,000 pounds?

25. Compute the deflection of a 15-pound 7-inch I-beam 10 feet long, when resting on end supports and sustaining a uniform load of 5,000 pounds.

26. Two bars of the same size, one of wrought iron and one of cast iron, rest on end supports and sustain equal central loads which are "safe" in each case. Which beam deflects most, and what is the ratio of the deflection?

27. A bar of wrought iron 10 feet long will shorten how many inches during a drop of 40 degrees Fahrenheit in its temperature?

28. If the bar of the preceding question is restrained and prevented from shortening, what unit-stress is produced in it by the drop in temperature?

REVIEW QUESTIONS

ON THE SUBJECT OF

GRAPHICAL STATICS.

1. Define concurrent and non-concurrent forces, equilibrant, and resultant.

2. What do you understand by the "Triangle law?"

3. Determine the magnitude and direction of the resultant of the 400- and 800-pound forces of Fig. 47.

4. Compute the magnitude and direction of the resultant of the 600- and 700-pound forces of Fig. 47.

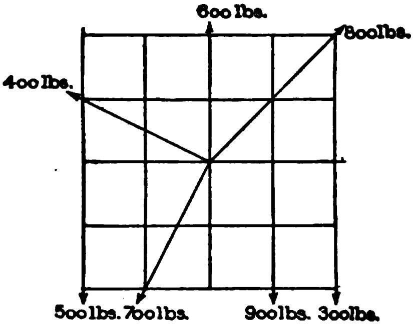


Fig. 47.

5. Determine the values of the horizontal and vertical components of the 700-pound force of Fig. 47.

6. Determine the magnitude and direction of the resultant of the four forces acting through the center of Fig. 47; also of their equilibrant.

7. Compute the resultant of the four parallel forces represented in Fig. 47.

8. Find the resultant of the 300-, 400-, 500-, and 800-pound forces of Fig. 47.

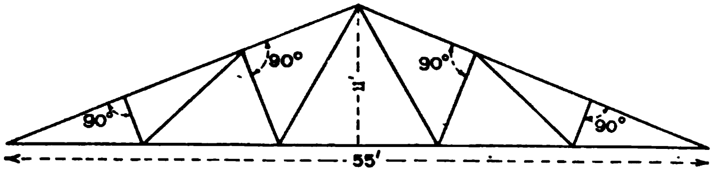
9. Suppose that the truss of Fig. 48 is one of several used to support a roof, the trusses being 16 feet apart. What is the

probable weight of each? What is the total roof load borne by each truss if the roofing weighs 18 pounds per sq. foot?

10. What is the total snow load for the truss if the snow weighs 20 pounds per square foot (horizontal)?

11. Compute the total wind load for the truss of Question 9, when the wind blows 75 miles per hour.

12. Supposing that the right end of the truss of Fig. 48 rests on rollers, and that the left end is fastened to its support, compute the values of the reactions (*a*) when the wind blows on the left; (*b*) when it blows on the right, 90 miles per hour.



Upper joints equally distant apart.

Fig. 48.

13. Construct stress diagram for the truss of Fig. 48 for the following cases:

(*a*) When sustaining dead load only as computed in answer to Question 9.

(*b*) When sustaining wind pressure on the left, the truss being supported as described in Question 12.

(*c*) When sustaining wind pressure on the right, the truss being supported as described in Question 12.

14. Make a complete record of the stresses as determined in answer to the preceding question for cases *a*, *b* and *c*, and for snow load as computed in answer to Question 10. Compute from the record the value of the greatest stress which can come upon each member due to combinations of loads, assuming that the wind and snow loads will not act at the same time.

15. Suppose that the truss of Fig. 49 is one of several used to support a roof, the trusses being 12 feet apart. What is the probable weight of a truss and that of the roofing supported by one truss, if the roofing weighs 15 pounds per square foot?

16. Compute the apex loads for the truss of Fig. 49 for snow if it weighs 20 pounds per square foot (horizontal).

REVIEW QUESTIONS

ON THE SUBJECT OF

MASONRY CONSTRUCTION.

PART I.

1. How may the effect of freezing of mortar be counteracted ?
2. What precautions are required in the use of asphaltic concrete ?
3. What precautions should be observed in forming a foundation on clay ?
4. In what situation do timber piles rapidly decay ?
5. Describe the requisites of a good building stone.
6. What effect has wind and water on stone ?
7. Describe the requisites of a good brick.
8. For what properties are cements tested ?
9. What is the essential requisite of a good sand for mortar ?
10. What is concrete ?
11. What qualities should clay for puddle possess ?
12. How is the bearing power of a soil increased ?
13. Describe the apparatus required and the manner of using a water-jet for sinking piles.
14. How are cribs put in place ?
15. Describe the structure of the two principal classes of stone and state in what way it affects the durability of the stone.
16. How is the approximate durability of a stone ascertained ?
17. How is lime distinguished from cement ?
18. Upon what does the quality of mortar depend ?
19. What is the effect of freezing on mortar ?
20. What precautions should be observed in depositing concrete ?
21. What is the effect of defective foundations ?

22. How are foundations formed in water?

23. What difficulties are encountered in constructing cofferdams?

24. Can stone be permanently preserved from decay?

25. State what ingredients are injurious to brick.

26. Describe the difference between "Natural" and "Portland" cements.

27. What is mortar used for?

28. How is mortar made?

29. How should the materials for concrete be prepared?

30. What is the essential requisite of a foundation?

31. What is the safe load, by the Engineering News formula, for a pile which sinks 1 inch when driven with a hammer weighing 3,000 lb. and falling 10 feet?

32. A wall having a weight of 12,000 lb. per running foot is to be built on two lines of piles spaced three feet apart transversely; how far apart must the piles be spaced longitudinally?

33. What is the use of a "footing"?

34. What is the proper offset for a hard brick footing under a pressure of two tons per square foot, using a factor of safety of 10?

35. What breadth must a Georgia pine beam have for a bearing power of 5000 lb. per sq. ft., a projection of 2 ft., a distance between centers of 1 ft., and an assumed depth of 12 inches?

REVIEW QUESTIONS

ON THE SUBJECT OF

MASONRY CONSTRUCTION.

PART II.

1. How is masonry classified?
2. How is brick masonry bonded?
3. For what purpose is a footing used? What precaution should be observed in its construction?
4. What precautions should be observed in the use of headers?
5. For what purpose is pointing used? How is the joint prepared for pointing?
6. How should sills be set?
7. For what purpose is a pean hammer used?
8. What tools are used for splitting stone?
9. What tool is used before the bush hammer?
10. What is the difference between a pitch faced stone and a drafted stone?
11. How is the bond of masonry formed; what purpose does it serve?
12. For what purpose is a coping used; what shape should it have; how should it be secured?
13. For what purpose should grout be used?
14. What is a bed joint? How should a joint be formed?
15. For what purpose is a rip-rap used?
16. How should the beds of a stone be dressed and what should be their relation to each other?
17. What is the point used for?
18. In dressing granite what is the first operation?
19. How are stones classified?